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IMPORTANCE OF FLOW CONDITION ON SEISMIC WAVES AT A SATURATED POROUS SOLID BOUNDARY

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The deformation of poroelastic saturated media due to passing seismic waves results in coupled solid–fluid motion. Energy losses arise due to the viscous flow of the fluid with respect to the solid matrix. This implies that the flow condition at a saturated porous solid boundary might play a key role in seismic reflection and transmission. A detailed investigation is therefore carried out in this paper to verify this inference. The problem treated herein corresponds to the reflection and transmission of waves obliquely incident at an interface between fluid-saturated porous media and one-phase elastic fluid media. The interface is considered to be either permeable or impermeable to include the effect of flow condition. Based on the theoretical formulation derived, numerical investigations are carried out on the variations of the reflection and transmission coefficients with the angle of incidence as well as the frequency for different interface flow conditions. In addition, the behavior of permeability and porosity effects in the two interface cases is discussed. The results indicate that the effect of the interface flow condition is significant on the reflection and transmission characteristics, especially on the generation of the second kind of compressional waves. A comparison between our theoretical results and the reported experimental observation shows an essential agreement though the physical models considered are different.

1. INTRODUCTION

Due to the existence of ground water, earth materials are often present in the form of porous solids saturated by fluid in nature. Ocean sediments may also be looked upon as materials of this category. Therefore, the study of wave propagation in fluid-saturated porous media is of considerable interest in several disciplines such as soil dynamics, geophysics and underwater acoustics. The theory of the wave propagation in such media was first developed by Biot [1, 2]. According to this widely accepted theory, a macroisotropic, homogeneous porous medium containing viscous fluid will sustain two compressional waves and one shear wave. The fast compressional wave (denoted as P-1 wave in this paper) and the shear wave (S wave) are analogous to the corresponding ones in

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elastic theory. The slow compressional wave (P-2 wave), however, is associated with a diffusion-type process at low frequency due to the fluid viscosity. Experimental observation of the slow compressional wave was not achieved until 1980 by Plona in artificial porous media at high frequencies because of its high attenuation [3]. More recently, this kind of wave was successfully measured in granular soils by Nakagawa *et al.* [4] using a pulse transmission system combined with a conventional triaxial testing system.

Since Biot's pioneering work, there have been many investigations on various aspects of wave propagation in such media. Among them reflection and transmission of waves from a saturated porous solid boundary is an important problem not only from the point of view of applications in geophysical exploration and non-destructive evaluation (NDE), but also at the theoretical level. The treatments of reflection of elastic waves at a normal incidence on a plane boundary between two saturated porous media were reported in references [5, 6]. The reflection and transmission of waves at normal incidence on the boundary between ordinary solids and saturated porous solids was discussed in reference [7]. For such a system Hajra and Mukhopadhyay [8] considered a more general case of seismic waves obliquely incident on the boundary, however, the dissipation resulting from the viscous flow of the fluid was neglected in their analysis. With a purpose of applications in underwater acoustics, Wu *et al.* [9] recently studied the reflection of elastic waves from the boundary of water and saturated porous solids. Similarly, the viscous flow effect was ignored due to the difficulty of calculation.

However, viscous flow plays a key role in wave propagation in saturated media because it makes wave propagation dispersive and is responsible for the dissipation of energy in propagation [2, 10]. More recently, its effect has been shown for the harmonic and transient vibrations of soil columns [11, 12] and seismic reflection and transmission [13]. Keeping in view this fact, it is consequently inferred that the flow condition at a saturated porous solid interface might play a key role in the reflection and transmission of waves. To the best knowledge of the author's, however, the available discussions on this issue are not adequate. It is only recently that Rasolofosaon [14] modified the well-known Plona experiments on saturated porous plates to investigate the importance of interface hydraulic condition. Therefore, the purpose of this paper is to examine this issue theoretically in detail for a general case, and it is expected that the investigation would provide a better understanding of the reflection and transmission in saturated porous media.

To this end, the problem of the reflection and transmission of waves obliquely incident at the interface between saturated porous solids and one-phase elastic media is investigated, which is of more interest in practice [8]. The boundary is considered to be either permeable or impermeable to include the flow condition effect. Based on Biot's theory, the formulation of the boundary value problem, which corresponds to a P-wave obliquely incident at the boundary after propagating through the elastic media, is briefly described. The viscous dissipation due to the fluid flow is included in the present study. As a result, all the three kinds of waves in saturated media are dispersive and dissipative, that is, the velocities are frequency dependent and the amplitudes undergo spatial attenuation. Numerical investigations on the variations of the reflection and transmission coefficients with the angle of incidence as well as the frequency for both permeable interface and impermeable interface are carried out. In addition, the behavior of permeability effect and porosity effect, which are recognized to be two important effects in dynamics of saturated porous media, is examined for the two cases of interface flow condition.

2. FIELD EQUATIONS FOR SATURATED POROUS MEDIA

In Biot's theory, a fluid-saturated porous medium is regarded as an interacting two-phase elastic system, one phase of which is a porous solid skeleton and the other phase is a fluid filling the voids. Hence, the deformation of the saturated porous medium due to passing seismic waves results in coupled solid–fluid motion. Energy losses arise due to the viscous motion of the fluid with respect to the solid skeleton. The equations of motion can be given in the form of displacements as [1, 2]

$$\mu\nabla^2\mathbf{u} + (\lambda + \mu + \alpha^2 M)\mathrm{grad}(\mathrm{div}\,\mathbf{u}) + \alpha M\,\mathrm{grad}(\mathrm{div}\,\mathbf{w}) = \rho\frac{\partial^2\mathbf{u}}{\partial t^2} + \rho_f\frac{\partial^2\mathbf{w}}{\partial t^2}, \qquad (1)$$

$$\alpha M\,\mathrm{grad}(\mathrm{div}\,\mathbf{u}) + M\,\mathrm{grad}(\mathrm{div}\,\mathbf{w}) = \rho_f\frac{\partial^2\mathbf{u}}{\partial t^2} + m\frac{\partial^2\mathbf{w}}{\partial t^2} + b\frac{\partial\mathbf{w}}{\partial t}, \qquad (2)$$

where $\mathbf{u}$ and $\mathbf{w}$ are, respectively, the displacement vectors of the solid skeleton and the fluid with respect to the solid phase; ρ_f is the mass density of pore fluid; ρ is the total density; $\rho = (1-n)\rho_s + n\rho_f$, ρ_s is the density of the grains; n is the porosity; λ, μ are the Lamé's constants of the solid skeleton; α, M are the Biot's parameters accounting for the compressibility of the grains and the fluid. They can be given by

$$\alpha = 1 - \frac{K_b}{K_s}, \quad M = \frac{K_s^2}{K_d - K_b}, \quad K_d = K_s\left[1 + n\left(\frac{K_s}{K_f} - 1\right)\right], \qquad (3)$$

in which K_s, K_f and K_b are the bulk moduli of solid grains, fluid and solid skeleton, respectively. It is noted that $0 \leqslant \alpha \leqslant 1$ and $0 \leqslant M \leqslant \infty$. For a completely dry material $M = 0$, whereas for a material with incompressible solid grains and pore fluid, $\alpha = 1$ and $1/M = 0$. It can be easily shown that this interpretation coincides with the one proposed by Zienkiewicz and Shiomi [15]. The parameter m describes the mass coupling effect which arises from the fact that for a typical porous system, the motion of the pore fluid is not uniform and in general does not take place in the direction of the macroscopic pressure gradient. It depends on the density of fluid and the geometry of the pores and can be expressed as $m = s\rho_f/n$; s is the experimentally determined parameter known as the structure factor. The parameter b accounts for the viscous coupling due to the relative motion between the solid frame and the pore fluid, which is defined as η/k, η being the fluid viscosity and k the permeability with the unit m^2. In the case of high frequency where the assumption of Poiseuille flow is not valid, the viscosity may depend upon frequency and pore structure in a complicated manner, as discussed by Biot [2] and Jonson *et al*. [16]. For most problems one is concerned with, the use of frequency-independent viscous coupling is justified and the structure factor is usually assumed to be unity [5, 11, 15].

It should be noted that k in our formulation is different from the permeability coefficient k' (m/s) that is used in soil mechanics

$$k = k' \frac{\eta}{\rho_f g}. \tag{4}$$

The constitutive equations for a homogeneous poroelastic material can be expressed as

$$\sigma_{ij} = \lambda e \delta_{ij} + 2\mu\varepsilon_{ij} - \alpha\delta_{ij} p_f, \quad p_f = M\varsigma - \alpha M e, \tag{5, 6}$$

where σ_{ij} is the total stress of the bulk material, and p_f is pore pressure; $e = \text{div}\,\mathbf{u}$ is the volumetric strain of soil skeleton and $\varsigma = -\text{div}\,\mathbf{w}$ gives the quantity of fluid that enters or leaves a unit volume attached to the skeletal frame; δ_{ij} is the Kronecker delta. By ignoring the compressibility of solid grains, Equation (5) can be reduced to the effective stress principle in soil mechanics [15].

