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DETERMINATION OF THE BLOCKAGE AREA FUNCTION OF A FINITE DUCT FROM A SINGLE PRESSURE RESPONSE MEASUREMENT

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1. INTRODUCTION

A technique is described for the reconstruction of the blockage area function of the duct from eigenfrequency and anti-resonance frequency shifts determined by using a single pressure response measurement in a finite length duct under one set of duct termination boundary conditions. It builds upon the earlier work of Wu and Fricke [1] which in its turn led on from initial research by Antonopoulos-Domis [2] into blockage detection in the subassembly wrappers of cooling systems for nuclear reactors. Wu and Fricke developed an inverse acoustic perturbation technique for reconstruction of the blockage area function within a finite length duct based upon measured blockage induced duct eigenfrequency shifts determined from pressure response measurements which required the use of two sets of termination boundary conditions.

While the blockage area function reconstruction results obtained by application of the Wu and Fricke method showed excellent accuracy the method could be deemed impractical, especially in the case of flow ducts, due to the requirement that acoustic pressure measurements be recorded under two separate sets of duct termination boundary conditions, each of which requires the closure of at least one end of the duct with a rigid termination.

The method described in this paper is made possible by the use of high noise immunity maximum length sequence techniques to reveal the locations of the anti-resonance residual pressures in the measured frequency response. The improved blockage reconstruction method is a considerable advance on the two boundary condition technique especially with a view to practical condition monitoring of flow ducts.

2. THE UNIQUE SOLUTION FOR THE BLOCKED DUCT

The one-dimensional acoustic equation governing the direct problem for the blockage perturbed duct is a formulation of the classic Webster horn equation for the lossless tract,

$$p''(x) + k^2 p(x) = -\frac{A'(x)}{A(x)} p'(x), \tag{1}$$

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where $A(x)$ is the variation of cross-sectional area function of the duct with longitudinal distance x.

The solution to equation (1) for the blockage perturbed finite length duct was formulated by Wu and Fricke [1] using perturbation theory [3]. By using their approach the new eigenfunction $p(x)$ and the new modal wavenumber k of the perturbed nth mode could be expressed in terms of the unperturbed modal solutions $\Phi_n(x)$ and k_n as

$$p(x) = \Phi_n(x) + \varepsilon \sum_{m \neq n} c_{mn}\Phi_m(x) + O(\varepsilon)^2 \quad (2)$$

and

$$k^2 = k_n^2 + \varepsilon\chi_n + O(\varepsilon)^2, \quad (3)$$

where $\Phi_n(x)$ and k_n^2 are respectively the eigenfunction and eigenvalue solutions for the unblocked duct, ε is the perturbation coefficient, χ_n is the blockage induced shift in eigenvalue for the nth mode, and $O(\varepsilon^2)$ is the higher order perturbation term. The terms in equation (2) represent a Fourier expansion of the other duct eigenfunctions which describes the perturbation in the mode shape.

The applied perturbation analysis, valid for obstacles of small cross section, assumes a first order approach and as such equations (2) and (3) can now be substituted into equation (1), less the higher order perturbation terms. Now, upon looking at the right side of equation (1), it may be expressed as a spatial Fourier series of all the unblocked eigenfunctions of the duct [including $\Phi_n(x)$], where each spatial Fourier component b_m is given by

$$b_m = \left(\frac{2}{L}\right)\int_0^L -\frac{A'(x)}{A(x)}\,\Phi_m'\Phi_m(x)\,\mathrm{d}x. \quad (4)$$

With the left side of equation (1) expanded in terms of equations (2) and (3), expressing the right side as a Fourier series of $\Phi_m(x)$ as in equation (4) and equating coefficients of ε yields an expression for the eigenvalue shift due to blockage perturbation χ_n namely

$$\chi_n = b_n, \quad (5)$$

where $n = 1, 2, 3, \ldots$ are the longitudinal mode orders of the duct.

