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SIMPLIFIED METHOD FOR EVALUATING FUNDAMENTAL NATURAL FREQUENCY OF SHEAR WALL WITH MULTIPLE BANDS OF OPENINGS

Y. WANG, C. ARNAOUTI AND S. GUO
Department of Aerospace, Civil and Mechanical Engineering, University of Hertfordshire, Hatfield, Hertfordshire AL10 9AB, England

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1. INTRODUCTION

In modern tall buildings shear walls are commonly used as a structural element for resisting the lateral loads that arise from the effects of wind and earthquakes. The resisting walls are generally located at the sides of the buildings, or arranged in the form of a core that houses the staircase and lifts. Wall openings are inevitably required for windows or doors and the resulting structure may be modelled as a number of solid walls connected by a system of horizontal spandrel beams, as shown in Figure 1.

Recently, based upon Bert's formula [1] and coupled wall theory [2], a simplified formula has been developed by Wong and Wang [3] to predict the fundamental transverse natural frequency for different types of uniform symmetric multi-storey building structures. The formula is

$$f_{y1} = \frac{0{\cdot}5595 D_y}{H^2}\sqrt{\frac{EI}{m}}, \tag{1}$$

in which

$$D_y = \frac{1}{\sqrt{F_{yH}}} \tag{2}$$

and

$$F_{yH} = 1 - \frac{1}{k^2}\left[1 - \frac{4}{(k\alpha H)^2} + \frac{8}{(k\alpha H)^4 \cosh k\alpha H}(1 + k\alpha H \sinh k\alpha H - \cosh k\alpha H)\right], \tag{3}$$

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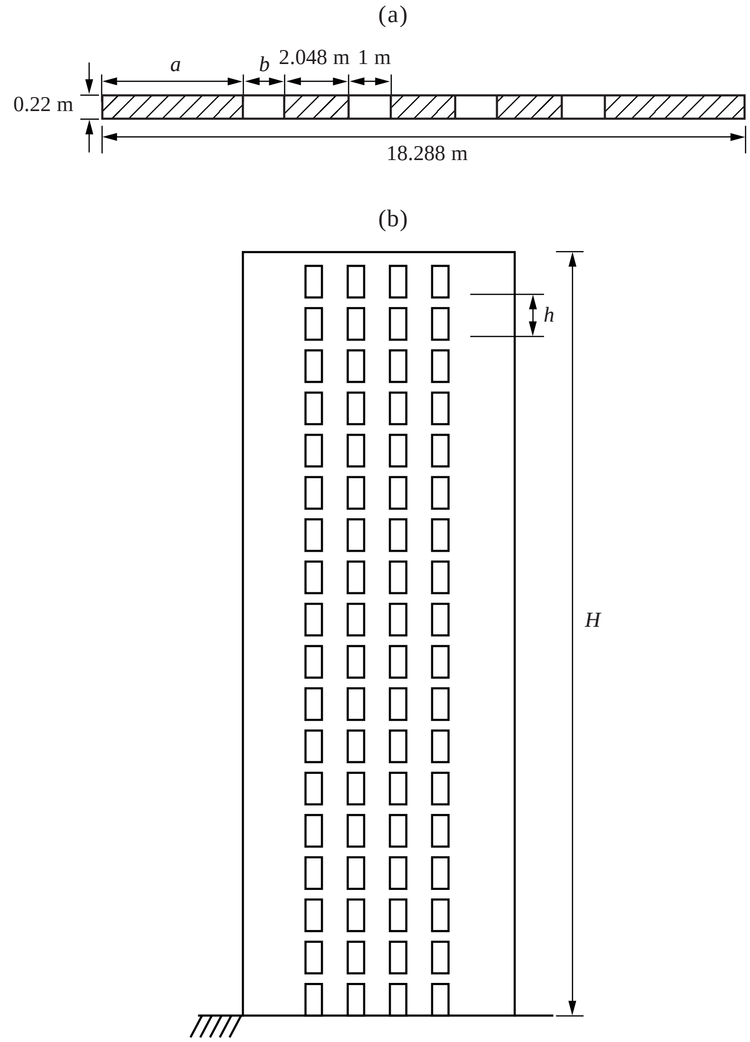

Figure 1. A shear wall with four bands of openings: (a) cross-section; (b) elevation.

where H is the building height, m is mass per unit length along the building height, EI is flexural rigidity contributed by different columns. The structural characteristic parameters α^2 and k^2 are as follows

$$\alpha^2 = \frac{(GA)}{EI}, \qquad k^2 = 1 + \frac{EI}{\sum (EAc^2)_j} = 1 + \frac{EI}{EAc^2}. \tag{4, 5}$$