Considering a Helmholtz resolution of each of the two displacement vectors in the form

$$\mathbf{u} = \text{grad}\,\varphi_s + \text{curl}\,\boldsymbol{\psi}_s, \quad \mathbf{w} = \text{grad}\,\varphi_f + \text{curl}\,\boldsymbol{\psi}_f, \tag{7, 8}$$

where φ_s, $\boldsymbol{\psi}_s$ are associated with the solid phase of the bulk material, while potentials φ_f, $\boldsymbol{\psi}_f$ are associated with the flow of the pore fluid relative to the solid, one can obtain the following two pairs of equations that determine the potentials

$$[M]\{\ddot{\varphi}\} + [C]\{\dot{\varphi}\} - [K_p][L]\{\varphi\} = \{0\}, \tag{9}$$

$$[M]\{\ddot{\psi}\} + [C]\{\dot{\psi}\} - [K_s][L]\{\psi\} = \{0\}, \tag{10}$$

where

$$\begin{aligned} &[M] = \begin{bmatrix} \rho & \rho_f \\ \rho_f & m \end{bmatrix}, && [C] = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix}, \\ &[K_p] = \begin{bmatrix} \lambda + 2\mu + \alpha^2 M & \alpha M \\ \alpha M & M \end{bmatrix}, && [K_s] = \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}, \\ &[L] = \begin{bmatrix} \nabla^2 & 0 \\ 0 & \nabla^2 \end{bmatrix}, && \{\varphi\} = \begin{Bmatrix} \varphi_s \\ \varphi_f \end{Bmatrix}, \quad \{\psi\} = \begin{Bmatrix} \psi_s \\ \psi_f \end{Bmatrix}. \end{aligned} \tag{11}$$

Obviously, equations (9) and (10) describe the propagation of the compressional and shear waves in saturated media, respectively.

Assuming the general solutions of equations (9) and (10) in the form

$$\{\varphi\} = \begin{Bmatrix} \varphi_s \\ \varphi_f \end{Bmatrix} = \begin{Bmatrix} A_s \exp[i(\omega t - \mathbf{l}_p \cdot \mathbf{r})] \\ A_f \exp[i(\omega t - \mathbf{l}_p \cdot \mathbf{r})] \end{Bmatrix}, \tag{12}$$

$$\{\psi\} = \begin{Bmatrix} \psi_s \\ \psi_f \end{Bmatrix} = \begin{Bmatrix} B_s \exp[i(\omega t - \mathbf{l}_s \cdot \mathbf{r})] \\ B_f \exp[i(\omega t - \mathbf{l}_s \cdot \mathbf{r})] \end{Bmatrix}, \tag{13}$$

where $\mathbf{l}_p$, $\mathbf{l}_s$ are the wave vectors of P wave and S wave, respectively, and $\mathbf{r}$ is the location vector, one can obtain the following characteristic equations

$$\begin{vmatrix} \rho\omega^2 - (\lambda + 2\mu + \alpha^2 M) l_p^2 & \rho_f \omega^2 - \alpha M l_p^2 \\ \rho_f \omega^2 - \alpha M l_p^2 & m\omega^2 - M l_p^2 - ib\omega \end{vmatrix} = 0, \tag{14}$$

$$\begin{vmatrix} \rho\omega^2 - \mu l_s^2 & \rho_f \omega^2 \\ \rho_f \omega^2 & m\omega^2 - ib\omega \end{vmatrix} = 0. \tag{15}$$

Equations (14) and (15) show that the small amplitude wave propagation in an unbounded poroelastic medium is associated with three types of waves: two compressional waves and a shear wave, all of which are dispersive and attenuative. The velocities and attenuation of the three waves are associated with the properties of the saturated medium.

3. FORMULATION OF THE BOUNDARY VALUE PROBLEM

The physical configuration considered in this paper is shown in Figure 1. The upper medium is treated as a saturated porous medium, whereas the lower half-space is assumed to be one-phase elastic medium. The plane of $z = 0$ defines the interface of the two media. The incident wave is taken to be a P wave with an angular frequency ω and an angle with the normal to the boundary θ_i from the lower medium. At the interface, the incident wave is transmitted into the saturated media, where P-2 wave and S wave are generated. The analysis of reflection and transmission is straightforward. For completeness and briefness of the paper, the framework of the formulation is presented as follows while the detailed algebra is omitted.

The incident, reflected and transmitted waves can be conveniently defined as follows. In region of $z < 0$: incident P wave

$$\varphi_i = A_i \exp[i(\omega t - l_{ix} x - l_{iz} z)]; \tag{16}$$

reflected P wave

$$\varphi_r = A_r \exp[i(\omega t - l_{rpx} x + l_{rpz} z)]; \tag{17}$$

reflected S wave

$$\psi_r = B_r \exp[i(\omega t - l_{rsx} x + l_{rsz} z)]. \tag{18}$$

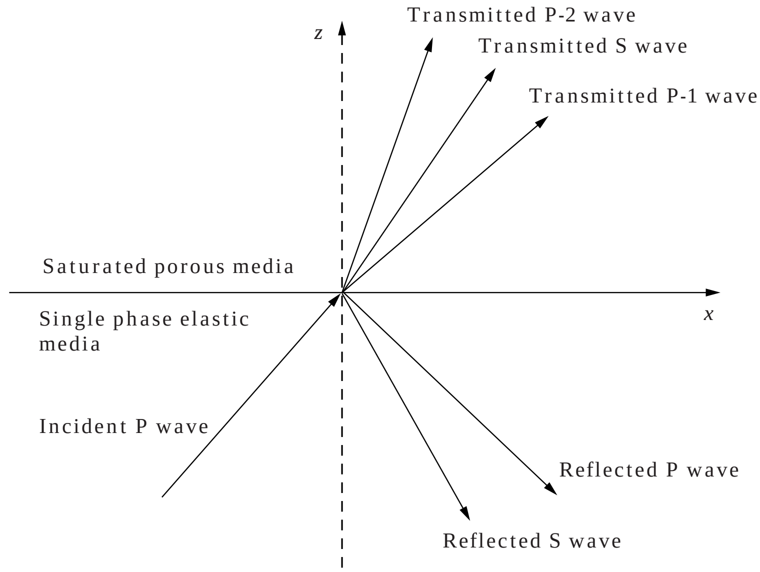

Figure 1. Reflection and transmission of a P wave at an interface between saturated porous media and single phase elastic media.

In region of $z > 0$: transmitted P waves (including P-1 and P-2 waves)

$$\varphi_s = A_{s1} \exp[i(\omega t - l_{1x}x - l_{1z}z)] + A_{s2} \exp[i(\omega t - l_{2x}x - l_{2z}z)], \tag{19}$$

$$\varphi_f = A_{f1} \exp[i(\omega t - l_{1x}x - l_{1z}z)] + A_{f2} \exp[i(\omega t - l_{2x}x - l_{2z}z)], \tag{20}$$

transmitted S wave

$$\psi_s = B_s \exp[i(\omega t - l_{sx}x - l_{sz}z)], \tag{21}$$

$$\psi_f = B_f \exp[i(\omega t - l_{sx}x - l_{sz}z)], \tag{22}$$

where the indices 1, 2, s denote P-1 wave, P-2 wave and S wave, respectively; the indices x, z denote, respectively, the x- and z-components of the wave vector; the subscripts i, r, t denote the incidence, reflection and transmission.

There are eight unknown amplitudes in equations (16)–(22). It can be readily shown that there exist the following relationships between the amplitudes associated with saturated media

$$\delta_1 = \frac{A_{f1}}{A_{s1}} = \frac{(\lambda_c + 2\mu)l_1^2 - \rho\omega^2}{\rho_f\omega^2 - \alpha M l_1^2}, \tag{23}$$

$$\delta_2 = \frac{A_{f2}}{A_{s2}} = \frac{(\lambda_c + 2\mu)l_2^2 - \rho\omega^2}{\rho_f\omega^2 - \alpha M l_2^2}, \tag{24}$$

$$\delta_3 = \frac{B_f}{B_s} = \frac{\mu l_s^2 - \rho\omega^2}{\rho_f \omega^2}, \tag{25}$$

in which l_1, l_2 are two complex roots of equation (14) and l_s is the root of equation (15), $\lambda_c = \lambda + \alpha^2 M$.

