Journal of Sound and Vibration (1999) **221**(3), 530–536
Article No. jsvi.1998.1967, available online at http://www.idealibrary.com on IDEAL

AN EVALUATION METHOD ON STOCHASTIC RESPONSE OF A LINEAR ACOUSTIC SYSTEM WITH NON-STATIONARY NOISE INPUT BY EMPLOYING A TIME-VARYING OPERATOR AND ITS EXPERIMENT

M. Ohta
Faculty of Engineering, Kinki University, 1 *Umenobe, Takaya, Higashi-Hiroshima City* 729-1715, *Japan*

AND

K. Nishimura
Faculty of Engineering, Kinki University, 5-1-3, *Hiro Koshingai, Kure City* 737-0112, *Japan*

(*Received* 18 *May* 1998, *and in final form* 2 *September* 1998)

1. INTRODUCTION

In a previous paper [1], a trial of statistical evaluation for a probability distribution of random variables on the power scale was proposed by introducing some differential operators with respect to distribution parameters reflecting hierarchically non-stationarity, non-gamma property and various type linear and/or non-linear auto-correlation information of input random signal. The weighting function (impulse response) of the system was treated in an expansion expression form taking a rectangular weighting type as the first term.

In this letter, once after noticing the importance on proper property of linear system mechanism like a transfer characteristic, the probabilistic evaluation of the system response to the standard input is first given by employing an inverse Laplace transformation with use of the residue theorem to reflect concretely the system characteristics into the evaluation. Then, by introducing some time-varying operator to the above expression of system response, probabilistic response evaluation of system can be found for an arbitrary random input. In the experiment, a single wall insulation system under a music noise excitation is employed and its theoretical evaluation of probabilistic response is compared with the experimentally observed data.

2. MODEL OF INSULATION SYSTEM

From the viewpoint of frequency analysis, let us consider a power linear type stochastic system with an input of N dimensional non-negative acoustic

0022–460X/98/130530 + 07 \$30.00/0

signal $\mathbf{X}(=(X_1, X_2, \ldots, X_N)^T, 0 \leqslant X_i < \infty$, for $i = 1, 2, \ldots, N)$ and a single output as follows:

$$Y = \sum_{i=1}^{N} a_i X_i, \tag{1}$$

where Y and X_i denote the system output and input of the ith frequency band with system parameter a_i of the insulation system and the number of frequency band N on input noise.

3. OUTPUT RESPONSE PROBABILITY EXPRESSION OF AN INSULATION SYSTEM WITH NON-STATIONARY INPUT

In a previous paper [1], a probability density function of non-stationary input $\mathbf{X}$ can be given by introducing a time-varying operator (parameter differential type) D as follows (details of its derivation are shown in reference [2]):

$$P(\mathbf{X}) = D \prod_{i=1}^{N} P_\gamma(X_i; m_i, s_i), \tag{2}$$

where

$$D \triangleq 1 + \sum_{n=1}^{\infty} \sum_{n_1 + \cdots + n_N = n} (-1)^n \langle b_n(n_1, \ldots, n_N) \rangle_{b_n}$$

$$\times \prod_{i=1}^{N} \frac{\Gamma(m_i) s_i^{m_i}}{\Gamma(m_i + n_i)} \left(\frac{\mathrm{d}}{\mathrm{d}s_i} \right)^{n_i}, \tag{3a}$$

$$P_\gamma(X_i; m_i, s_i) \triangleq X_i^{m_i - 1} \, \mathrm{e}^{-X_i/s_i} / \Gamma(m_i) s_i^{m_i}, \tag{3b}$$

$$b_n(n_1, \ldots, n_N) \triangleq \left\langle \prod_{i=1}^{N} L_{n_i}^{(m_i - 1)} \left(\frac{X_i}{s_i} \right) \right\rangle, \tag{3c}$$

$$m_i = \langle X_i \rangle^2 / \sigma_{X_i}^2, \qquad s_i = \langle X_i \rangle / m_i. \tag{3d}$$

On the other hand, the probability distribution function of output response Y under an arbitrary random input can be given by operating D to only the basic probability density distribution $P_0(Y)$ of the output response with excitation by a test signal of only gamma type probability distribution given *a priori* by pre-experiment with an intensity X_i of white noise input (here, X_is are statistically independent). That is,

$$P(Y) = DP_0(Y). \tag{4}$$

Here, $P_0(Y)$ can be estimated by an inverse Laplace transformation of the following moment generating function on a standard gamma type input:

$$M_0(\theta) \triangleq \int_0^\infty e^{\theta Y} P_0(Y)\,dY$$

$$= \prod_{i=1}^{N} (1 - a_i s_i \theta)^{-m_i} \tag{5}$$

with the use of equation (1).