Consider now a rigid walled duct length L with one closed rigid termination and one open termination. The duct is excited by a driver mounted at $x = 0$, which is the closed rigid end. The spatial eigenfunction $\Phi_n(x)$ and wavenumber k_n are given by

$$\Phi_n(x) = \cos\left[(2n-1)\pi x/(2L)\right], \qquad k_n = \left[(2n-1)\pi/(2L)\right]. \quad (6, 7)$$

Substituting equation (6) into equation (4) and using a simple trigonometric identity leads to collapse of the term $\Phi_n'(x)\Phi_n(x)$ into a single sine expression. The exact form of b_n in equation (4) depends on the form of the eigenfunction $\Phi_n(x)$

in equation (6) for the unblocked finite duct. For the closed/open termination conditions described for the duct of length L,

$$b_n = \left(\frac{(2n-1)\pi}{2L^2}\right)\int_0^L \frac{A'(x)}{A(x)} \sin\left(\frac{(2n-1)\pi x}{L}\right)\mathrm{d}x, \qquad n = 1, 2, 3, \ldots . \tag{8}$$

Substituting χ_n in equation (5) for b_n in equation (8), following some algebraic manipulation, yields

$$\chi_n = \frac{(2n-1)\pi}{4L} a_{2n-1}, \qquad n = 1, 2, 3, \ldots, \tag{9}$$

where a_{2n-1} is the $(2n-1)$th Fourier coefficient of the expansion [overlength L] of $A'(x)/A(x)$. The full analysis has been given by Wu and Fricke [1] using the unperturbed cosine eigenfunction $\Phi_n(x)$ and wavenumber k_n at resonance from equations (6) and (7).

The above procedure yields only the odd components of the Fourier series and it is necessary to devise a procedure for obtaining the even components, thus completing the expansion of $A'(x)/A(x)$. In the Wu and Fricke work this was achieved by applying the previously described perturbation analysis to the blockage perturbed duct eigenfrequency shifts obtained under a second set of boundary conditions: i.e., for the closed-closed duct. It is this requirement to modify boundary conditions which limits the practical application of their work.

However, it can be shown that the completion of the Fourier expansion is possible by using measurements obtained under a single set of boundary conditions. For the closed–open duct discussed above, as the driven frequency approaches the anti-resonance condition, the longitudinal pressure distribution will tend to a sine function (albeit one with a very small amplitude) with its origin at $x = 0$. At this limit, the theoretical distribution of pressure, $\Phi_{(a)n}(x)$, that exists in the duct, and the associated wavenumber $k_{(a)n}$ will be given by

$$\Phi_{(a)n}(x) = \sin[n\pi x/L], \qquad k_{(a)n}(x) = (n\pi/L), \tag{10, 11}$$

where $f_{(a)n} = ck_{(a)n}/(2\pi)$ describes the position of the residual or anti-resonance pressures in the duct pressure frequency response function. Upon using the previous argument, the relationship in equation (5) now becomes

$$\mu_{(a)n} = b_n, \tag{12}$$

where $\mu_{(a)n}$ is the shift in the value of $k^2_{(a)n}$ at anti-resonance in the pressure response function. The new formulation of the duct pressure function $\Phi_{(a)n}(x)$ from equation (10) can now be substituted into equation (4). Expanding equation (4) and using an argument similar to the one cited previously yields

$$\mu_{(a)n} = b_n = -(n\pi/2L)a_{2n}, \tag{13}$$

where a_{2n} is the $(2n)$th Fourier coefficient of the expansion [overlength L] of $A'(x)/A(x)$.

Thus an expression for the unique solution of $A'(x)/A(x)$ may be obtained in terms of χ_n and $\mu_{(a)n}$ by using equations (9) and (13). With further integration to obtain the form of $A(x)$, the blockage area function $A_b(x)/A_0(x) = (A_0(x) - A(x))/A_0(x)$, where $A_0(x)$ is the area function of the unblocked duct, is given by

$$A_b(x)/A_0(x) = \left\{1 - \exp\left[\sum_{n=1}\left[\frac{L_e}{n\pi}\right]^2 \mu_{(a)n}\cos\left(\frac{2n\pi x}{L_e}\right) - \sum_{n=1}\left[\frac{2L_e}{(2n-1)\pi}\right]^2 \chi_n \cos\left(\frac{(2n-1)\pi x}{L_e}\right) - a_0\right]\right\}, \qquad (14)$$

where a_0 is an added DC component equal to the ratio of blockage to duct volume [1], and L_e is the end-corrected length of the closed/open condition duct. The blockage area function in equation (14) is thus described by using the eigenvalue shifts χ_n and anti-resonance value shifts $\mu_{(a)n}$ obtained under a single set of boundary conditions.