The remaining unknown amplitudes of the potentials are related to each other via the following boundary conditions [17]: continuity of the normal and tangential stresses ($\bar{\lambda}$, $\bar{\mu}$ are Lamé's constants of elastic media)

$$\bar{\sigma}_z = \sigma_z, \quad \bar{\tau}_{xz} = \tau_{xz}, \tag{26, 27}$$

or in terms of the potentials these require that

$$\bar{\lambda}\nabla^2(\varphi_i + \varphi_r) + 2\bar{\mu}\left(\frac{\partial^2\varphi_i}{\partial z^2} + \frac{\partial^2\varphi_r}{\partial z^2} + \frac{\partial^2\psi_r}{\partial x \partial z}\right) = \lambda_c \nabla^2 \varphi_s + \alpha M \nabla^2 \varphi_f + 2\mu\left(\frac{\partial^2\varphi_s}{\partial z^2} + \frac{\partial^2\psi_s}{\partial x \partial z}\right), \tag{28}$$

$$2\bar{\mu}\left(\frac{\partial^2\varphi_i}{\partial x \partial z} + \frac{\partial^2\varphi_r}{\partial x \partial z}\right) + \bar{\mu}\left(\frac{\partial^2\psi_r}{\partial x^2} - \frac{\partial^2\psi_r}{\partial z^2}\right) = 2\mu\frac{\partial^2\varphi_s}{\partial x \partial z} + \mu\left(\frac{\partial^2\psi_s}{\partial x^2} - \frac{\partial^2\psi_s}{\partial z^2}\right), \tag{29}$$

continuity of the normal and tangential displacements

$$\bar{u}_z = u_z, \quad \bar{u}_x = u_x \tag{30, 31}$$

or

$$\frac{\partial\varphi_i}{\partial z} + \frac{\partial\varphi_r}{\partial z} + \frac{\partial\psi_r}{\partial x} = \frac{\partial\varphi_s}{\partial z} + \frac{\partial\psi_s}{\partial x}, \quad \frac{\partial\varphi_i}{\partial z} + \frac{\partial\varphi_r}{\partial x} + \frac{\partial\psi_r}{\partial z} = \frac{\partial\varphi_s}{\partial x} - \frac{\partial\psi_s}{\partial z}. \tag{32, 33}$$

To investigate the effect of the interface flow condition, an additional boundary condition is now introduced by considering the interface as permeable or impermeable. For the permeable interface, it is assumed that the pore fluid pressure vanishes at the interface, while for the impermeable interface, it is assumed that the normal velocity of the fluid is zero. The same interface conditions have been used in reference [18] for analysis of vibration of a layered saturated system. It should be mentioned that the consideration of the two interface flow conditions is of significance not only at the theoretical level, but also in reality. It is well known in geotechnical engineering that in natural deposits, there may exist a thin layer of clay, the permeability of which is very small, or a thin layer of coarse sand or gravel, whose permeability (in plane) is very high. Such a thin layer of clay may perform as an impermeable interface, whereas the thin layer of coarse sand or gravel may be regarded as a permeable interface.

Thus, for the impermeable interface, the z-component of the relative fluid displacement vector is zero

$$w_{z,z=0} = 0 \quad \text{or} \quad \frac{\partial \varphi_f}{\partial z} + \frac{\partial \psi_f}{\partial x} = 0; \tag{34, 35}$$

for the permeable interface

$$p_{f,z=0} = 0, \quad \text{or} \quad M(\nabla^2 \varphi_f + \alpha \nabla^2 \varphi_s) = 0. \tag{36, 37}$$

By substitution of equations (16)–(22) into the stress and displacement boundary conditions, and the hydraulic condition (35) or (37), together with Snell's law

$$l_{ix} = l_{rx} = l_{rsx} = l_{1x} = l_{2x} = l_{sx} = l_x, \tag{38}$$

one can obtain six homogeneous equations in the following matrix form by setting $A_i = 1$

$$[P]_i \{Q\} = \{R\} \text{ (impermeable interface)} \tag{39}$$

or

$$[P]_p \{Q\} = \{R\} \text{ (permeable interface),} \tag{40}$$

where

$$\{Q\} = [A_r \quad B_r \quad A_{s1} \quad A_{s2} \quad B_s]^T, \quad \{R\} = [\bar{\lambda} l_i^2 + 2\bar{\mu} l_{iz}^2 \quad 2\bar{\mu} l_x l_{iz} \quad l_{iz} \quad l_x \quad 0]^T. \tag{41}$$

The elements of matrices $[P]_i$ and $[P]_p$ are given in the Appendix. By solving equations (39) and (40), one can obtain the amplitude reflection and transmission coefficients for the impermeable interface and permeable interface, respectively.

4. NUMERICAL INVESTIGATIONS

So far, a general theoretical formulation has been briefly presented of the reflection and transmission at the interface between saturated porous media and single phase elastic media. In this section, detailed numerical investigations will be carried out on the influence of interface flow condition in various situations. The objectives are: (1) to compute the reflection and transmission coefficients as a function of the angle of incidence as well as frequency for permeable and impermeable interfaces, and to compare these results correspondingly; (2) to examine the behavior of the permeability and porosity effects in the two cases of flow condition. Permeability and porosity are two important properties which characterize saturated porous media. In general, they may produce an influence on the characteristics of wave propagation in saturated porous media. Thus, this consideration is of interest and is worth mentioning.

The properties of the saturated porous media, which are typical of what would be expected in clean sand, are listed in Table 1, together with the properties of elastic media.

TABLE 1
Properties of two media

Saturated porous media
Bulk modulus of solid skeleton $K_b = 43{\cdot}6$ MPa
Bulk modulus of solid grains $K_s = 36$ GPa
Bulk modulus of fluid $K_f = 20$ GPa
Shear modulus of solid skeleton $\mu = 26{\cdot}1$ MPa
Mass density of fluid $\rho_f = 1000$ kg/m^3
Mass density of grains $\rho_s = 2650$ kg/m^3
Permeability $k = 10^{-10}$ m^2
Viscosity of fluid $\eta = 10^{-3}$ Pa s
Porosity $n = 0{\cdot}47$
Single phase media
Lame constants $\bar{\lambda} = 2{\cdot}48$ GPa, $\bar{\mu} = 1{\cdot}28$ GPa
Mass density $\bar{\rho} = 2000$ kg/m^3

4.1. REFLECTION AND TRANSMISSION COEFFICIENTS AS A FUNCTION OF THE ANGLE OF INCIDENCE

First, the variations of the reflection and transmission coefficients with the angle of incidence in the aforementioned two cases of interface flow condition are studied. The results are shown in Figure 2, in which the frequency is taken to be 10 Hz. Clearly, it is observed that there exists quite a difference in the behavior for the two cases for all the reflected and transmitted waves, especially for the transmitted P-2 wave. The amplitudes of the P-2 wave in the case of the impermeable interface are much less than the corresponding ones in the permeable case. This implies that for the case of the impermeable interface, generation of the P-2 wave is difficult. This phenomenon may be explained as follows: because the P-2 wave corresponds to the relative fluid motion with respect to the solid frame, the impermeable interface prevents the fluid flow and subsequently causes the difficulty of generation of the P-2 wave. Meanwhile, due to the conservation of energy, this fact will produce an influence on other waves, as can be seen in the figure, that is, the transmission coefficients for the P-1 wave and S-wave in impermeable cases are greater than those in permeable cases. Though the physical models considered are different, the important feature obtained in our study essentially agrees with the experimental results by Rasolofosaon [14], who modified Plona's experiments on saturated porous plates to investigate the interface hydraulic condition. It was found in his experiments that the generation of the second kind of compressional wave was observed on coated plates (impermeable interface).

On the other hand, it can be seen from Figure 2 that the reflection and transmission coefficients vary with the angle of incidence in either case. The nature of the variation with the angle of incidence is, however, different for different reflected and transmitted waves. When the angle of incidence is 0°, say normal incidence, there are no S waves generated in the two media, while when the angle of incidence is 90° (grazing incidence), there is only a reflected P wave.

