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USE OF THE DIFFERENTIAL QUADRATURE METHOD WHEN DEALING WITH TRANSVERSE VIBRATIONS OF A RECTANGULAR PLATE SUBJECTED TO A NON-UNIFORM STRESS DISTRIBUTION FIELD

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1. INTRODUCTION

Due to the efforts of Bert and associates, the method of differential quadrature is already well established in the technical and scientific literature [1–3]. Recently [4], the DQ technique was employed in the analysis of transverse vibrations of a thin rectangular plate subjected to a non-uniform stress field due to the presence of distributed loading of the type

$$f(y) = S\left(1 - \frac{y^2}{b^2}\right) \tag{1}$$

applied to plate edges parallel to the y-axis. The components of the plane stress tensor are approximately known [5], and the governing vibrations partial differential equation was solved by means of the DQ method for several combinations of plate boundary conditions.

The present study deals with a numerical investigation of the relative accuracy of the DQ method by comparing the frequency coefficients obtained in the case of the structural system shown in Figure 1 with extensive numerical results obtained by Carnicer *et al*. [6]. This investigation used the Galerkin method with two different sets of co-ordinate functions (sinusoids and polynomials), and the finite elements technique to obtain the fundamental frequency coefficients of the structural element shown in Figure 1, in the case of simply supported edges.

2. APPROXIMATE SOLUTION OF THE PROBLEM

The general governing partial differential equation is

$$D\left(\frac{\partial^4 W}{\partial \bar{x}^4} + 2\frac{\partial^4 W}{\partial \bar{x}^2\,\partial \bar{y}^2} + \frac{\partial^4 W}{\partial \bar{y}^4}\right) - \left(N_x\frac{\partial^2 W}{\partial \bar{x}^2} + 2N_{xy}\frac{\partial^2 W}{\partial \bar{x}\,\partial \bar{y}} + N_y\frac{\partial^2 W}{\partial \bar{y}^2}\right) - \rho h\omega^2 W = 0. \tag{2}$$

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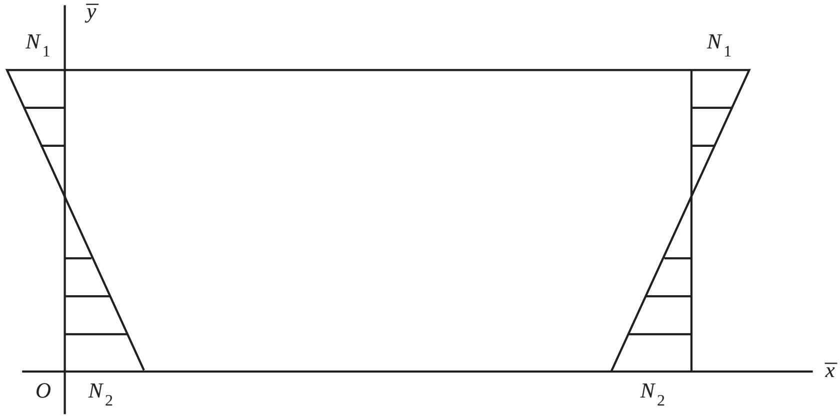

Figure 1. Rectangular plate subjected to a non-uniform stress field and executing transverse vibrations.

In the case of the system shown in Figure 1 one has

$$N_x = N_2 + (N_1 - N_2)\left(\frac{\bar{y}}{b}\right), \qquad N_{xy} = 0, \qquad N_y = 0. \tag{3}$$

Substituting equation (3) in equation (2) yields

$$D\left(\frac{\partial^4 W}{\partial \bar{x}^4} + 2\,\frac{\partial^4 W}{\partial \bar{x}^2\,\partial \bar{y}^2} + \frac{\partial^4 W}{\partial \bar{y}^4}\right) - \left(N_2 + (N_1 - N_2)\,\frac{\bar{y}}{b}\right)\frac{\partial^2 W}{\partial \bar{x}^2} - \rho h \omega^2 W = 0. \tag{4}$$

Introducing the dimensionless variables $\bar{x} = ax$, $\bar{y} = by$, and defining

$$\lambda = \frac{a}{b}\,, \qquad S_1 = \frac{N_1 a^2}{D}\,, \qquad S_2 = \frac{N_2 a^2}{D}\,,$$

$$g(y) = S_2 + (S_1 - S_2)\,\frac{y}{b}\,, \qquad \Omega^2 = \frac{\rho h a^4}{D}\,\omega^2,$$

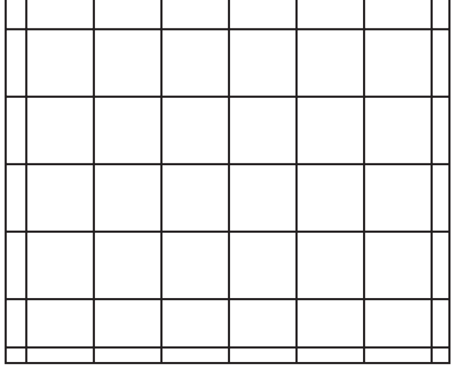

Figure 2. Partition of the domain.

