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FREE VIBRATION ANALYSIS OF ROTATING TWISTED CYLINDRICAL THIN PANELS

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Blades are idealized as twisted cylindrical thin panels and the free vibrations of rotating blades are studied in this paper. First the steady deformation due to rotation is studied by using the principle of virtual work for deformation, then considering the initial deformations and the initial stress resultants, the vibration of the panel is analyzed by using the principle of virtual work for free vibration. The numerical procedure for analyzing the free vibrations of rotating twisted cylindrical thin panels is presented by the Rayleigh–Ritz method. The solutions are shown in non-dimensional frequency parameters and compared with the previous results. Finally, the effects of twist angle, center angle, setting angle, rotating speed and radius of rotating disc on the vibrations are investigated.

1. INTRODUCTION

As one knows, it is very important to study the vibrations of blades which rotate at high speed. There are many studies on the vibrations of rotating blades. In most of these, the blades are generally idealized as twisted cantilever plates and shells, and the finite element technique has been used to analyze the vibrations of the rotating blades [1–6]. Another useful and widely used method is the Rayleigh–Ritz method. Leissa *et al.* [7] proposed a shallow shell model of blades and derived a numerical method for analyzing the vibrations of rotating blades by using the Rayleigh–Ritz method. It is not adequate for analyzing shells having large curvature and twist, because it is based on the thin shallow shell theory.

By using general thin shell theory, Tsuiji [8] presented a numerical procedure for the vibration analysis of rotating twisted thin plates by applying the Rayleigh–Ritz method. For typical twisted thin plates, the effects of twist angle, rotating speed, setting angle and radius of rotating disc on the vibration characteristics were studied in detail. It can be seen that the procedure is applicable to the analysis of the vibrations of plates having a large twist.

The main purpose of this paper is to present a numerical procedure which is adequate for analyzing the vibrations of rotating twisted cylindrical thin panels,

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even if the panels have large twist and curvature. By using reference [9], non-linear strain–displacement relations of rotating twisted cylindrical thin panels are proposed with considering the initial displacements due to centrifugal forces. The governing equations for the free vibrations of rotating twisted cylindrical thin panels are presented by the principle of virtual work and the Rayleigh–Ritz method assuming two dimensional algebraic polynomials as displacement functions. The procedure consists of two steps: first, the deformations caused by centrifugal forces are evaluated by an iteration method; second, by considering the initial displacements and initial stress resultants, the vibration characteristics of rotating twisted cylindrical thin panels are analyzed. The results obtained by the present procedure are compared with previous data [7, 8]. The effects of twist angle, center angle, setting angle, rotating speed and radius of rotating disc on the vibration characteristics are investigated.

2. STRAINS WITH INITIAL DEFORMATION

A blade is treated as a twisted cylindrical thin panel mounted on the periphery of a rotating disc of radius x_0 with a setting angle ϕ, as shown in Figure 1. There are two rectangular Cartesian right hand co-ordinate systems employed, one is a (x, y, z') with the origin O which is a point on the z'-axis. The x-axis is along the length of the panel and the twisting center axis, the z'-axis takes up the radial direction where the cylindrical arc is equally divided into two parts, and the y-axis is selected in such a way that the x-, y- and z'-axes form a right hand system. The other system is (X, Y, Z) where the X- and Y-axes are in the horizontal plane, and the panel system rotates around the Z-axis at an angular velocity Ω (rad/s). Ω_1, r and b are the center angle, average radius and arc length of the cylindrical panel respectively. O_1 is the center of the cylinder and θ is the angle measured from the z'-axis. t is thickness, l is the length along the x-axis, k is the twist angle per unit length and kl is the twist angle at the free end of the panel. e is the distance between the two points O and O_1, and the z-axis is a normal of the middle surface and the outward direction is defined as positive. $\mathbf{i}_1$, $\mathbf{i}_2$ and $\mathbf{i}_3$ are the unit vectors of the co-ordinate system (x, y, z').