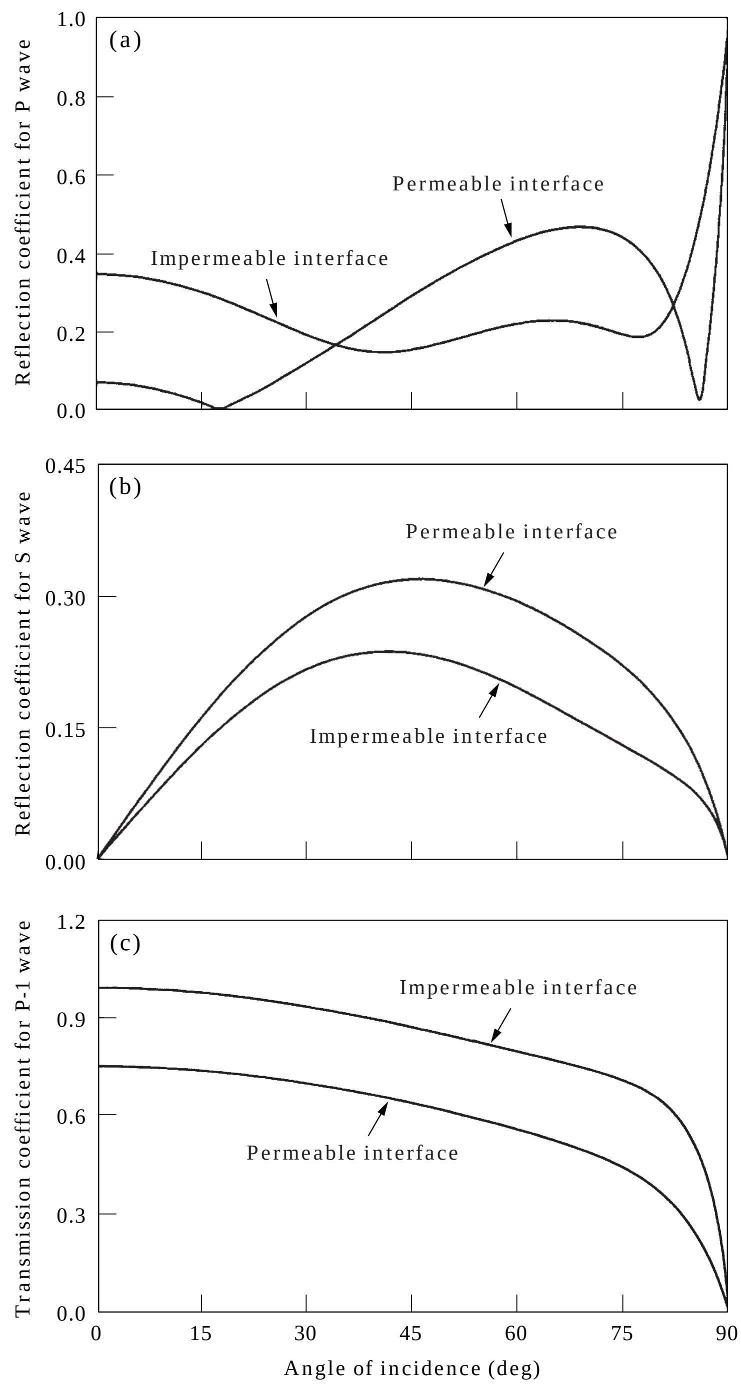
(a)
Reflection coefficient for P wave
1.0
0.8
0.6
0.4
0.2
0.0
Permeable interface
Impermeable interface
(b)
Reflection coefficient for S wave
0.45
0.30
0.15
0.00
Permeable interface
Impermeable interface
(c)
Transmission coefficient for P-1 wave
1.2
0.9
0.6
0.3
0.0
Impermeable interface
Permeable interface
0
15
30
45
60
75
90
Angle of incidence (deg)

Fig. 2.

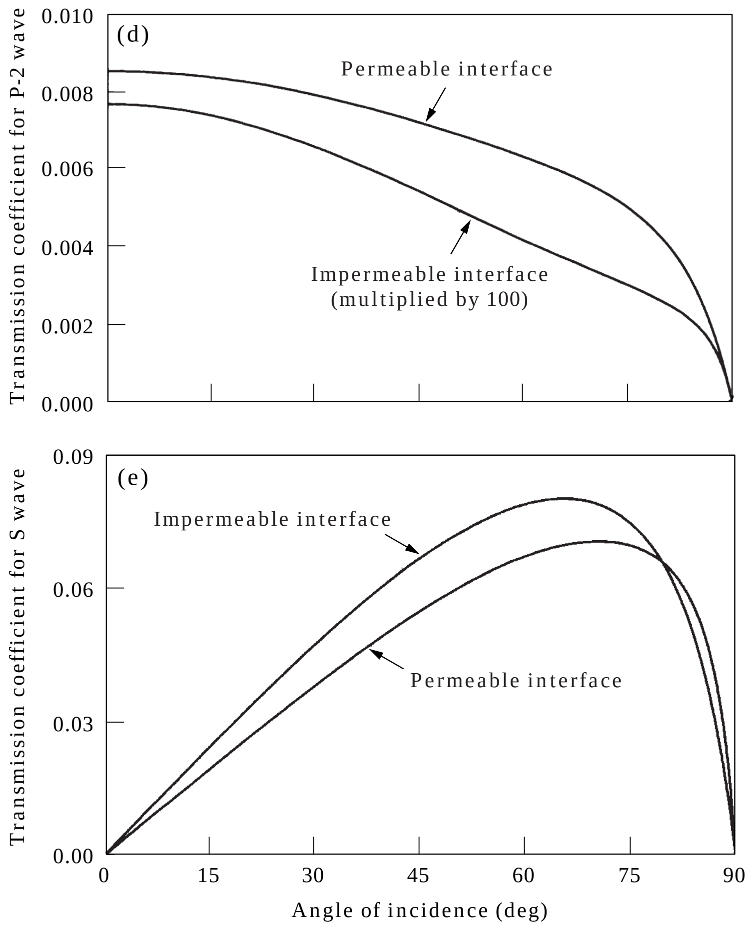

Figure 2. Reflection and transmission coefficients as a function of the angle of incidence for permeable and impermeable interfaces: (a) reflected P wave; (b) reflected S wave; (c) transmitted P-1 wave; (d) transmitted P-2 wave; (e) transmitted S wave.

4.2. REFLECTION AND TRANSMISSION COEFFICIENTS AS A FUNCTION OF FREQUENCY

The results presented above are for the case where the frequency is low (10 Hz). As mentioned before, due to the inclusion of the viscous flow in the present study, all the three kinds of waves in saturated media are dispersive, namely, frequency-dependent. To examine the effect of frequency, the reflection and transmission coefficients at various frequencies are calculated for the two cases of flow condition. The frequency range considered herein covers the frequencies generally used in the field and laboratory experiments. The angle of incidence of the P wave is taken to be 30°. The computed results are shown in Figure 3, in which the frequency is normalized by the characteristic frequency ($f_c = 748$ Hz) of the saturated medium, which is defined by Biot [1, 2] as $f_c = bn/2\pi\rho_f$. From Figure 3 it can be observed that, the frequency-dependence of the reflection and transmission in the two cases is different. For the impermeable case, the reflection and transmission coefficients show a small

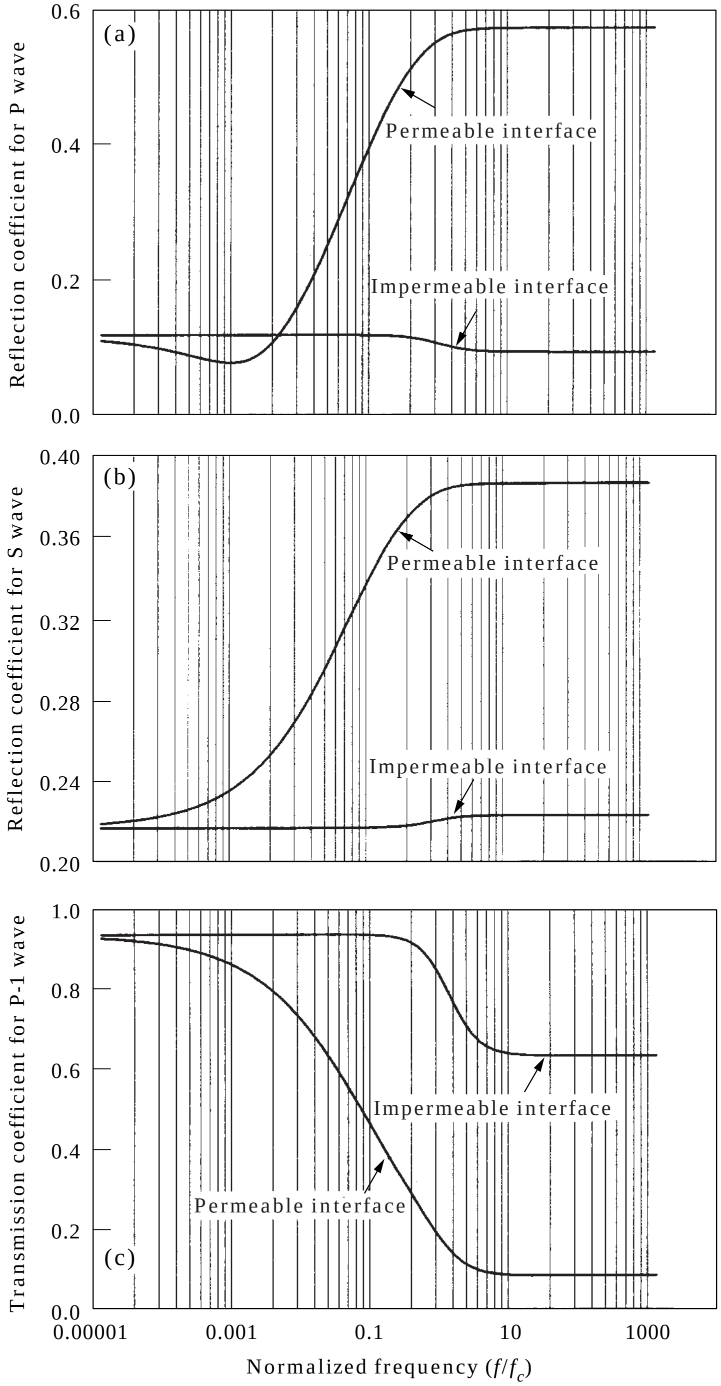
0.6
(a)
0.4
0.2
0.0
Reflection coefficient for P wave
Permeable interface
Impermeable interface
0.40
(b)
0.36
0.32
0.28
0.24
0.20
Reflection coefficient for S wave
Permeable interface
Impermeable interface
1.0
0.8
0.6
0.4
0.2
0.0
(c)
Transmission coefficient for P-1 wave
Impermeable interface
Permeable interface
0.00001
0.001
0.1
10
1000
Normalized frequency (f/f_c)