TABLE 1

Comparison of fundamental frequency coefficients in the case of a simply supported rectangular plate (Figure 1)

λ	S_1	S_2	*DQ*	Reference [6] Polynomials	Reference [6] Fourier series	Reference [6] Finite elements
2/3	25·0	25·0	21·213	21·218	21·213	21·350
	25·0	12·5	19·684	19·711	19·682	19·823
	25·0	0	17·976	18·078	17·973	18·130
	25·0	−25·0	13·765	14·263	13·751	14·002
	0	−25·0	8·741	8·947	8·734	8·939
	−12·5	−25·0	4·163	4·284	4·157	4·447
	−25·0	−25·0	–	–	–	–
1	25·0	25·0	25·227	25·234	25·227	25·431
	25·0	12·5	23·969	23·980	23·968	24·175
	25·0	0	22·630	22·658	22·628	22·844
	25·0	−25·0	19·647	19·748	19·641	19·895
	0	−25·0	16·291	16·328	16·288	16·525
	−12·5	−25·0	14·296	14·314	14·295	14·536
	−25·0	−25·0	11·955	11·967	11·954	12·214
2	25·0	25·0	51·751	51·814	51·788	53·126
	25·0	12·5	51·192	51·215	51·189	52·537
	25·0	0	50·585	50·609	50·582	51·942
	25·0	−25·0	49·348	49·357	49·345	50·729
	0	−25·0	48·084	48·100	48·081	49·489
	−12·5	−25·0	47·439	47·464	47·436	48·857
	−25·0	−25·0	46·785	46·809	46·781	48·216

equation (4) becomes

$$\frac{\partial^4 W}{\partial x^4} + 2\lambda^2 \frac{\partial^4 W}{\partial x^2\,\partial y^2} + \lambda^4 \frac{\partial^4 W}{\partial y^4} - g(y)\frac{\partial^2 W}{\partial y^2} - \Omega^2 W = 0. \tag{5}$$

Following references [1–3] the plate domain is partitioned, as shown in Figure 2. For all the situations considered, the number of nodal points in each direction was $N = 9$.

Using the notation introduced by Bert and co-workers [1–3] one obtains, in the case of a simply supported rectangular plate,

$$\sum_{k_1=2}^{N-1} D_{ik_1} W_{k_1 j} + 2\lambda^2 \sum_{k_1=2}^{N-1}\sum_{k_2=2}^{N-1} B_{ik_1} B_{jk_2} W_{k_1 k_2} + \lambda^4 \sum_{k_2=2}^{N-1} D_{jk_2} W_{ik_2}$$

$$- g_j \sum_{k_1=2}^{N-1} B_{ik_1} W_{k_1 j} - \Omega^2 W_{ij} = 0,$$

$$(i, j = 3, \ldots, N-2);$$

$$\sum_{k_1=2}^{N-1} B_{2k_1} W_{k_1 j} = 0, \qquad (j = 3, \ldots, N-1);$$

$$\sum_{k_2=2}^{N-1} B_{2k_2} W_{ik_2} = 0, \qquad (i = 2, \ldots, N-2);$$

$$\sum_{k_1=2}^{N-1} B_{(N-1)k_1} W_{k_1 j} = 0, \qquad (j = 2, \ldots, N-2),$$

$$\sum_{k_2=2}^{N-1} B_{(N-1)k_2} W_{ik_2} = 0, \qquad (i = 3, \ldots, N-1).$$

3. NUMERICAL RESULTS

Table 1 depicts values of the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho h/D}\,\omega_1 a^2$ obtained by means of the DQ method which can be compared with the results obtained by Carnicer *et al*. [6]. The comparison is performed for several combinations of values of λ, S_1 and S_2. One observes for all the situations an excellent agreement between the results of the DQ method and those obtained by means of the Galerkin approach coupled with a double Fourier series [6] which are upper bounds with respect to the exact solution. The differences are, in general, less than 0·1%.

Table 2 shows values of Ω_1 for different combinations of boundary conditions, and values of λ, S_1 and S_2. The plate edges are defined in the table, starting from

TABLE 2

Values of fundamental frequency coefficients of the structural system shown in Figure 1 for different combinations of boundary conditions

	λ	$S_1 = 25{\cdot}0$ $S_2 = 25{\cdot}0$	25·0 12·5	25·0 0	25·0 −25·0	0 −25·0	12·5 −25·0	−25·0 −25·0
SS–C–SS–SS	2/3	22·122	20·818	19·399	16·088	11·383	7·956	–
	1	28·388	27·407	26·386	24·197	21·201	19·521	17·647
	2	71·094	70·715	70·333	69·563	68·557	68·048	67·535
C–C–SS–SS	2/3	26·031	24·789	23·446	20·362	16·315	13·734	10·415
	1	31·755	31·779	29·761	27·579	24·625	22·794	21·167
	2	72·958	72·555	72·149	71·329	70·256	69·711	69·160
SS–C–SS–C	2/3	23·430	22·067	20·596	17·222	13·321	10·809	7·450
	1	32·949	31·998	31·015	28·941	26·743	25·568	24·384
	2	96·574	96·254	95·933	95·287	94·638	94·312	93·984
C–C–C–SS	2/3	31·156	30·039	28·847	26·491	22·063	21·089	18·967
	1	36·231	35·307	34·349	32·316	29·638	28·180	26·623
	2	75·424	75·002	74·576	73·716	72·592	72·021	71·444
C–C–C–C	2/3	32·123	30·929	29·664	26·876	23·982	22·342	20·530
	1	39·949	39·007	38·036	35·991	33·845	32·704	31·509
	2	99·863	99·499	99·133	98·395	97·651	97·275	96·898

$x = 0$ and following the plate contour in a counter-clockwise fashion [7]. For these combinations of boundary conditions no results are available in the open literature but judging from the excellent accuracy achieved in the case of simply supported edges, one hopefully expects at least good engineering accuracy in the case of the frequency coefficients contained in Table 2.

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