Journal of Sound and Vibration (1999) **222**(1), 85–98
Article No. jsvi.1998.2048, available online at http://www.idealibrary.com on IDEAL

STRUCTURE-BORNE SOUND TRANSMISSION FROM MACHINES IN BUILDINGS, PART 1: INDIRECT MEASUREMENT OF FORCE AT THE MACHINE–RECEIVER INTERFACE OF A SINGLE AND MULTI-POINT CONNECTED SYSTEM BY A RECIPROCAL METHOD

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(*Received* 15 *May* 1998 *and in final form* 2 *November* 1998)

The analysis of structure-borne sound transmission is obscured by the complicated nature of the dynamic interaction between source and receiver at each contact point. There is little measured data because of the practical difficulties in directly measuring forces and moments at the contacts. This is particularly so in the case of moments, since there is not yet available an accepted method of registration. Even in the case of forces, where commercially transducers are available, it is seldom practical to insert transducers between the source and receiver without altering the installed condition. A method is described whereby the force and moment induced powers from an installed machine are measured indirectly by a reciprocal method. In the companion paper, the validated method is used to measure the moment induced power produced by a base sub-assembly. Hence, the relative importance of force and moment with respect to variation in machine excitation position on a receiving structure is examined.

1. INTRODUCTION

The analysis of structure-borne sound transmission through the contacts between machines and supporting structures is hindered by the complexity of the process and the large amount of theoretical or experimental data required. The data can be in the form of complex free velocities and source and receiver mobilities, which will yield the contact velocities and forces and thus the component powers. Alternatively, the contact velocities and forces (and moments) might be measured directly when the machine is installed.

A major difficulty with both approaches arises from the non-availability of moment transducers. In addition, in the second approach, it is seldom possible to insert force transducers to measure at the machine contacts without altering

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the installed condition. To circumvent these problems, a method is proposed in this paper where forces and moments at the machine supporting contacts are obtained indirectly by reciprocal measurement of the transfer and cross-transfer mobilities on the receiving structure, without the need for attaching in-line transducers at the machine mounting points. Hence, the forces and vibrational power flow associated with each component can be obtained for analysis. In this study, a class of installation, fan base assemblies on concrete floors is considered as multi-point sources, in this paper, and as multi-component sources in the companion paper.

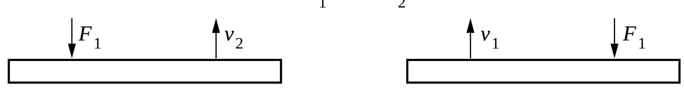
2. RECIPROCITY

It has been more than a century since Rayleigh [1] formulated the general principle of reciprocity for vibrating systems in succession to earlier work by Helmholtz [2]. The development of this principle and the practical applications are, however, not as widespread as one would expect. The most important and recognised application of the principle is in the field of reciprocal calibration of electro-acoustical transducers. Of all the methods for calibrating microphones, the reciprocal technique is the most accurate when the wavelength is large compared with the dimension of the microphone element [3].

Some of the discussions on the validity and application of reciprocity in acoustical and vibro-acoustical systems can be found in references [4–10]. In most cases, the study was confined to vibro-acoustical reciprocity involving measurement of sound radiation and transfer functions. However, Heckl pointed to a use of reciprocity in measurement of the strength of structure-borne sources [11]. A comprehensive review of the development and some applications of the principle were undertaken by Fahy [12]. More recently, Verheij advocated the use of this principle to characterise sources and quantify sound paths [13].

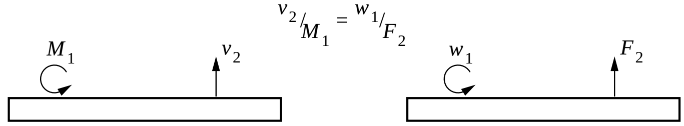
The application of reciprocity in vibration involving forces, moments and translational and rotational velocities is realised as follows.

For Type 1, $$v_2/F_1 = v_1/F_2$$



This relationship states that in a reciprocal system, the transmission of vibration from position 1 to 2 is equal to that from 2 to 1. The source and receiver positions are interchangeable.

For Type 2, $$v_2/M_1 = w_1/F_2$$



The transfer mobility of a translational velocity at position 2, excited by a moment at position 1, is equivalent to that of a force excited rotational velocity in the opposite position.

A common application of Type 1 is in the construction of mobility matrices, where the elements above and below the diagonal are assumed to be symmetrical. The use of Type 2 is relatively rare, for the reason that moments are not usually considered.

The two reciprocal relationships above can be exploited to circumvent the following practical measurement problems: (i) application of a force at the machine footing in mobility measurement; (ii) generation of a known moment in mobility measurement; (iii) direct acquisition of forces and moments generated by vibrating machines at contacts.