3. EXPERIMENTAL ANALYSIS

The above analysis suggests that information similar to that obtained by Wu and Fricke with two sets of duct termination conditions can be achieved with one set of termination conditions from determination of the frequency shifts of pressure maxima and minima. However, determination of the precise frequencies corresponding to pressure minima by using a conventional swept sine technique would be affected by background noise. This would have the effect of making the minima very poorly defined and thus make it impossible to locate the frequency with any accuracy. In this work background noise problems were overcome by making measurements with a deterministic maximum length sequence used as the signal driving the loudspeaker.

The experimental set-up is shown in Figure 1. The duct was made of rigid walled polypropylene and was 2 m in length and 0·1 m in diameter. The induct blockage was made of hardwood and had the dimensions 45 mm square by 500 mm long. The test duct was excited by using a 16 384 point maximum length sequence of 2 kHz bandwidth. The sequence, generated in the acoustics analysis system MLSSA, was averaged over a period of 1 min. The pressure response within the

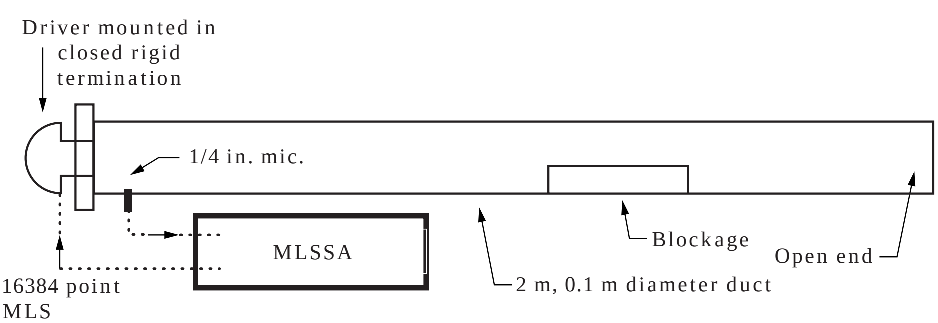

Figure 1. Experimental set-up showing blockage perturbed condition for a closed/open duct.

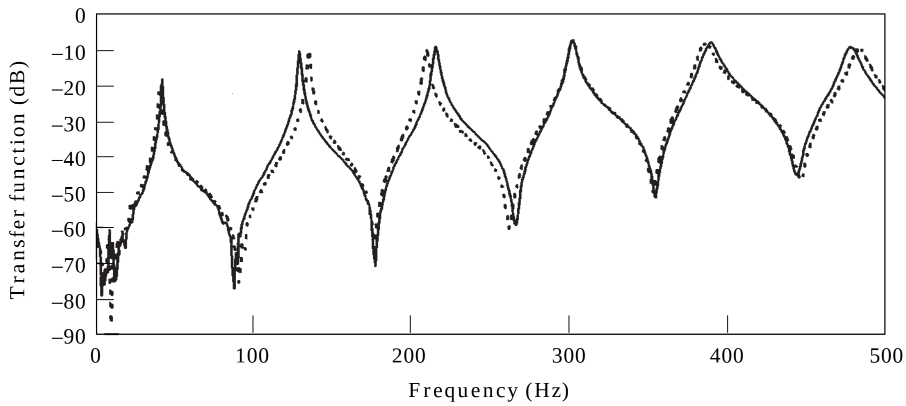

Figure 2. Unblocked (——) and blockage perturbed (· · · · ·) duct transfer functions.