Fig. 3.

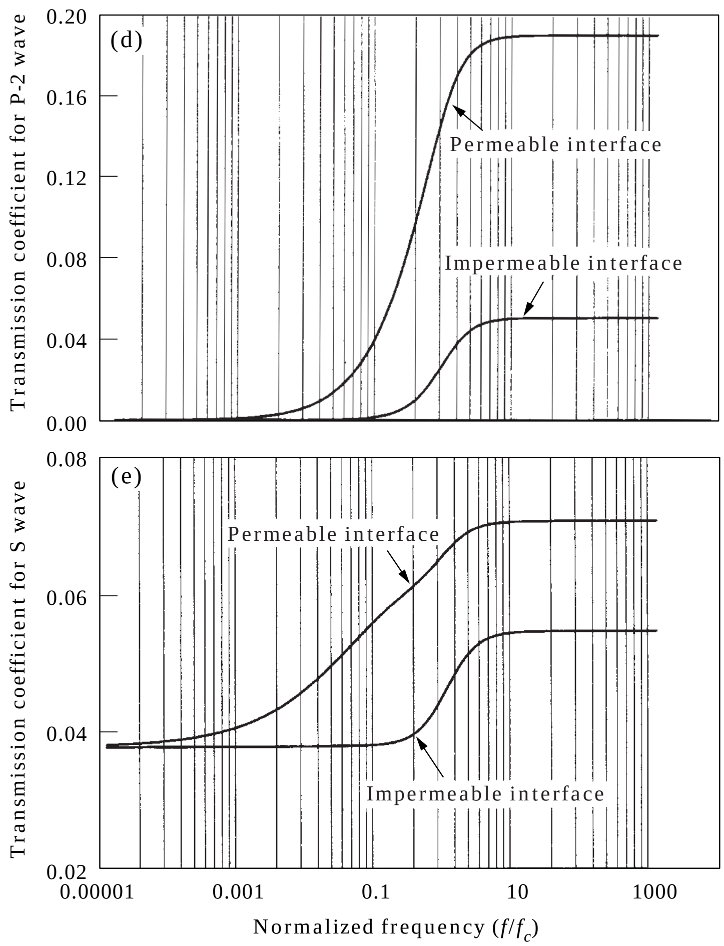

Figure 3. Reflection and transmission coefficients as a function of the frequency for permeable and impermeable interfaces: (a) reflected P wave; (b) reflected S wave; (c) transmitted P-1 wave; (d) transmitted P-2 wave; (e) transmitted S wave.

dispersion over the frequency band considered, whereas for the permeable case, the dispersion is found to be large. Between the low and high frequency limits, these coefficients exhibit an obvious transition in either interface case. It is found that the intermediate frequency band is around the characteristic frequency of the saturated media. Moreover, it is noted from Figure 3, that for permeable interface, even in a very low frequency range, the reflection and transmission coefficients are affected by frequency. This behavior is contrary to the case of the impermeable interface.

On the other hand, it is seen again, that the frequency-dependent transmission coefficients for the P-2 wave in the case of the permeable interface is much higher than the corresponding ones in the impermeable case. In both cases, they increase with a rise in the frequency until they reach the high-frequency limit, and they approach zero in the zero-frequency limit. This behavior is due to the

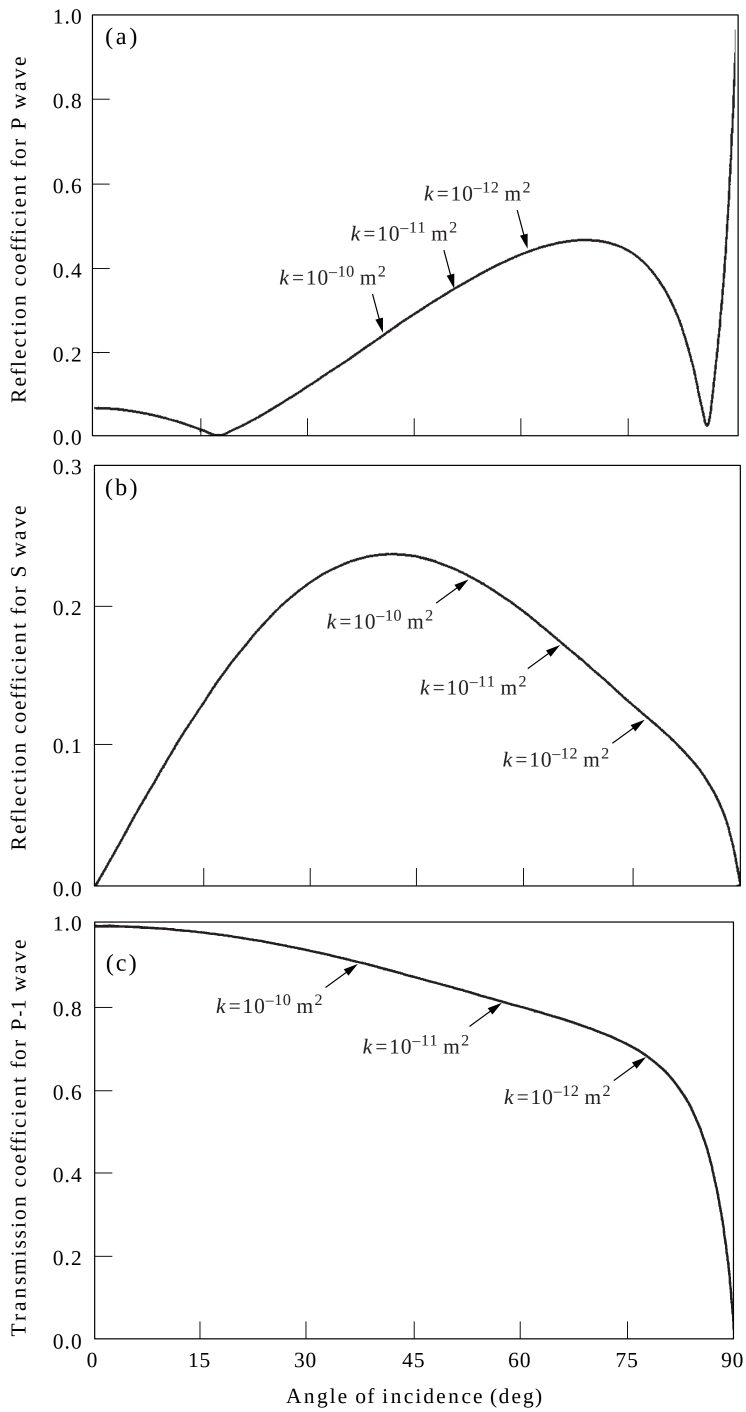
(a)
Reflection coefficient for P wave
$k=10^{-12}$ m^2
$k=10^{-11}$ m^2
$k=10^{-10}$ m^2
1.0
0.8
0.6
0.4
0.2
0.0
(b)
Reflection coefficient for S wave
$k=10^{-10}$ m^2
$k=10^{-11}$ m^2
$k=10^{-12}$ m^2
0.3
0.2
0.1
0.0
(c)
Transmission coefficient for P-1 wave
$k=10^{-10}$ m^2
$k=10^{-11}$ m^2
$k=10^{-12}$ m^2
1.0
0.8
0.6
0.4
0.2
0.0
0
15
30
45
60
75
90
Angle of incidence (deg)

Fig. 4.