3. THEORY AND MEASUREMENT PRINCIPLES

3.1. SINGLE POINT AND DEGREE OF FREEDOM

This section describes a general measurement method which yields the power flow through each excitation point and component by means of an indirect approach. First, consider the simplest case where a machine is connected to a supporting structure at a single mounting point at x_e where the machine can excite only a vertical translational force F_e during operation. Under the action of this force, the response velocity at point x_e is v_e. Following this, the power transmitted via this mounting point is [14]

$$P = \tfrac{1}{2}\,\mathrm{Re}(F_e v_e^*). \tag{1}$$

From the above it can be seen that the power is expressed as a co-spectrum, and can be calculated directly if the cross spectrum of force and velocity at the excitation point is known. In practice, the velocity can be obtained by means of accelerometers but direct measurement of force at the mount is difficult and often impossible. If equation (1) is rearranged, it gives

$$P = \tfrac{1}{2}\,\mathrm{Re}(F_e v_e^*) = \tfrac{1}{2}\,\mathrm{Re}(F_e v_e^* v_r / v_r), \tag{2}$$

where v_r is the response velocity of the supporting plate due to excitation force F_e at point x_e, measured at an arbitrary remote point x_r, when the machine is in operation. Since the power at the excitation point is governed by the dynamic characteristics of the excited structure, the expression now becomes

$$P = \tfrac{1}{2}\,\mathrm{Re}(v_e^* v_r / Y_{vrFe}). \tag{3}$$

The ratio Y_{vrFe} is termed the complex transfer mobility from point x_e to x_r. In the measurement of this transfer function, registration of force at a machine contact point is difficult or not possible, since this point is usually not accessible to force exciters. By the principle of reciprocity, $Y_{vrFe} = Y_{veFr}$ and equation (3) can be written as

$$P = \tfrac{1}{2}\,\mathrm{Re}(v_e^* v_r / Y_{veFr}). \tag{4}$$

The numerator in equation (4) is obtained from the cross spectrum of v_e and v_r. The reciprocal of transfer mobility from x_e to point Y_{veFr}, can be measured without much difficulty. Thus, this arrangement converts the problem of direct measurement of force at the machine and floor interface to a simpler floor transfer mobility measurement.

If there is a sufficiently large mobility mismatch between the source and receiving structure, then Y_{veFr} can be approximated by $\check{Y}_{veFr}$, and the power imparted by the machine to the structure can be calculated without removing the existing installed machine [15]. Y_{veFr} is measured without machine loading and thus point x_e is under the condition of free movement whereas $\check{Y}_{veFr}$ is obtained when point x_e is imposed on by the loading of a machine. If uncorrupted transfer mobilities can be obtained in a laboratory measurement environment or on a typical test structure, then the same mobility matrix can be used for other machines, thereby allowing comparison of the structure-borne "noisiness" of different machines on the same receiving structure.

3.2. MULTIPLE POINTS AND DEGREES OF FREEDOM

Direct measurement of force is difficult as force transducers are required to be attached at the machine and floor interface, causing disruption to the normal installed condition. Also, real installations rarely consist of one contact point with pure (vertical) force excitation only and the problem arises of the measurement of moments, aggravated by the non-availability of a transducer to directly register moment. A cross-spectral intensity method proposed by Pinnington and White [16] eliminates the need for direct measurement of force. In their approach, force is estimated from the displacement at the top of the mount and the mount dynamic stiffness. However, this method is restricted to resiliently mounted machines.

This section describes an expansion of the method proposed earlier to the case of multiple points and degrees of freedom system, where the rotational components can be taken into account to estimate the excitation moments and their respective power. Consider now a structure excited by a combined force and moment through one contact point, resulting in a translational and rotational response. Power is expressed as

$$P = \tfrac{1}{2}\mathrm{Re}(F_e v_e^* + M_e w_e^*) = \tfrac{1}{2}\mathrm{Re}\left(\{v_e^* \quad w_e^*\}\begin{Bmatrix} F_e \\ M_e \end{Bmatrix}\right). \tag{5}$$

F_e and M_e are respectively the force and moment excited at the point by the machine. The translational and rotational velocities, v_e and w_e at the mounting point are the results of combined force and moment contributions. Again, two arbitrary points x_{r_1} and x_{r_2} are selected for measurement of remote translational velocities v_{r_1} and v_{r_2} when the machine is in operation. With the introduction of the mobility matrix, it follows that

$$\begin{pmatrix} Y_{v_{r_1}F_e} & Y_{v_{r_1}M_e} \\ Y_{v_{r_2}F_e} & Y_{v_{r_2}M_e} \end{pmatrix}\begin{Bmatrix} F_e \\ M_e \end{Bmatrix} = \begin{Bmatrix} v_{r_1} \\ v_{r_2} \end{Bmatrix}, \quad \begin{Bmatrix} F_e \\ M_e \end{Bmatrix} = \begin{bmatrix} Y_{v_{r_1}F_e} & Y_{v_{r_1}M_e} \\ Y_{v_{r_2}F_e} & Y_{v_{r_2}M_e} \end{bmatrix}^{-1}\begin{Bmatrix} v_{r_1} \\ v_{r_2} \end{Bmatrix}. \tag{6}$$

Substituting equation (6) into equation (5) yields

$$P = \tfrac{1}{2}\mathrm{Re}\left(\{v_e^* \quad w_e^*\}\begin{bmatrix} Y_{v_{r_1}F_e} & Y_{v_{r_1}M_e} \\ Y_{v_{r_2}F_e} & Y_{v_{r_2}M_e} \end{bmatrix}^{-1}\begin{Bmatrix} v_{r_1} \\ v_{r_2} \end{Bmatrix}\right). \tag{7}$$