In this letter, $P_0(Y)$ is derived from equation (5) by inverse Laplace transformation by using the well-known Heaviside expansion theorem [3] as follows:

$$P_0(Y) = \mathscr{L}^{-1}\left[\prod_{i=1}^{N} (1 - a_i s_i \theta)^{-m_i}\right]$$

$$= \sum_{i=1}^{N} \frac{1}{(m_i - 1)!}\left(\frac{d}{d\theta}\right)^{m_i - 1}\left\{e^{\theta Y}\left(\theta + \frac{1}{a_i s_i}\right)^{m_i} \prod_{i=1}^{N} (1 + a_i s_i \theta)^{m_i}\right\}$$

$$= \sum_{i=1}^{N} \sum_{k_1 + k_2 + \cdots + k_N = m_i - 1} \prod_{\substack{j=1 \\ j \neq i}}^{N} G\left(k_j, m_j, \frac{a_j s_j}{a_i s_i}\right) P_\gamma(Y; k_i + 1, a_i s_i), \tag{6}$$

where

$$G(k, m, s) \triangleq \frac{\Gamma(m + k)}{\Gamma(m) k!} \frac{s^k}{(1 + s)^{k + m}}.$$

The output probability density distribution for an arbitrary input signal having the non-stationary and non-gamma properties can be simply found by applying the previous time-varying operator D to equation (6) as:

$$P(Y) = D P_0(Y)$$

$$= \sum_{i=1}^{N} \sum_{k_1 + k_2 + \cdots + k_N = m_i - 1} \prod_{\substack{j=1 \\ j \neq i}}^{N} G\left(k_j, m_j, \frac{a_j s_j}{a_i s_i}\right) P_\gamma(Y; k_i + 1, a_i s_i)$$

$$+ \sum_{i=n}^{\infty} \sum_{n_1 + n_2 + \cdots + n_N = n} \prod_{i=1}^{N} (a_i s_i)^{n_i} \langle b_n(n_1, n_2, \ldots, n_N)\rangle_b$$

$$\times \sum_{i=1}^{N} \sum_{k_1 + k_2 + \cdots + k_N = m_i - n_i - 1} \left\{ \sum_{j=0}^{k_i} \frac{(-1)^{n + j - k_1} n!}{(n + j - k_i)!(k_j - j)!}\right.$$

$$\left. \times P_\gamma(Y; j + 1, a_i s_i) \prod_{\substack{j=1 \\ j \neq i}}^{N} G\left(k_j, m_j, \frac{a_j s_j}{a_i s_i}\right)\right\}. \tag{7}$$

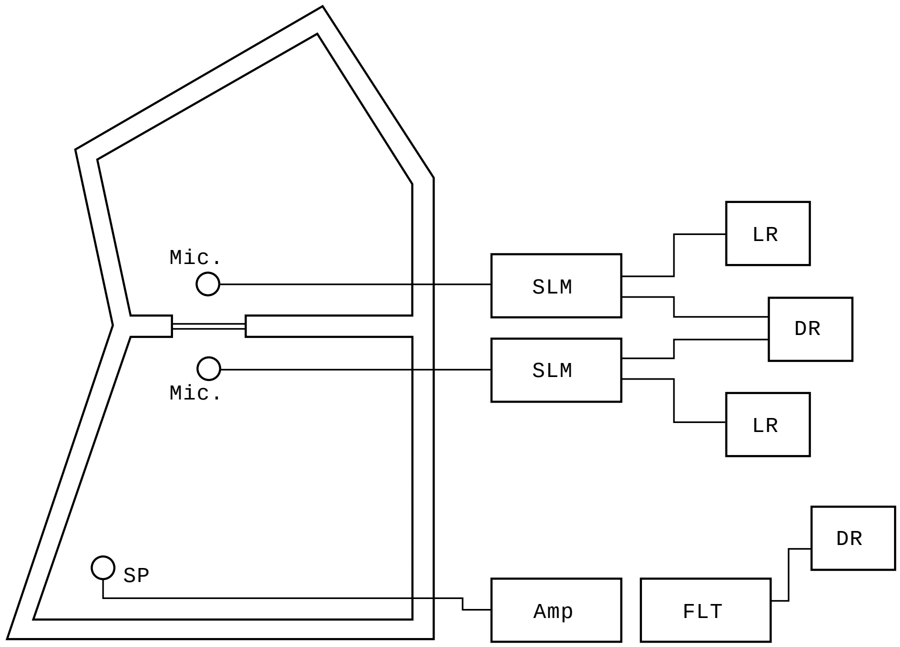

Figure 1. Experimental arrangement in two reverberation rooms. Abbreviations: Mic., microphone; SLM, sound level meter; LR, level recorder; DR, data recorder; SP, speaker; Amp, amplifire; FLT, filter.