duct was recorded via a 1/4 in. pressure response microphone wall mounted close to the driver, and the captured signal was Fast Hadamard Transformed to yield the impulse response of the duct system. Once the impulse response had been processed the first 8192 points in the time history were Fast Fourier Transformed to give the transfer function frequency response within the duct at 1 Hz resolution. The analysis of this initial part of the time domain captured the entire impulse response while rejecting much of the extraneous noise which was evenly spread over the time history. As a result an appreciable gain in signal to noise ratio was realized. The resulting measured transfer function revealed the residuals, or zeros, as sharp points in the frequency spectrum. This was in contrast to previous work [1], where the background noise distortion would have precluded location of the residual pressures. The transfer function frequency response between 1 and 500 Hz for the unblocked and partially blocked duct are shown in Figure 2. The first six

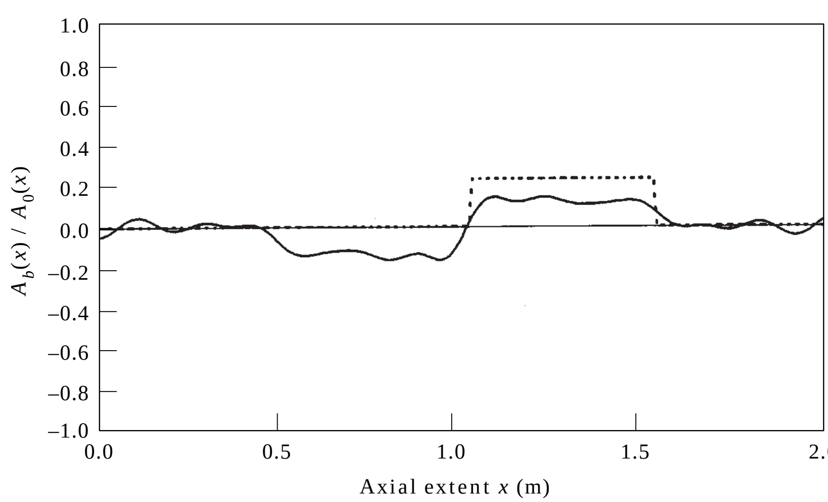

Figure 3. Blockage area reconstruction obtained by using closed/open duct eigenvalue shifts χ_n alone. ——, Reconstruction; · · · · · , actual.

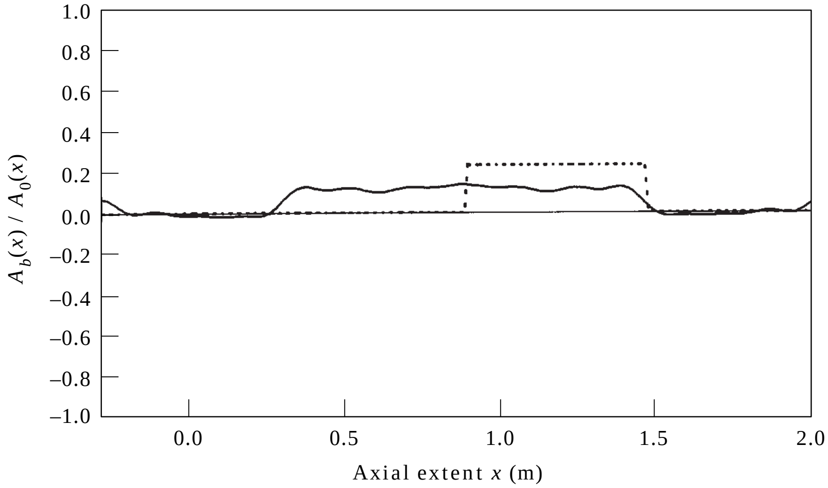

Figure 4. As Figure 3 but reconstruction obtained by using anti-resonance value shifts $\mu_{(a)n}$ alone.

eigenfrequency poles and the first five anti-resonance zeros of the unblocked and blockage perturbed duct are clearly discernible.