Two of the mobility elements in the above 2×2 matrix are cross-transfer mobilities which would normally involve generation of moments during measurements. This can be avoided if the reciprocity relationships are considered, which yields

$$Y_{v_{r_1}F_e} = Y_{v_eF_{r_1}} \approx \check{Y}_{v_eF_{r_1}}, \quad Y_{v_{r_1}M_e} = Y_{w_eF_{r_1}} \approx \check{Y}_{w_eF_{r_1}},$$
$$Y_{v_{r_2}F_e} = Y_{v_eF_{r_2}} \approx \check{Y}_{v_eF_{r_2}}, \quad Y_{v_{r_2}M_e} = Y_{w_eF_{r_2}} \approx \check{Y}_{w_eF_{r_2}}.$$

In order to obtain the phase relationships between the velocities at different points and components, x_{r_1} is arbitrarily selected as the reference. Any point or component can be chosen as the reference but for numerical and practical convenience, a vertical translational response in the far field is preferred. With the machine in operation, the velocity transfer functions between v_{r_1} and all other points are measured:

$$v_e/v_{r_1} = \varphi(v_{r_1}, v_e), \quad v_e^* = \varphi^*(v_{r_1}, v_e)v_{r_1}^*,$$
$$w_e/v_{r_1} = \varphi(v_{r_1}, w_e), \quad w_e^* = \varphi^*(v_{r_1}, w_e)v_{r_1}^*,$$
$$v_{r_2}/v_{r_1} = \varphi(v_{r_1}, v_{r_2}), \quad v_{r_2}^* = \varphi^*(v_{r_1}, v_{r_2})v_{r_1}^*.$$

Here φ represents the complex frequency dependent transfer or cross-transfer functions between velocities and angular velocities of points indicated in parentheses. The asterisk denotes complex conjugate. From equation (7),

$$P = \frac{1}{2}\mathrm{Re}\left(|v_{r_1}|^2\{\varphi^*(v_{r_1}, v_e) \quad \varphi^*(v_{r_1}, w_e)\}\begin{bmatrix} Y_{v_{r_1}F_e} & Y_{w_eF_{r_1}} \\ Y_{v_{r_2}F_e} & Y_{w_eF_{r_2}} \end{bmatrix}^{-1}\begin{Bmatrix} 1 \\ \varphi(v_{r_1}, v_{r_2}) \end{Bmatrix}\right). \quad (8)$$

By virtue of reciprocity, the moment cross-transfer mobilities have been replaced by their associated force cross-transfer mobilities. The mobilities required in the above matrix are not difficult to measure and the direct measurement of moment mobility is avoided.

If the above condition is expanded to n contact points, each with two components, say, then the number of remote points required will be $2\times n$ with, as before, one point as the reference:

$$P = \tfrac{1}{2}\mathrm{Re}$$
$$\times\left(|v_{r_1}|^2\{\varphi^*(v_{r_1},v_{e_1}),\varphi^*(v_{r_1},w_{e_1})\ldots\varphi^*(v_{r_1},v_{e_n}),\varphi^*(v_{r_1},w_{e_n})\}[Y]_{2n\times 2n}^{-1}\begin{Bmatrix} 1 \\ \varphi(v_{r_1}, v_{r_2}) \\ \cdot \\ \cdot \\ \varphi(v_{r_1}, v_{r_{2n}}), \end{Bmatrix}\right). \quad (9)$$

In principle, six degrees of freedom can be considered for each point and this will require $6\times n$ remote points.

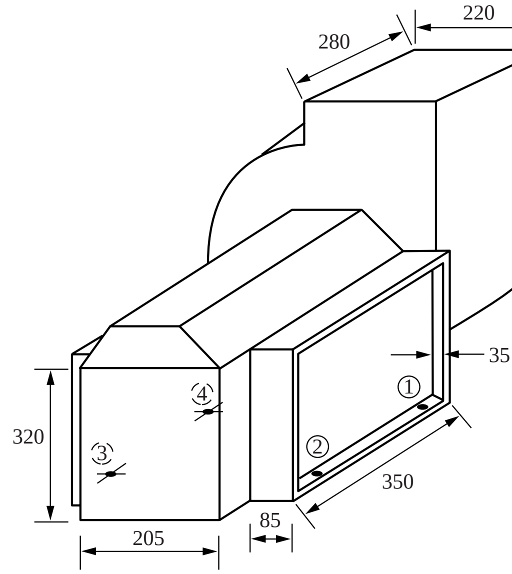


Figure 1. Centrifugal fan. Dimensions in mm.

4. EXPERIMENTAL VALIDATION

To examine the practicality of the indirect method, experiments were carried out on a 120-mm concrete plate of dimensions $4 \cdot 8 \times 3$ m. This method entails the acquisition of transfer mobilities between the excitation and selected remote points. For realistic representation of field measurement conditions, a centrifugal fan, shown in Figure 1, was rigidly fixed to the structure at its four connection points prior to the measurement of the receiving structure transfer mobilities.