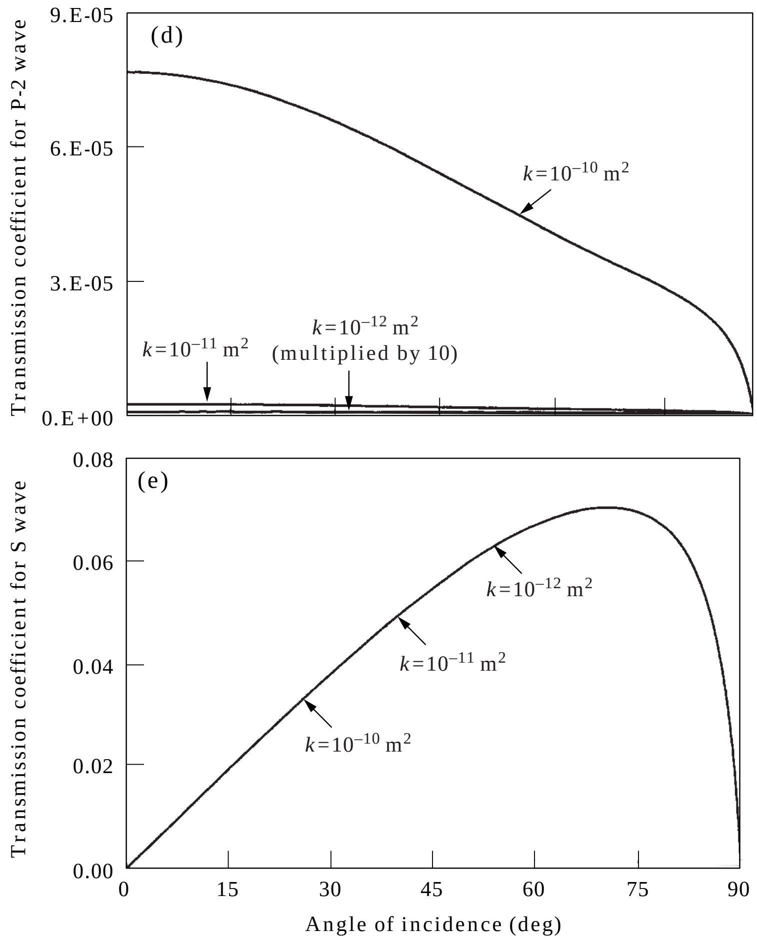

Figure 4. Reflection and transmission coefficients as a function of the angle of incidence for different values of permeability for the impermeable interface: (a) reflected P wave; (b) reflected S wave; (c) transmitted P-1 wave; (d) transmitted P-2 wave; (e) transmitted S wave.

characteristics of the P-2 wave, which is associated with a diffusion-type process in the low frequency limit and takes the character of a true wave in the high frequency range.

4.3. EFFECT OF PERMEABILITY IN PERMEABLE AND IMPERMEABLE CASES

Permeability is an important property which characterizes the porous media. It reflects the viscous coupling between the solid skeleton and the pore fluid [12, 19]. Its behavior in different interface flow conditions is interesting and worth mentioning. Figure 4 depicts the reflection and transmission coefficients as a function of the angle of incidence for several different values of permeability for the case of the impermeable interface. These values (in m^2) are typical of what would be expected in clean sand. The corresponding permeability coefficients in soil mechanics are 10^{-3} m/s, 10^{-4} m/s and 10^{-5} m/s, respectively,

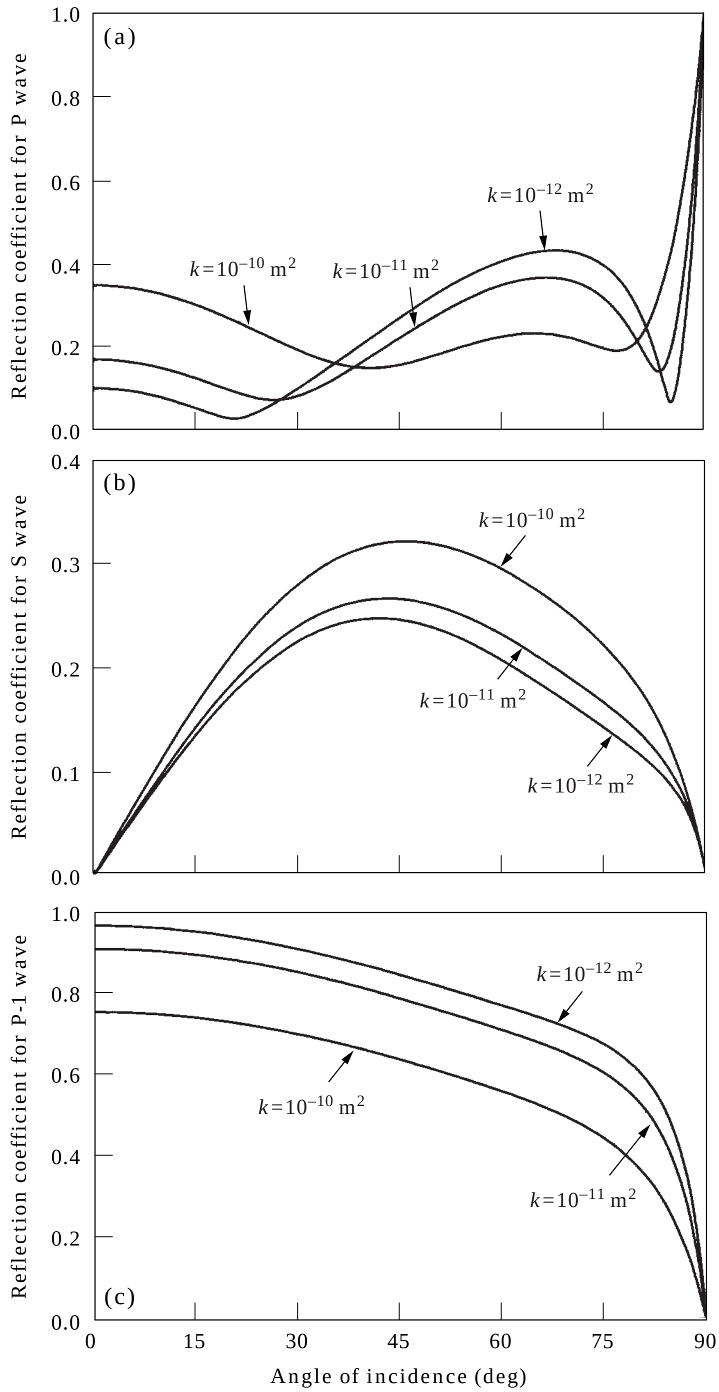
(a)
Reflection coefficient for P wave
k=10^{-10} m^2
k=10^{-11} m^2
k=10^{-12} m^2
(b)
Reflection coefficient for S wave
k=10^{-10} m^2
k=10^{-11} m^2
k=10^{-12} m^2
(c)
Reflection coefficient for P-1 wave
k=10^{-12} m^2
k=10^{-10} m^2
k=10^{-11} m^2
Angle of incidence (deg)

Fig. 5.

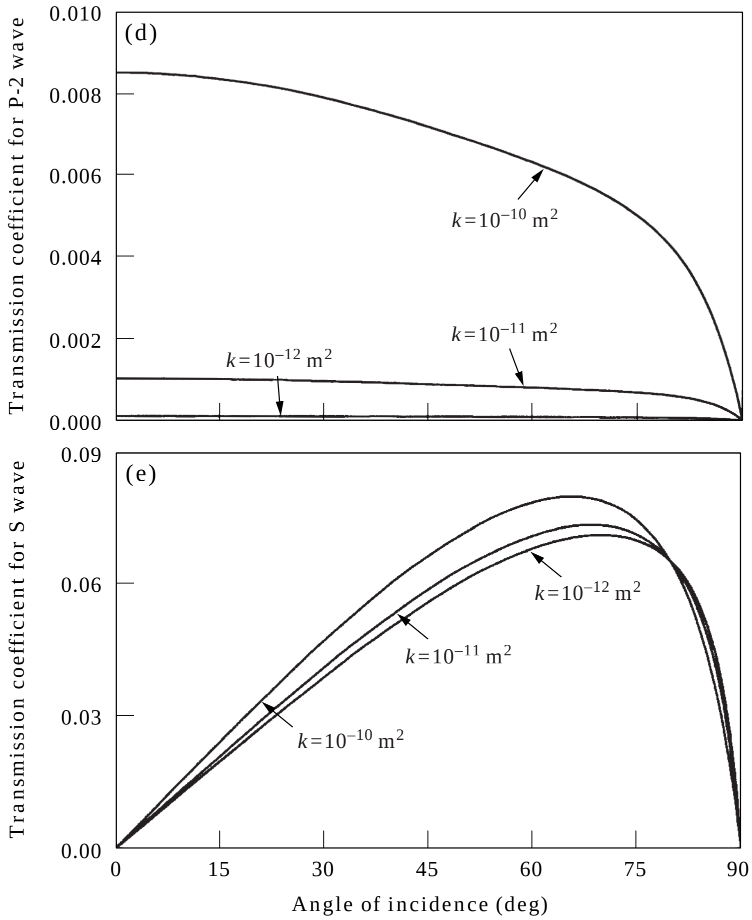

Figure 5. Reflection and transmission coefficients as a function of the angle of incidence for different values of permeability for the permeable interface: (a) reflected P wave; (b) reflected S wave; (c) transmitted P-1 wave; (d) transmitted P-2 wave; (e) transmitted S wave.

by use of equation (4). The frequency of the incident wave is taken to be 10 Hz and the porosity of the saturated media is 0·47. From Figure 4 it can be seen that the reflection and transmission is nearly unaffected by the permeability at all angles of incidence, except for the transmitted P-2 wave, for which the amplitude ratios increase with the increasing permeability. This is understandable, since as the permeability becomes greater, the resistance between the solid and fluid becomes smaller and the relative fluid flow larger; subsequently the P-2 wave becomes obvious.