To obtain the blockage area function the first fifteen poles and first fifteen zeros of the transfer functions for the blocked and unblocked closed/open duct were utilized. An iterative Matlab routine was employed to select the poles and zeros and the resulting shifts χ_n and $\mu_{(a)n}$ were determined and subsequently processed by using equation (14).

4. RESULTS

The DC corrected reconstruction results for polar shift and zero shift results for a closed open duct similar to one of the configurations used by Wu and Fricke [1] are given in Figures 3 and 4, respectively. The true blockage function is shown

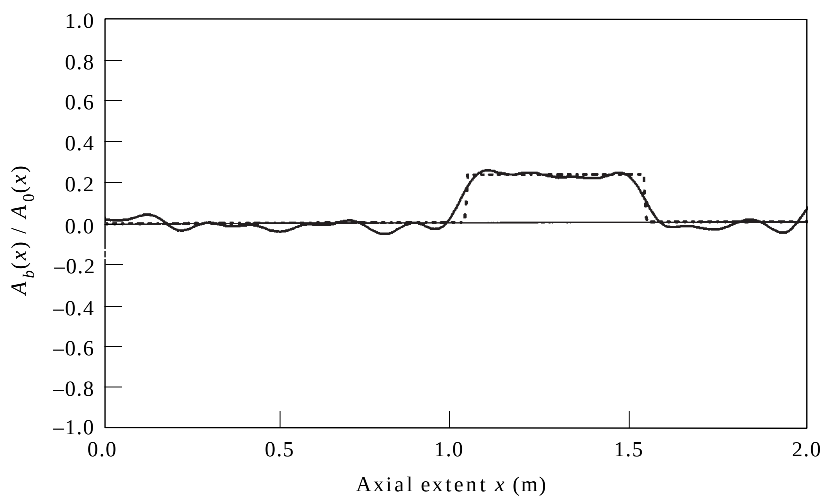

Figure 5. As Figures 3 and 4 but reconstruction obtained by using both χ_ns and $\mu_{(a)n}$s.

by the dotted line and it can be seen that neither single set of shifts can describe the blockage area function within the duct as each yields two apparent blockages. However, applying both sets of shifts in equation (14) yields the single blockage within the duct as shown in Figure 5.

This approach has also been successfully applied to ducts incorporating multiple obstacles, blockages added to ducts that have non-uniform area function $A(x)$ and ducts of increased length and cross-sectional area ratio. Further reconstructions have been realized for a finite element model of an open/open end duct excited at the walls close to one of the duct terminations. This particular result paves the way towards the development of an unobtrusive condition monitoring system for flow ducts.

5. CONCLUSIONS

A technique for determining the blockage area function of a duct by using a single set of duct termination conditions has been proposed. This technique has been tested by means of measurements made by using a finite duct of length 2 m and diameter 0·1 m with closed/open end conditions and excited via a maximum length sequence amplified through a driver mounted at the closed end. The pressure response for the unblocked duct and that of the duct incorporating a small blockage was measured via a wall mounted microphone close to the driver end. Due to the high noise immunity of the maximum length sequence technique the locations of the pressure residuals in the frequency response of the duct could be determined along with the modal poles. It was found that the shifts in the poles and zeros of the measured transfer function for the single set of boundary conditions could be employed to obtain a spatial Fourier series describing the area function of the blockage within the duct. The method is an advancement on previous techniques [1] developed from research into nuclear reactor cooling systems [2], where measurements under multiple sets of boundary conditions were required to determine the blockage area function within a duct.

REFERENCES

1. Q. Wu and F. Fricke 1990 *Journal of the Acoustical Society of America* **87**(1), 67–75. Determination of blocking locations and cross-sectional area in a duct by eigenfrequency shifts.
2. M. Antonopoulos-Domis 1980 *Journal of Sound and Vibration* **72**(4), 443–450. Frequency dependence of acoustic resonances on blockage position in a fast reactor subassembly wrapper.
3. R. Bellman 1968 *Perturbation Techniques in Mathematics, Physics and Engineering*. New York: Holt, Rinehart & Winston, 25–26.