The parameter k^2 accounts for the effect of axial deformations of the columns and walls on the overall flexure and α^2 for coupling effects of the connecting beams between the columns. $(EAc^2)_j$ for a bent is the flexural rigidity of the column and wall sectional areas acting about their common centroid. (GA) is the racking shear

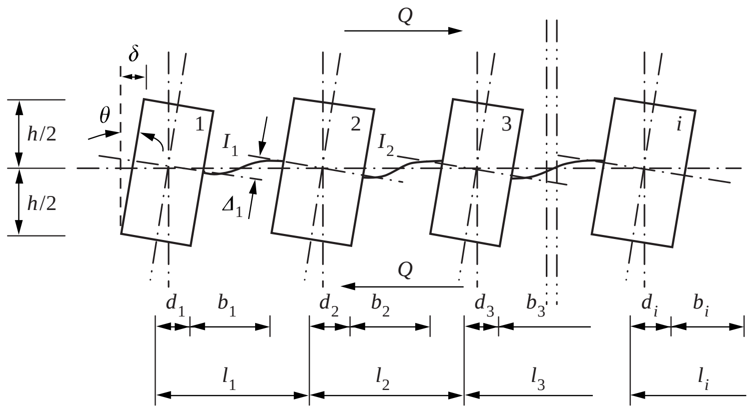

Figure 2. Storey height deflection of a shear wall with multiple bands of openings related to effective shear rigidity.

rigidity that depends on the structural forms. For coupled shear walls, the shear wall with one band of openings, the racking shear rigidity is given by

$$(GA) = \frac{12EI_b l^2}{b^3 h}. \tag{6}$$

The purpose of this letter is to extend the developed formula equation (1) from its application to coupled shear walls to a more complicated structural component, the shear wall with multiple bands of openings. To achieve this, the shear rigidity (GA) in terms of structural parameters valid for a shear wall with multiple bands of openings needs to be established.

2. ANALYSIS

For a shear wall with multiple bands of openings, it is assumed that the racking shear rigidity (GA) of the bent depends on the flexural rigidity of the connecting beams between the walls, and the width and spacing of the walls. It is also assumed that the points of contraflexure occur at the mid-span of the connecting beams.

Considering a storey-height segment of a shear wall and half-storey-height walls above and below each joint, as shown in Figure 2, the moment acting on wall 1 due to the contraflexure of the connecting beam I_{b1} is given by

$$M_1 = \frac{6EI_{b1}}{b_1^2}\Delta_1 + \frac{12EI_{b1}d_1}{b_1^3}\Delta_1,$$

in which $\Delta_1 = \theta l_1$. Thus, the moment M_1 can be rewritten as

$$M_1 = \left[\frac{6EI_{b1}\, l_1}{b_1^2}\left(1 + \frac{2d_1}{b_1}\right)\right]\cdot\theta, \tag{7}$$

where b_1 is a clear span of connecting beams between wall 1 and wall 2; l_1 is the distance between centrodial axes of the wall 1 and wall 2; d_1 is the half width of wall 1; and I_{b1} is the second moment of area of the connecting beam between wall 1 and wall 2.

The moment acting on wall 2 due to the contraflexures of the connecting beam I_{b1} and I_{b2} is given by

$$M_2 = \frac{6EI_{b1}}{b_1^2}\Delta_1 + \frac{6EI_{b2}}{b_2^2}\Delta_2 + \frac{12EI_{b1}d_2}{b_1^3}\Delta_1 + \frac{12EI_{b2}d_2}{b_2^3}\Delta_2$$

in which $\Delta_2 = \theta l_2$, and thus the moment M_2 can be rewritten as

$$M_2 = \left[\frac{6EI_{b1}l_1}{b_1^2}\left(1 + \frac{2d_2}{b_1}\right) + \frac{6EI_{b2}l_2}{b_2}\left(1 + \frac{2d_2}{b_2}\right)\right]\cdot\theta. \tag{8}$$

Similarly, for any internal wall i, the moments acting on it due to the contraflexure of the connecting beam $I_{b,i-1}$ and $I_{b,i}$ is

$$M_i = \left[\frac{6EI_{b,i-1}l_{i-1}}{b_{i-1}^2}\left(1 + \frac{2d_i}{b_{i-1}}\right) + \frac{6EI_{b,i}l_i}{b_i^2}\left(1 + \frac{2d_i}{b_i}\right)\right]\cdot\theta \tag{9}$$

and for the last external wall m, the moment is

$$M_m = \left[\frac{6EI_{b,m-1}l_{m-1}}{b_{m-1}^2}\left(1 + \frac{2d_m}{b_{m-1}}\right)\right]\cdot\theta. \tag{10}$$