The angle-dependent reflection and transmission coefficients for different values of permeability for the permeable interface are shown in Figure 5. One interesting feature is that in contrast to the situation in the case of the impermeable interface, the influence of permeability is notable not only on the transmitted P-2 wave, but also on the other reflected or transmitted waves. For the transmitted P-2 wave, similarly, the amplitudes increase with an increase in permeability.

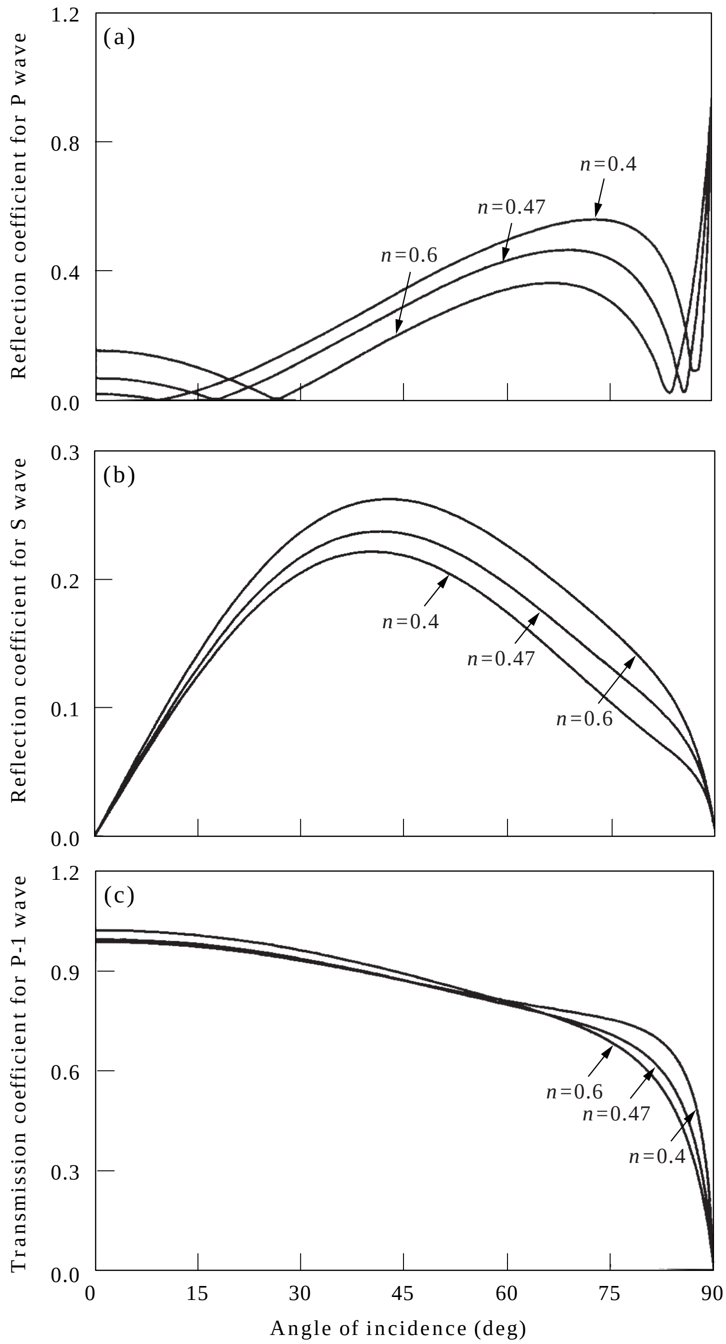
(a)
Reflection coefficient for P wave
1.2
0.8
0.4
0.0
n=0.4
n=0.47
n=0.6
(b)
Reflection coefficient for S wave
0.3
0.2
0.1
0.0
n=0.4
n=0.47
n=0.6
(c)
Transmission coefficient for P-1 wave
1.2
0.9
0.6
0.3
0.0
n=0.6
n=0.47
n=0.4
0
15
30
45
60
75
90
Angle of incidence (deg)

Fig. 6.

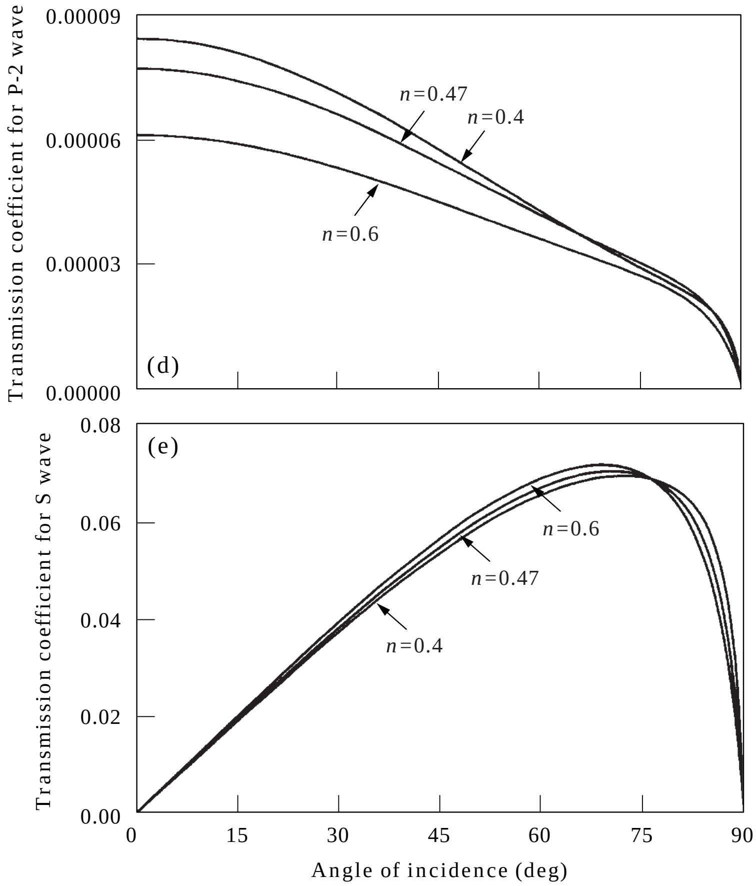

Figure 6. Reflection and transmission coefficients as a function of the angle of incidence for different values of porosity for the impermeable interface: (a) reflected P wave; (b) reflected S wave; (c) transmitted P-1 wave; (d) transmitted P-2 wave; (e) transmitted S wave.

4.4. EFFECT OF POROSITY IN PERMEABLE AND IMPERMEABLE CASES

Porosity is another characteristic property of saturated porous materials. Figure 6 shows the reflection and transmission coefficients as a function of the angle of incidence for different values of porosity for the impermeable interface. The frequency is assumed to be 10 Hz and the permeability is taken to be 10^{-10} m^2. Figure 7 describes the angle-dependent reflection and transmission coefficients for these several values of porosity for the permeable interface case. It is seen from Figures 6 and 7 that, in general the influence of porosity on the generated P-2 wave is more significant than the other waves. Since the P-2 wave is due to the presence of fluid in the porous media, it is reasonable that the porosity has a large influence on it. In addition, it is noted that the influence of porosity depends strongly on the angle of incidence in either case, and the dependence is different for different reflected and transmitted waves.