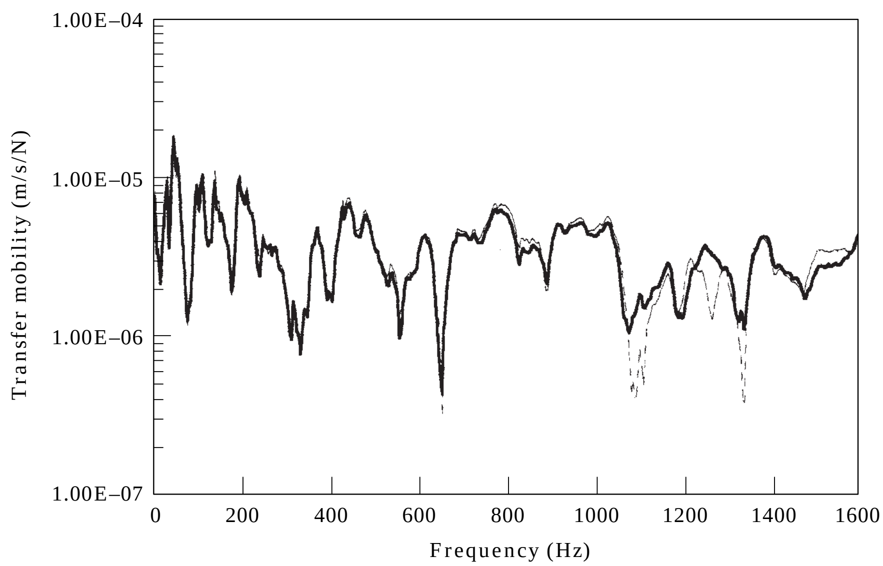


Figure 2. Magnitudes of transfer mobility between excitation and remote point. —, Directly (without fan loading); - - -, reciprocally (with fan loading).

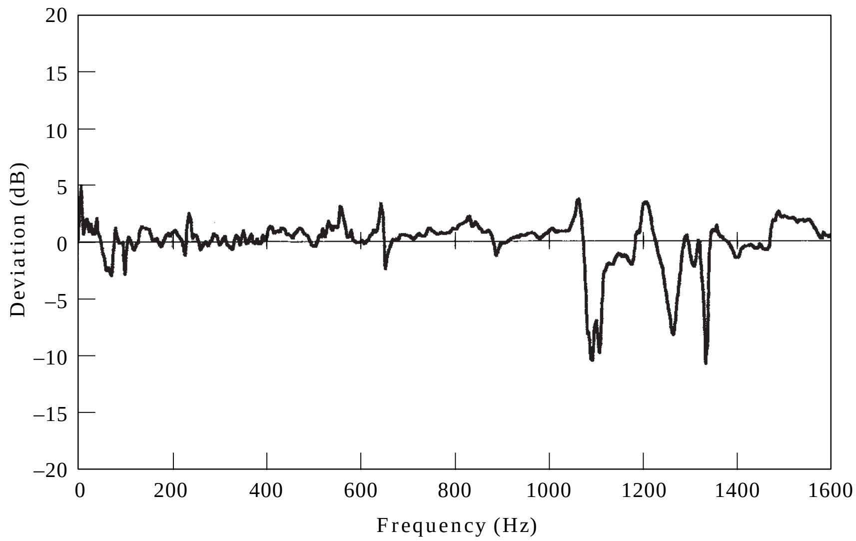


Figure 3. Deviation between the transfer mobilities.

The floor was then excited reciprocally at the arbitrary remote points and the response velocity was acquired by means of accelerometer pairs at the connection points.

Figure 2 shows the spectra of Y_{vrFe} and Y_{veFr} measured by an impulse hammer. The transfer mobility $\check{Y}_{veFr}$ is generally reciprocal within the margin of experimental error. The deviation between the two spectra is bounded by ± 2 dB, as seen in Figure 3 except in the region between 1050 and 1350 Hz which was possibly due to the excitation or response being in the vicinity of a nodal point on the plate. This occurred because a mode cannot be excited by a force at a node, though the force registered by the transducer appears large. This discrepancy will also arise when the response transducer is at a node. This hypothesis is confirmed in Figure 4, where it is seen that although the spectrum of the force applied by the impulse hammer was "flat", the response velocity at x_e shows dips between 1·1 and 1·4 kHz. Not shown are measurement results which were taken for four remote points, all of which indicated that the repeatability of the measurements was within ± 1 dB, up to 1100 Hz.

4.1. SINGLE CONTACT

In the initial validation measurement, it was necessary to eliminate the effect of airborne, moment and multipoint excitation. Therefore, it was of the simplest case of a single contact excitation, with a vertical translational component only and without airborne excitation of the floor by the fan. Measurement in this case was relatively simple as only one transfer mobility between a selected remote point and the mounting point was required. The fan was then freely suspended by an isolated frame with one of the feet in contact to impart vibrational energy into the concrete structure. Figure 5 shows the experimental set-up. A 12-mm timber board with 20 mm foam lining was used as a seating base of the suspended fan and the fan was enclosed by a foam lined box to reduce airborne excitation. In order to eliminate extraneous moment excitation, force from the