The strain–displacement relations for twisted cylindrical thin panels can be obtained from reference [9] where the curvature of the x-axis becomes zero, which are given by the following:

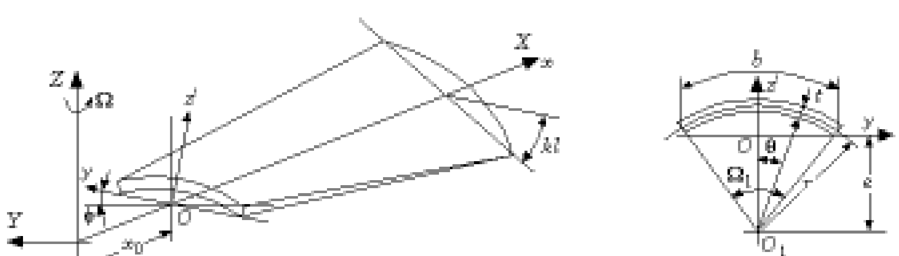

Figure 1. A model of rotating twisted cylindrical thin panels for rotating blades.

$$\varepsilon'_{\xi\xi} = (1/F)\mathbf{Z}\mathbf{G}_{x1}\mathbf{U} + \tfrac{1}{2}(\mathbf{G}_{x2}\mathbf{U})^{\mathrm{T}}(\mathbf{G}_{x2}\mathbf{U}), \qquad \varepsilon'_{\eta\eta} = (1/F)\mathbf{Z}\mathbf{G}_{\theta 1}\mathbf{U} + \tfrac{1}{2}(\mathbf{G}_{\theta 2}\mathbf{U})^{\mathrm{T}}(\mathbf{G}_{\theta 2}\mathbf{U}),$$

$$\gamma'_{\xi\eta} = (1/F)\mathbf{Z}\mathbf{G}_{x\theta}\mathbf{U} + (\mathbf{G}_{x2}\mathbf{U})^{\mathrm{T}}(\mathbf{G}_{\theta 2}\mathbf{U}), \qquad \gamma'_{\xi\zeta} = 0, \qquad \gamma'_{\eta\zeta} = 0, \qquad \varepsilon\zeta = 0, \quad (1)$$

where (ξ, η, ζ) is a local co-ordinate system with respect to a point. Matrices $\mathbf{Z}$, $\mathbf{G}_{x1}$, $\mathbf{G}_{\theta 1}$, $\mathbf{G}_{x\theta}$ and $\mathbf{U}$ are expressed as

$$\mathbf{Z} = \begin{bmatrix} 1 & \dfrac{z}{l} & \dfrac{z^2}{l^2} \end{bmatrix},$$

$$\mathbf{U}^{\mathrm{T}} = \begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial \theta} & u & \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial \theta} & v & \dfrac{\partial^2 w}{\partial x^2} & \dfrac{\partial^2 w}{\partial \theta^2} & \dfrac{\partial^2 w}{\partial x \partial \theta} & \dfrac{\partial w}{\partial x} & \dfrac{\partial w}{\partial \theta} & w \end{bmatrix},$$

$$\mathbf{G}_{x1} = [{}_{x1}G_{i,j}], \quad \mathbf{G}_{\theta 1} = [{}_{\theta 1}G_{i,j}], \quad \mathbf{G}_{x\theta} = [{}_{x\theta}G_{i,j}], \quad (i = 1, 2, 3;\ j = 1, \ldots, 12), \quad (2)$$

the non-zero elements of matrices $\mathbf{G}_{x1}$, $\mathbf{G}_{\theta 1}$ and $\mathbf{G}_{x\theta}$ are

$${}_{x1}G_{1,1} = B, \quad {}_{x1}G_{1,2} = \frac{Bk}{r}(r - e\cos\theta), \quad {}_{x1}G_{1,6} = \frac{e^2k^2}{Br}\sin\theta\cos\theta,$$

$${}_{x1}G_{1,12} = -\frac{ek^2}{B^2r}(r - e\cos\theta)\cos\theta, \quad {}_{x1}G_{2,1} = \frac{1}{r}, \quad {}_{x1}G_{2,2} = \frac{k}{r},$$

$${}_{x1}G_{2,4} = -\frac{ek}{B^2r}\cos\theta, \quad {}_{x1}G_{2,5} = \frac{ek^2}{B^2r^2}(e\cos\theta - r)\cos\theta,$$