4. EXPERIMENTAL CONSIDERATION

Figure 1 shows an experimental arrangement in two reverberation rooms. A single wall of aluminum with thickness 0·8 mm has been employed especially as one of principal experiments for a linear acoustic system on intensity scale, and music noise has been employed as a non-stationary input. In order to study the effect of the non-stationary fluctuation of input noise on the output response probability, the total observation period of 1000 s has been equally divided into 10 locally stationary sub-intervals of 100 s. Expansion coefficients $b_n(n_1, n_2, \ldots, n_N)$s of input noise have been experimentally observed and then calculated along equation (3c) in each locally stationary sub-interval. The system parameters, a_i, of the linear acoustic system have been determined *a priori* by another standard pre-experiment, and their measured values are shown in Table 1. The system order N has been set in advance as $N = 4$ due to the number of frequency bandwidths used in the usual octave band analysis of the experiment.

TABLE 1

Estimation results of system parameters

i	1	2	3	4
Center freq. (Hz)	250	500	1000	2000
$a_i \times 10^{-3}$	115	72·5	22·2	6·59

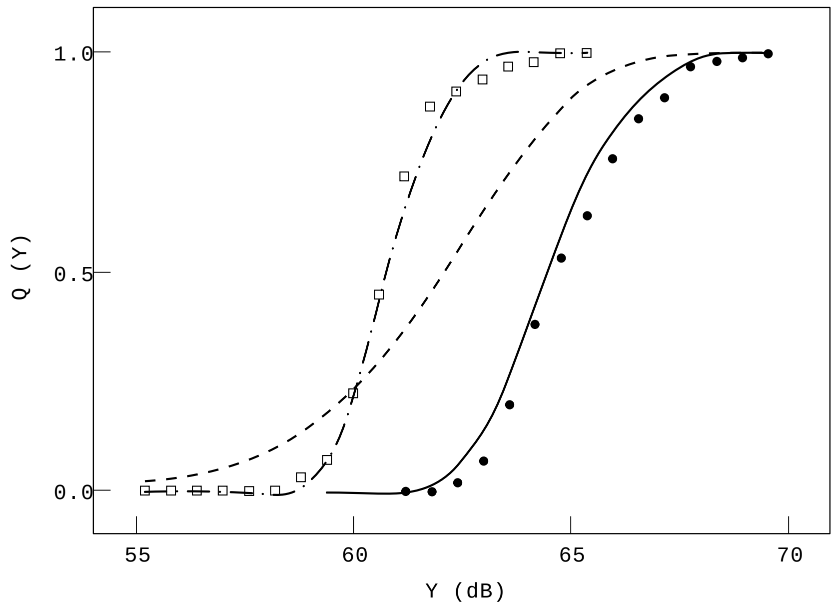

Figure 2. A comparison between experimentally sampled points and theoretically estimated curves for the output probability distribution of a sound insulation system in specific locally stationary time intervals under a non-stationary random input. Experimentally sampled points are marked by □ and ● observed in the 4th and 8th locally stationary time intervals, and theoretically estimated curves are shown with the degree ℓ of approximation in equation (8) as follows: – – –, $\ell = 0$; — - — and ——, $\ell = 4$ for 4th and 8th locally stationary sub-intervals.

The probability distribution of the output response was expressed in advance by the well-known Laguerre type expansion series expression [4] as:

$$P_0(Y) = P_\gamma(Y; M, S)\left\{1 + \sum_{\ell=1}^{\infty} C_\ell \frac{\Gamma(M)\ell!}{\Gamma(M+\ell)} L_\ell^{(M-1)}\left(\frac{Y}{S}\right)\right\}, \tag{8}$$

where

$$M \triangleq \frac{\langle Y \rangle^2}{\sigma_Y^2}, \qquad S \triangleq \frac{\sigma_Y^2}{\langle Y \rangle}, \qquad C_\ell \triangleq \left\langle L_\ell^{(M-1)}\left(\frac{Y}{S}\right)\right\rangle.$$

Here, this distribution parameters M, S and C_ℓ can be evaluated based on the statistics of Y predicted by using equation (7) with correction terms up to $n = 3$. The parameters m_i and s_i with $i = 1$ are determined in the first locally stationary sub-interval.

Figure 2 shows theoretically estimated results with correction terms up to $\ell = 4$ in equation (8) for the 4th and 8th time sub-intervals of local stationarity. In this figure, the theoretically estimated results on the cumulative probability distribution of the output response for each locally stationary sub-interval become closer to the experimentally observed ones by reflecting only the expansion coefficient $b_n(\cdot)$ obtained in each locally stationary sub-interval.