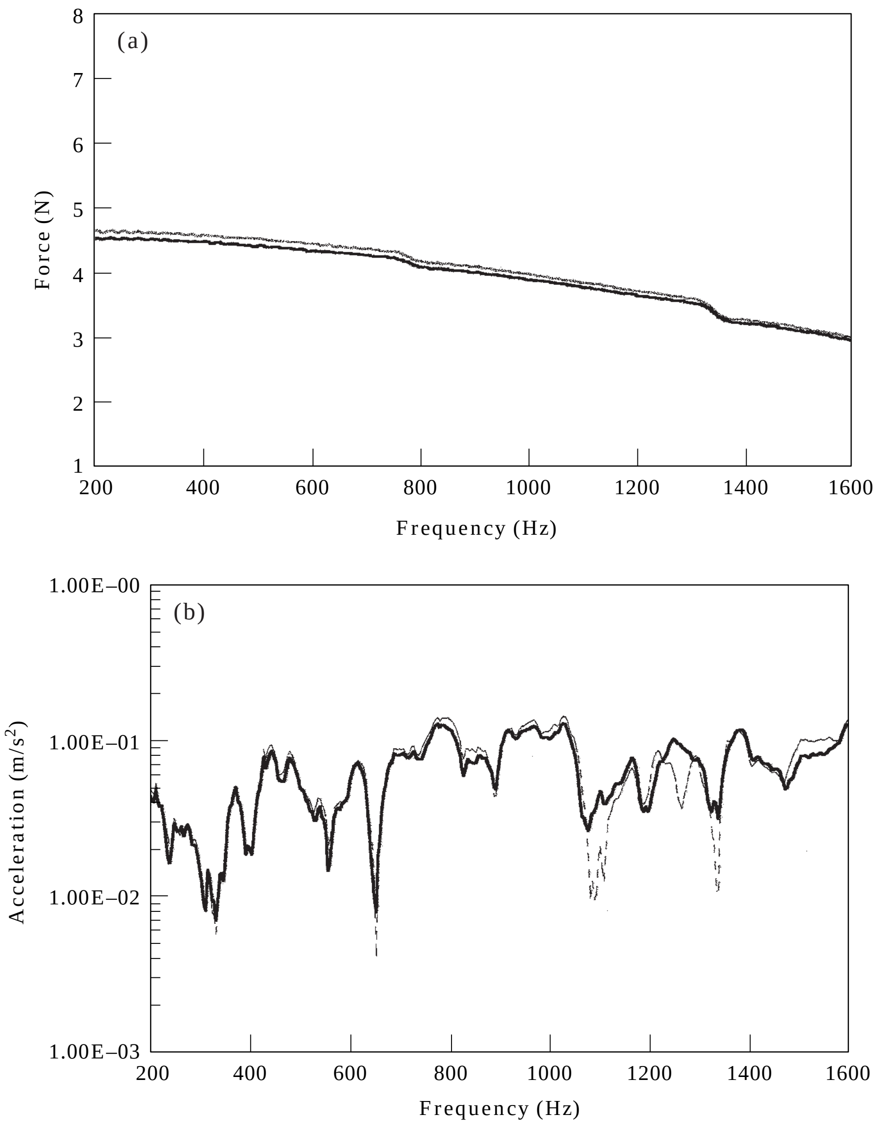


Figure 4. Magnitudes of (a) force and (b) acceleration, in the directly (excitation at x_e, —) and reciprocally excitation at x_r, - - -) measured transfer mobilities.

operating fan was applied to the concrete structure via a driving rod of diameter 1 mm, to a force transducer which was attached to the floor. Hence, the power transmitted into the structure was measured directly for comparison with that obtained indirectly. This is referred to as the direct method which represents the ideal case where an installed machine can be lifted up enabling a force transducer to be fixed at the mounting point. In indirect measurement, instead of registering the force spectrum directly, the force transducer simply acts as a passive bridge between the machine and the floor.

The machine was set to run and the cross-spectrum of acceleration between the two points was obtained. The power transmitted into the structure was calculated by using equation (1) for the case of direct measurement. By using the

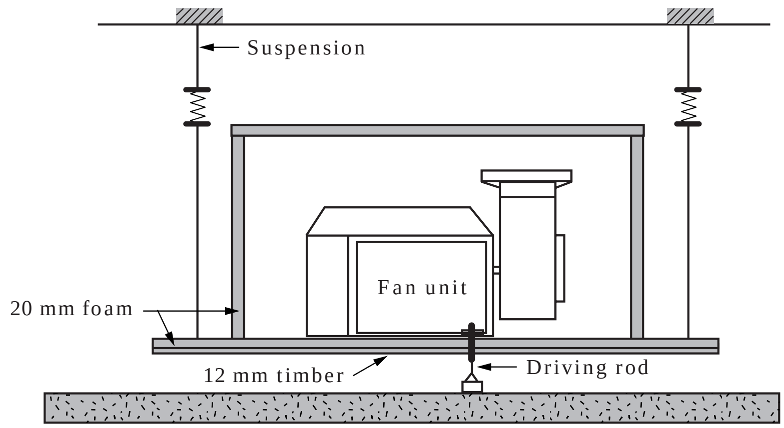


Figure 5. Fan on sound-absorptive foam and timber board.

measured transfer functions in the indirect approach, power was also calculated from equation (4). Shown in Figures 6 and 7 are the force and power spectra obtained by both methods. It is shown in Figure 8 that the deviation between directly and indirectly estimated power in general falls within ± 3 dB, up to 1 kHz and the fluctuation is evenly distributed between the positive and negative regions. Above 1 kHz the discrepancy between the indirect and direct measurements increases with negative power flow increasingly indicated, represented by the discontinuities in the power spectrum.