$${}_{x1}G_{2,6} = \frac{ek^2}{B^2r}\left[1 + \frac{2e^2k^2}{B^2r}(r - e\cos\theta)\cos^2\theta\right]\sin\theta, \quad {}_{x1}G_{2,7} = -\frac{1}{B},$$

$${}_{x1}G_{2,8} = -\frac{k^2}{Br^2}(e\cos\theta - r)^2, \quad {}_{x1}G_{2,9} = \frac{2k}{Br}(e\cos\theta - r),$$

$${}_{x1}G_{2,10} = \frac{e^2k^3}{B^3r}(r - e\cos\theta)\sin\theta\cos\theta,$$

$${}_{x1}G_{2,11} = \frac{ek^2}{Br}\left[\frac{ek^2}{B^2r}(e\cos\theta - r)^2\cos\theta - 1\right]\sin\theta, \quad {}_{x1}G_{2,12} = -\frac{ek^2}{B^3r}\cos\theta,$$

$${}_{x1}G_{3,4} = -\frac{ek}{B^3r^2}\cos\theta, \quad {}_{x1}G_{3,5} = -\frac{ek^2}{B^3r^2}\cos\theta, \quad {}_{x1}G_{3,7} = -\frac{1}{B^2r},$$

$${}_{x1}G_{3,8} = \frac{k^2}{B^2r^2}(e\cos\theta - r), \quad {}_{x1}G_{3,9} = \frac{k}{B^2r^2}(e\cos\theta - 2r),$$

$${}_{x1}G_{3,10} = \frac{e^2k^3}{B^4r}\sin\theta\cos\theta, \quad {}_{x1}G_{3,11} = -\frac{ek^2}{B^2r^2}\left[1 + \frac{ek^2}{B^2}(e\cos\theta - r)\cos\theta\right]\sin\theta,$$

$${}_{\theta 1}G_{1,2} = \frac{Bk}{r}(e\cos\theta - r), \quad {}_{\theta 1}G_{1,5} = \frac{B}{r}, \quad {}_{\theta 1}G_{1,12} = \frac{1}{r},$$

$$ {}_{\theta 1}G_{2,1} = \frac{ek^2}{B^2 r}(e\cos\theta - r)\cos\theta, \quad {}_{\theta 1}G_{2,2} = -\frac{k}{r}, \quad {}_{\theta 1}G_{2,4} = \frac{ek}{B^2 r}\cos\theta, $$

$$ {}_{\theta 1}G_{2,5} = \frac{1}{r^2}, \quad {}_{\theta 1}G_{2,6} = -\frac{2e^2k^2}{B^2r^2}\sin\theta\cos\theta, \quad {}_{\theta 1}G_{2,8} = -\frac{B}{r^2}, $$

$$ {}_{\theta 1}G_{2,10} = -\frac{ek}{Br}\sin\theta, \quad {}_{\theta 1}G_{2,11} = \frac{ek^2}{Br^2}(e\cos\theta - r)\sin\theta, $$

$$ {}_{\theta 1}G_{2,12} = -\frac{ek^2}{B^3 r}\cos\theta, \quad {}_{\theta 1}G_{3,1} = -\frac{ek^2}{B^3 r}\cos\theta, \quad {}_{\theta 1}G_{3,4} = \frac{ek}{B^3 r^2}\cos\theta, $$

$$ {}_{\theta 1}G_{3,6} = \frac{e^3k^4}{B^5r^2}\sin\theta\cos^2\theta, \quad {}_{\theta 1}G_{3,9} = -\frac{ek}{B^2r^2}\cos\theta, $$

$$ {}_{\theta 1}G_{3,10} = \frac{e^2k^3}{B^4 r}\sin\theta\cos\theta, \quad {}_{\theta 1}G_{3,11} = -\frac{e^2k^4}{B^4r^2}(e\cos\theta - r)\sin\theta\cos\theta, $$