When a segment of a coupled wall system is subjected to a shear force Q, the equilibrium of the internal resultant moment and external moment applied to the segment is

$$M_e = Qh = M_1 + M_2 + \cdots + M_i + \cdots + M_m \tag{11}$$

and its deflection is given by

$$\delta = \frac{Qh}{(GA)} = h\theta, \tag{12}$$

from which the shear rigidity of the shear wall with multiple bands of openings is obtained by

$$\begin{aligned}(GA) = \frac{Qh}{\delta} = \frac{12E}{h}\Bigg\{&\frac{I_{b1}l_1}{b_1^2}\left(1 + \frac{d_1 + d_2}{b_1}\right) + \frac{I_{b2}l_2}{b_2^2}\left(1 + \frac{d_2 + d_3}{b_2}\right)\\ &+ \cdots + \frac{I_{b,i}l_i}{b_i^2}\left(1 + \frac{d_i + d_{i+1}}{b_i}\right)\\ &+ \cdots + \frac{I_{b,m-1}l_{m-1}}{b_{m-1}^2}\left(1 + \frac{d_{m-1} + d_m}{b_{m-1}}\right)\Bigg\}.\end{aligned} \tag{13}$$

In the case where only two walls couple together, the formula (13) will become identical with equation (6) for the shear rigidity of coupled shear walls.

Since it has been assumed that the connecting beams are rigidly connected to the walls, the effects of any local elastic deformations at the beam-to-wall junctions on the flexibility of the lamina are ignored. However, such additional flexibility may be included by simply expanding the beam length. Michael's correction [4] in which the "equivalent beam length" should be equal to the beam clear span plus its depth when such effects are taken into account is adopted in this study.

3. NUMERICAL STUDIES

3.1. *Shear wall with four bands of openings*

The symmetrical shear wall with four bands of openings, as shown in Figures 1(a) and (b) with $a = 4{\cdot}072$ m and $b = 1$ m, is a full scale cross wall of B.R.E. structural model idealised from the Ronan Point building in London [5] with 18 storeys at 2·62 m. The elastic modulus E of $3{\cdot}3\text{E} + 10$ N/m^2 and density of material of 2200 kg/m^3 are assumed. The thickness of the wall is 0·22 m.

After obtaining the shear rigidity (GA) of the example shear wall from equation (13) and then substituting it into equations (4) and (5) the dimensionless characteristic parameters, k and $k\alpha H$, can be determined. The fundamental frequency of this wall can be calculated using equation (1) combining equations (2) and (3).

The fundamental frequencies obtained from the proposed method and that from finite element analysis using the ANSYS software package (using 2772 eight-node quadrilateral elements, Plane82, for plane stress and plane strain problems) are given in Table 1.

Based upon the above example structure, more numerical studies have been carried out to validate the proposed method for the wall with different structural parameters and different storey numbers.

3.2. *The wall with different storey numbers*

The shear wall with four bands of openings and the same structural dimensions as the above example, except for changing the numbers of storeys, have been analysed using the proposed method and finite element analysis. The results are listed in Table 2, and shown in Figure 3. It is shown that the more numbers of storeys the wall has, the better the agreement between the proposed method and finite element analysis. The differences are less than 7% for shear walls with more than six storeys and results from the proposed method are always slightly larger than those from FEM.

TABLE 1

Comparison between results of proposed analysis and that from ANSYS FEM analysis for shear wall with four bands of openings

Methods	Fundamental frequency (Hz)
Proposed method	4·265
ANSYS FEM	4·175

TABLE 2

Comparison between results of proposed analysis and those from ANSYS FEM analysis for shear wall with four bands of openings and different numbers of storeys

Numbers of storeys	$k = 1{\cdot}0147$ $k\alpha H$	Fundamental frequency (Hz) Proposed method	Fundamental frequency (Hz) ANSYS FEM
18	14·911	4·265	4·175
17	14·082	4·691	4·578
16	13·254	5·182	5·041
15	12·426	5·753	5·578
14	11·597	6·422	6·203
13	10·769	7·212	6·939
12	9·941	8·156	7·814
11	9·112	9·301	8·870
10	8·284	10·707	10·162
9	7·455	12·471	11·776
8	6·627	14·736	13·841
7	5·798	17·741	16·568
6	4·970	21·898	20·326
5	4·142	28·01	25·823
4	3·314	37·842	34·621