The increased discrepancy above 1 kHz is due to a combination of factors: low signal to noise, unavoidable excitation by other components and paths, and increasing errors in the transfer functions. The thin connecting rod, used to eliminate moment, had reduced the bandwidth of excitation at the same time. In Figure 7, the power above 1 kHz is more than 30 dB down on the maximum value at low frequencies. Figure 8 shows that, above 1 kHz, the power is

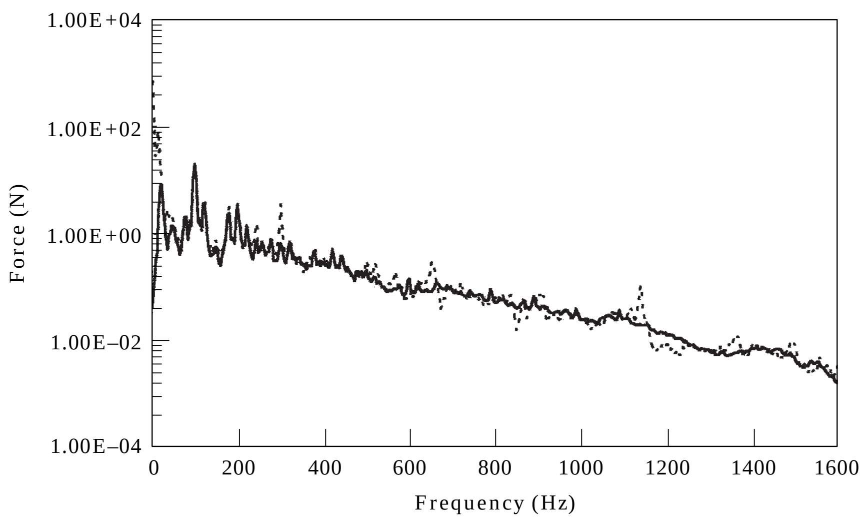


Figure 6. Magnitudes of directly (—) and indirectly (- - -) measured force.

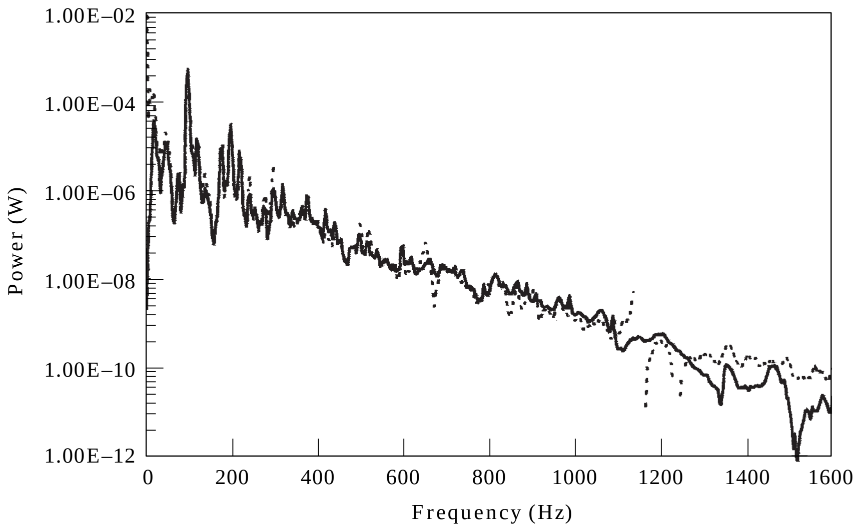


Figure 7. Force induced power transmitted through a driving rod. Key as Figure 6.

overestimated by the indirect method. As the force induced power decreases, other forms of excitation will assume increased relative importance and the floor plate will be energised other than through the contact point. This can lead to negative power flow at some frequencies. Observation from several measurements concluded that a persistent negative flow near 300 Hz was attributable to a flanking path from a support frame from which the fan was suspended. It was suspected that the large discrepancies above 1.3 kHz, were the result of combined force and moment excitation since the floor is more receptive to moment excitation.

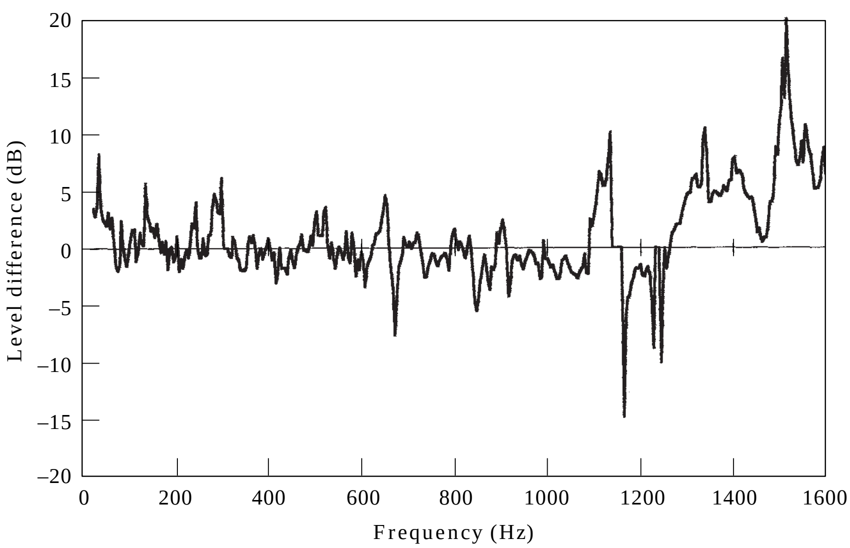


Figure 8. Level difference between directly and indirectly measured powers.