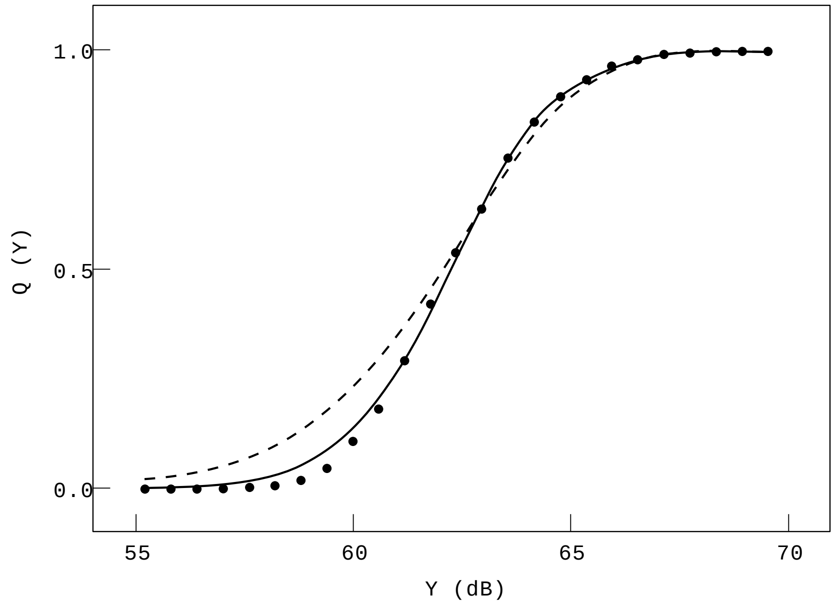

Figure 3. A comparison between experimentally sampled points and theoretically estimated curves for the output probability distribution of a sound insulation system with a non-stationary random input over a whole observation time interval. Experimentally sampled points are marked by ●, and theoretically estimated curves are shown with the degree ℓ of approximation in equation (8) as follows: – – –, $\ell = 0$; ——, $\ell = 4$.

The prediction results for cumulative probability distribution of the output response over a long time period are shown in Figure 3 by reflecting statistically the temporal change of expansion coefficients observed in each stationary sub-interval. In this figure, the first expansion term in the theoretical expression (which coincides with the basic output probability distribution function) is rather different from the experimentally sampled points. However, it is obvious that theoretically estimated curves become closer to the experimentally sampled points as the number of correction terms reflecting the statistics of temporal change of $b_n(\cdot)$ increases. Therefore, the output probability distribution of an arbitrary linear system on intensity scale can be predicted by observing only the coefficients $b_n(\cdot)$s reflecting the statistical properties of input signal in each locally stationary sub-interval and accumulating those statistics over a long time period with help of the probabilistic sum calculation of exclusive events.

5. CONCLUSIONS

In this letter, the probability distribution of transmitted intensity fluctuation has been easily derived by applying the time-varying operator to the basic probability distribution of output response derived by using the well-known residue theorem and statistical parameters obtained by a pre-experiment with a test signal of standard gamma type (which can be easily constructed as an intensity fluctuation of usually standard white noise). As an application to the actual noise environment, the proposed evaluation method has been employed for the evaluation on a sound insulation system of a single wall excited by a non-stationary music noise.

ACKNOWLEDGMENT

The authors would like to express their cordial thanks to Professor A. Ikuta, Messrs T. Tanimura and K. Yakushiji for their helpful discussions and valuable assistance.

REFERENCES

1. M. OHTA, K. NISHIMURA and K. HATAKEYAMA 1991 *Trans. of IEICE Japan* **E74**, 2547–2554. An evaluation method for energy stochastic system with nonstationary random input and a prediction of response probability distribution for the acoustic insulation wall excited by a road traffic noise.
2. M. OHTA, K. HATAKEYAMA, A. IKUTA and B. CHANG 1988 *Trans. IEE Japan* **108-C**, 39–46. A hierarchical establishment of digital filter for the state estimation of stochastic energy system and acoustic environment (in Japanese).
3. J. BROWN and E. V. D. GLAZIER 1964 *Signal Analysis*. New York: Reinhold; Chap. A.7.6, 332.
4. M. OHTA and T. KOIZUMI 1968 *IEEE Trans. IT* **IT-14**, 595–598. General statistical treatment of response of nonlinear rectifying device to stationary random input.
5. S. S. WILKS 1950 *Mathematical Statistics*. Princeton, NJ: Princeton University Press; Chap. 7.8, 183.