3.3. *The wall with different depths of connecting beams*

An 18-storey shear wall with four bands of openings and the same structural dimensions as in section 3.1, except for changing the depth of connecting beams, has been analysed using the proposed method and finite element analysis. The results are listed in Table 3, and shown in Figure 4. It is shown that when the depth of the connecting beam reaches one half of the storey height the coupled wall will behave like a pierced wall. Beyond that point, the effect of the depth increase of connecting beams on the fundamental frequency becomes very small. It is also noted that when the depth of the connecting beams is less than about 0·55 m the results from the proposed method become smaller than those from FEM. The reason will be considered in a further investigation.

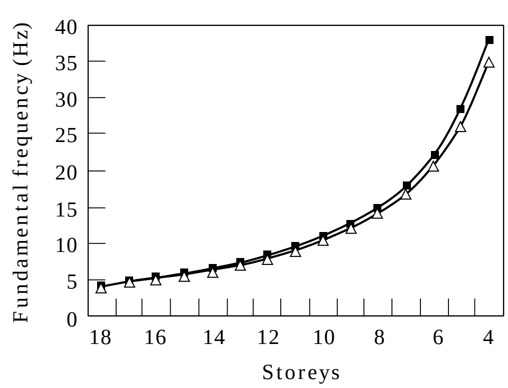

Figure 3. Comparison between results of proposed analysis and those from ANSYS FEM analysis for different numbers of storeys. –■–, Proposed method; –△–, ANSYS FEM.

TABLE 3

Comparison between results from proposed method and those from ANSYS FEM for the shear wall with four bands of openings and different depths of connecting beams

Depth of connecting beams (m)	$k = 1{\cdot}0147$ $k\alpha H$	Fundamental frequency (Hz)	
		Proposed method	ANSYS FEM
0·4	9·366	3·561	3·762
0·6	14·428	4·223	4·149
0·8	19·052	4·528	4·346
1·0	23·241	4·672	4·456
1·2	27·048	4·741	4·519
1·4	30·527	4·770	4·558

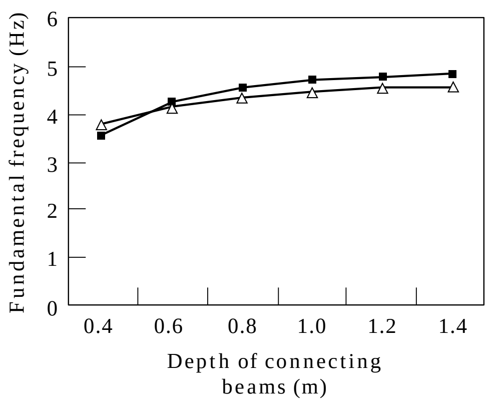

Figure 4. Comparison between results from proposed method and those from ANSYS FEM for the shear wall with four bands of openings and different depths of connecting beams. Key as for Figure 3.

TABLE 4

Comparison between results from proposed method and those from ANSYS FEM for asymmetrical shear walls

First wall depth and opening span	Fundamental frequency (Hz)	
	Proposed	ANSYS FEM
$a = 3{\cdot}572$ m, $b = 1{\cdot}5$ m	4·143	4·088
$a = 3{\cdot}048$ m, $b = 2{\cdot}024$ m	4·076	3·937

3.4. *The asymmetrical wall*

The following two examples are given for a shear wall with four bands of openings with asymmetrical opening span and wall width. All other structural dimensions are the same as in section 3.1, except for the first wall width and opening span. The results from the proposed method and those from FEM are listed in Table 4. Good agreement has been achieved between the two methods.

4. CONCLUSION

A shear wall with multiple bands of openings, which is relatively more complicated than a single-band structural component, may also be classified into shear flexure cantilever in which deflection and action are governed by its stiffness in bending and racking shear. The coupled shear wall, a shear wall with one band of openings, may be considered as a special case of multiple bands. It is reasonable to assume that the racking shear rigidity (GA) of the shear wall with multiple bands of openings depends on the flexural rigidity of connecting beams between the walls. The previously developed formula has been extended to determine the fundamental lateral frequency of a shear wall with multiple bands of openings. Numerical studies have shown that the proposed method can be applied to such shear walls with asymmetrical wall and opening span. Compared with FEM, it is much more efficient with high accuracy for such shear walls with more than six storeys. The accuracy becomes lower for a low-rise building. This is because equation (3), derived from coupled wall theory, tends to have inherent errors for low-rise structures.

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