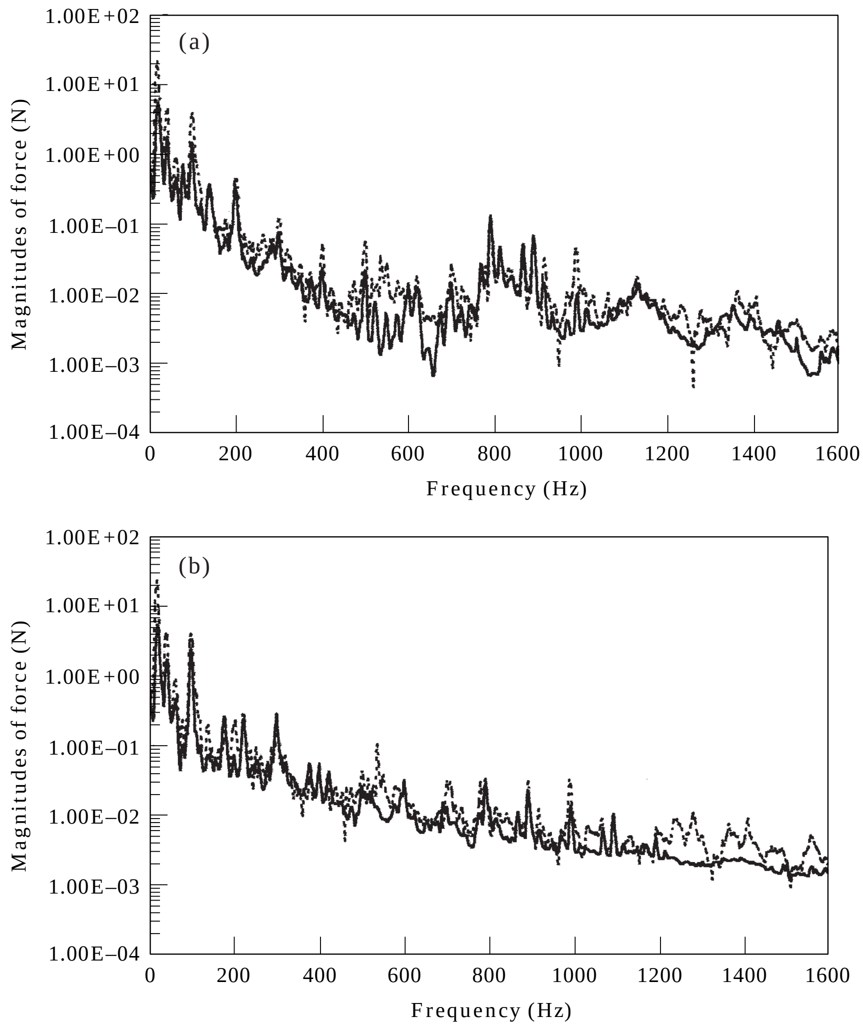


Figure 9(a) and (b)—*Caption on next page.*

The indirect method requires uncorrupted measured transfer mobilities for the estimate of force. Figure 3 shows that discrepancies between direct and reciprocally measured transfer mobility is greatest above 1 kHz. This was the case for all remote accelerometer positions selected.

To summarise: for the bandwidth of excitation, 0–1 kHz, the agreement between directly and indirectly measured power is within ± 3 dB.

4.2. MULTIPLE CONTACTS

The results for single point, single component excitation were encouraging, and it remained to explore the practicalities of the method by considering vertical forces at multiple supports. This again allowed comparison of direct and indirect measurements before considering moments where such comparisons are not possible. In the multi-point measurement, the four feet were connected to the