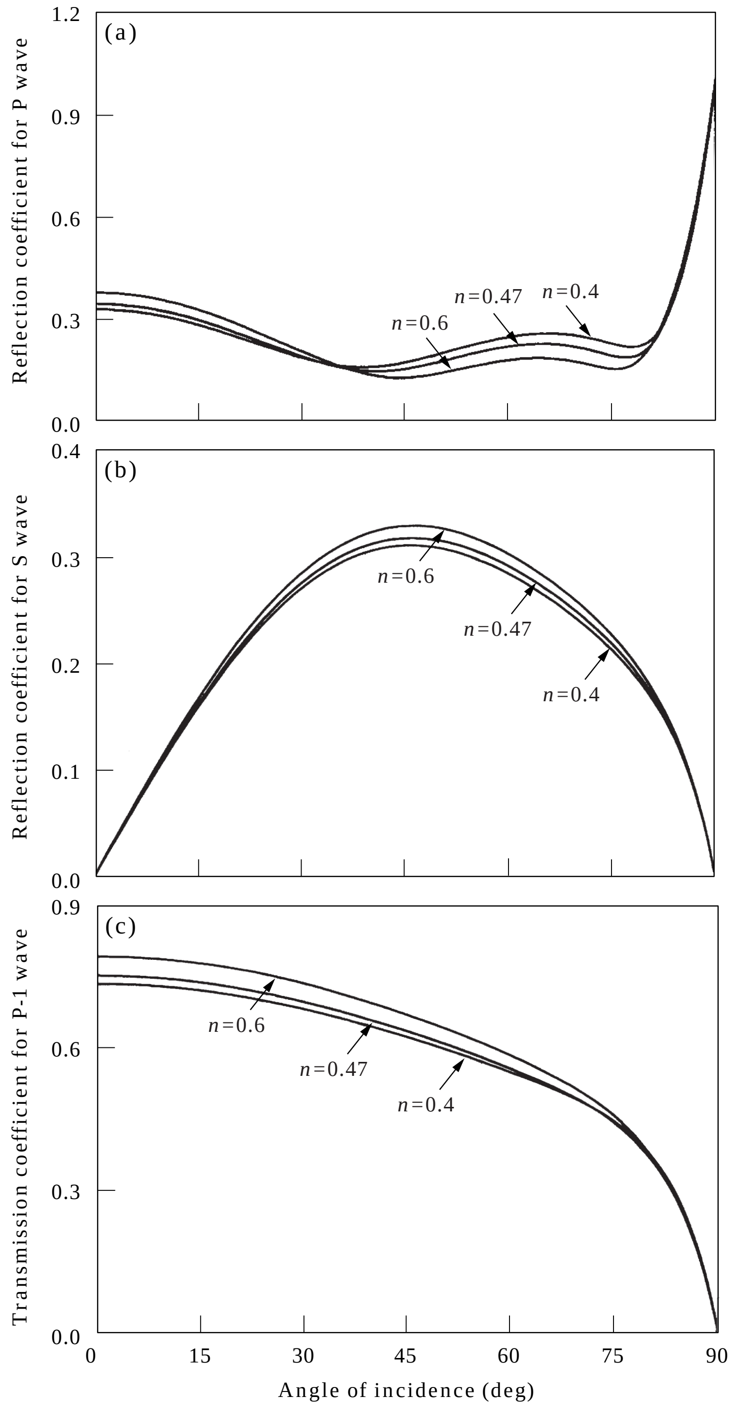
(a)
Reflection coefficient for P wave
1.2
0.9
0.6
0.3
0.0
n=0.6
n=0.47
n=0.4
(b)
Reflection coefficient for S wave
0.4
0.3
0.2
0.1
0.0
n=0.6
n=0.47
n=0.4
(c)
Transmission coefficient for P-1 wave
0.9
0.6
0.3
0.0
n=0.6
n=0.47
n=0.4
0
15
30
45
60
75
90
Angle of incidence (deg)

Fig. 7.

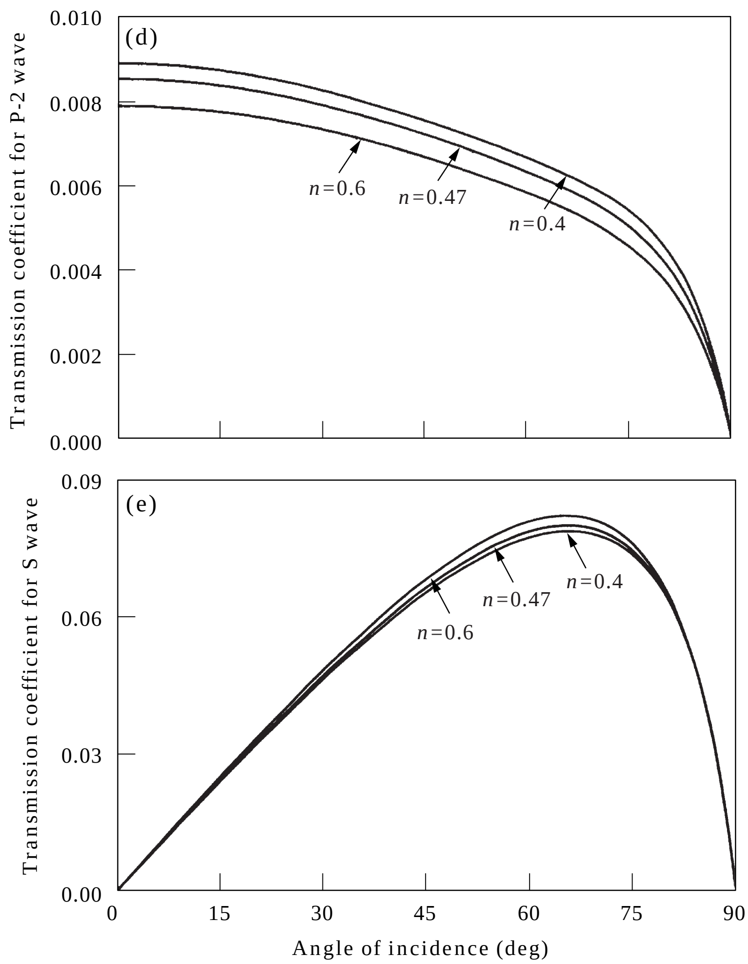

Figure 7. Reflection and transmission coefficients as a function of the angle of incidence for different values of porosity for the permeable interface: (a) reflected P wave; (b) reflected S wave; (c) transmitted P-1 wave; (d) transmitted P-2 wave; (e) transmitted S wave.

5. CONCLUSIONS

In this paper the influence of the interface flow condition on the reflection and transmission from a boundary between saturated porous media and one-phase elastic media is analyzed. One important feature found is that there exists a significant difference in the reflection and transmission characteristics for different interface flow conditions. In particular, the transmission coefficients of the slow compressional wave in the case of the permeable interface are much greater than those in the case of the impermeable interface. This indicates that the interface flow condition plays an important role in the generation of the slow compressional wave in the porous saturated media, and consequently, on the energy dissipation since the slow wave is highly attenuated. Moreover, the behavior of permeability effect in permeable and impermeable cases is quite different: for the impermeable interface, the effect on the angle-dependent reflection and transmission coefficients is negligible, except for the transmitted P-2 wave, whereas for the permeable interface, the effect of permeability is significant. It is also found that the porosity effect is generally more significant on the slow compressional wave in both interface cases.

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6. APPENDIX

The elements of $[P]_i$ are as follows

$$P_{11} = -\bar{\lambda} l_i^2 - 2\bar{\mu} l_{iz}^2, \qquad P_{12} = 2\bar{\mu} l_x l_{rz},$$

$$P_{13} = (\lambda_c + 2\mu + \alpha M \delta_1) l_1^2 - 2\mu l_x^2, \qquad P_{14} = (\lambda_c + 2\mu + \alpha M \delta_2) l_2^2 - 2\mu l_x^2,$$

$$\begin{array}{lll}
P_{15} = 2\mu l_x l_{sz}, & P_{21} = 2\bar{\mu} l_x l_{iz}, & P_{22} = \bar{\mu}(l_{rz}^2 - l_x^2), \\
P_{23} = 2\mu l_x l_{1z}, & P_{24} = 2\mu l_x l_{2z}, & P_{25} = \mu(l_x^2 - l_{sz}^2), \\
P_{31} = l_{iz}, & P_{32} = -l_x, & P_{33} = l_{1z}, \\
P_{34} = l_{2z}, & P_{35} = l_x, & P_{41} = -l_x, \\
P_{42} = -l_{rz}, & P_{43} = l_x, & P_{44} = l_x, \\
P_{45} = -l_{sz}, & P_{51} = 0, & P_{52} = 0, \\
P_{53} = l_{1z}\delta_1, & P_{54} = l_{2z}\delta_2, & P_{55} = l_x \delta_3.
\end{array}$$

Except for the fifth line, the elements of $[P]_p$ are same with the corresponding ones in $[P]_i$

$$\begin{array}{lll}
P_{51} = 0, & P_{52} = 0, & P_{53} = l_1^2 \delta_1 + \alpha l_1^2, \\
P_{54} = l_2^2 \delta_2 + \alpha l_2^2, & P_{55} = 0. &
\end{array}$$