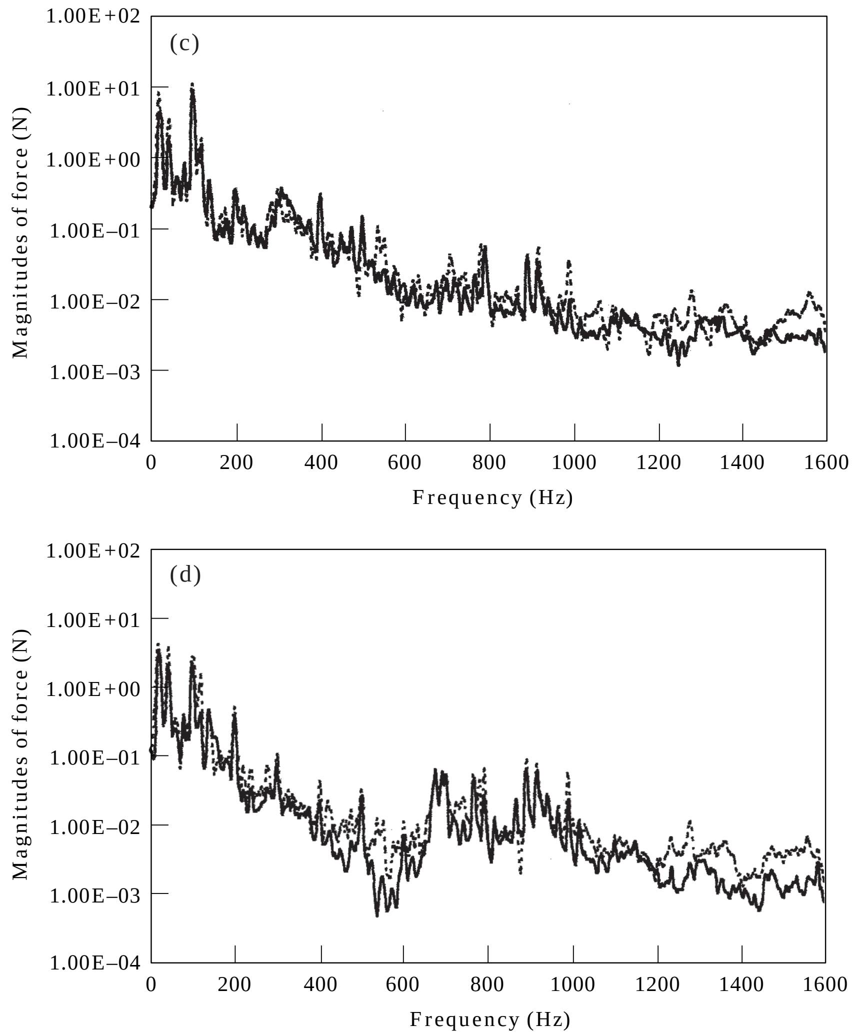


Figure 9. Magnitudes of directly (—) and indirectly (- - -) measured force. (a) Foot 1; (b) foot 2; (c) foot 3; (d) foot 4.

concrete floor via thin driving rods. Again, airborne excitation of the floor was reduced by acoustically enclosing the fan unit. A 4×4 transfer mobility matrix was required and three velocity transfer functions between the reference and the remaining arbitrary points were obtained when the fan was operating. Comparison of force rather than the power spectra is more appropriate in this case, as powers obtained by both methods are likely to encompass the response due to other excitation sources or components not considered here.

Figures 9(a) to (d) show the force spectra at the four mount points. It is seen that in all cases, the signatures of the directly and indirectly measured forces are similar up to 1 kHz. Contacts 2 and 3, which were nearer to the fan motor than contacts 1 and 4, produce higher forces, with less variation with frequency.

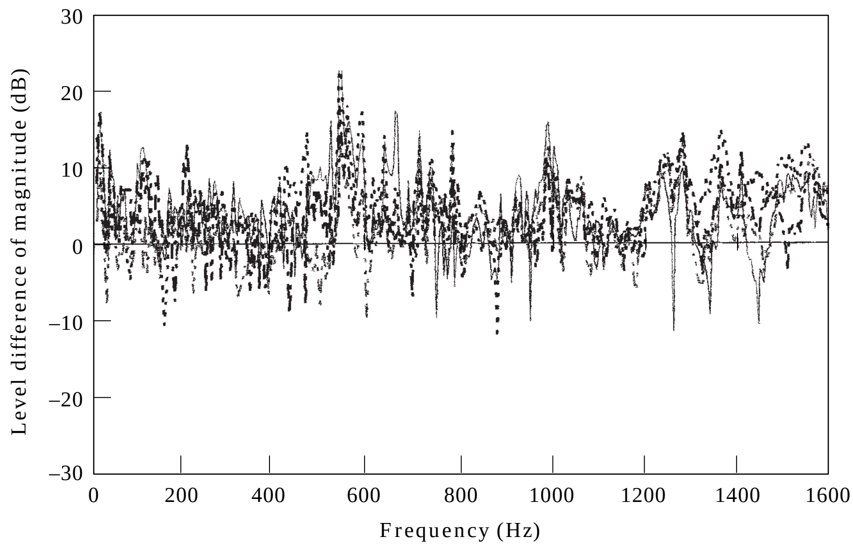


Figure 10. Level differences between directly and indirectly measured forces. —, Foot 1; -·-·-, foot 2; -···-, foot 3; ----, foot 4.

Discrepancies are again interpreted as due to the contribution from the simultaneously excited moment components. Complete elimination of moment was not possible, as relatively thick connecting rods (2 mm diameter) were used to ensure strong response signals. In addition, alignment of the four rods with the force transducers was difficult to maintain, due to the fan motion when in operation. In short, generation of four pure forces at the mount points was practically difficult.

Figure 10 depicts the deviation of the four estimated forces in comparison with the forces acquired directly by the force transducers. The deviations are positively biased by approximately 3 dB. Below 1 kHz, the deviations are within ± 8 dB, except at frequencies near 600 Hz. These frequencies corresponded to the condition when the difference between the contact forces was a maximum. Deviations above 1 kHz are within ± 10 dB and have a positive bias of approximately 5 dB.

5. CONCLUSIONS

The reciprocity principle, when applied in conjunction with the simple impulse response method, allows forces and moments at the feet of a machine to be measured indirectly. In the absence of a direct method for measurement of moment, this method is particularly useful. In addition, the need to insert a force transducer at the interface of machine and structure is obviated.

Validation of this method was undertaken by considering the simple case of a single point and component of excitation (vertical force) from a centrifugal fan. The practical feasibility of this method was confirmed when the results showed that discrepancies between directly and indirectly measured power were within ± 3 dB for the bandwidth of interest.

The presence of unavoidable airborne excitation and other components of excitation at the contacts, is highlighted by negative power flows, indicating that the floor is energised predominantly by these other transmission paths.

The main experimental difficulty, for both single point and multi-point contacts, was the application of a pure force via a driving rod. A thin rod will not generate a sufficiently large force whereas a thick rod will generate an unwanted moment simultaneously.

In the following companion paper (Part 2) the method is developed to include moment and force excitation in an investigation of the effect of source location on the moment excitation.

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