$$ {}_{x\theta}G_{1,1} = k(e\cos\theta - r), \quad {}_{x\theta}G_{1,2} = \frac{1}{r}[B^2 - k^2(e\cos\theta - r)^2], \quad {}_{x\theta}G_{1,4} = 1, $$

$$ {}_{x\theta}G_{1,5} = \frac{k}{r}(r - \cos\theta), \quad {}_{x\theta}G_{1,6} = -\frac{ek}{r}\sin\theta, \quad {}_{x\theta}G_{1,12} = -\frac{2ek}{Br}\cos\theta, $$

$$ {}_{x\theta}G_{2,1} = \frac{2k}{Br}(e\cos\theta - r), \quad {}_{x\theta}G_{2,2} = -\frac{2k^2}{Br}(r - e\cos\theta), \quad {}_{x\theta}G_{2,4} = \frac{2}{Br}, $$

$$ {}_{x\theta}G_{2,5} = \frac{2k}{Br^2}(r - e\cos\theta), \quad {}_{x\theta}G_{2,6} = \frac{4e^3k^3}{B^3r^2}\sin\theta\cos^2\theta, $$

$$ {}_{x\theta}G_{2,8} = \frac{2k}{r^2}(e\cos\theta - r), \quad {}_{x\theta}G_{2,9} = -\frac{2}{r}, \quad {}_{x\theta}G_{2,10} = \frac{2e^2k^2}{B^2 r}\sin\theta\cos\theta, $$

$$ {}_{x\theta}G_{2,11} = -\frac{2e^2k^3}{B^2r^2}(e\cos\theta - r)\sin\theta\cos\theta, \quad {}_{x\theta}G_{3,7} = -\frac{ek}{B^3 r}\cos\theta, $$

$$ {}_{x\theta}G_{3,8} = -\frac{k}{Br^2}, \quad {}_{x\theta}G_{3,9} = \frac{1}{Br^2}\left[\frac{ek^2}{B^2}(e\cos\theta - r)\cos\theta - 1\right], $$

$$ {}_{x\theta}G_{3,11} = -\frac{e^2k^3}{B^3r^2}\sin\theta\cos\theta, \tag{3} $$

and matrices $\mathbf{G}_{x2}$, $\mathbf{G}_{\theta 2}$ are expressed as

$$ \mathbf{G}_{x2} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad (ek/B^2r)\cos\theta \quad 0 \quad 0 \quad 0 \quad (1/B) \ (k/Br) \ (r - e\cos\theta) \quad 0], $$
$$ \mathbf{G}_{\theta 2} = [0 \quad 0 \quad k/B \quad 0 \quad 0 \quad -1/Br \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0], \tag{4} $$

where u, v and w are the displacement components of a point on the middle surface of the panel, and coefficients B, F and p are defined as follows:

$$B = 1 + e^2k^2 \sin\theta, \quad F = B(1 + zp), \quad p = (1/Br)[1 - (ek^2/B^2)(r - e\cos\theta)\cos\theta]. \tag{5}$$

It is necessary to consider the strains with initial deformations for analyzing the vibrations of rotating blades. From equation (1), the strains of the twisted panel having initial deformations are given by

$$\varepsilon_{\xi\xi} = \varepsilon^L_{\xi\xi} + \varepsilon^N_{\xi\xi}, \quad \varepsilon_{\eta\eta} = \varepsilon^L_{\eta\eta} + \varepsilon^N_{\eta\eta}, \quad \gamma_{\xi\eta} = \gamma^L_{\xi\eta} + \gamma^N_{\xi\eta}, \quad \gamma_{\xi\zeta} = 0, \quad \gamma_{\eta\zeta} = 0, \quad \varepsilon_{\zeta\zeta} = 0, \tag{6}$$

where $()_L$, $()_N$ denote linear and non-linear strain components respectively, represented by

$$\begin{aligned}
\varepsilon^L_{\xi\xi} &= (1/F)\mathbf{Z}[\mathbf{G}_{x1} + B\mathbf{H}(\mathbf{U}^0)^{\mathrm{T}}\mathbf{G}^{\mathrm{T}}_{x2}\mathbf{G}_{x2}]\mathbf{U} = (1/F)\mathbf{Z}\bar{\mathbf{G}}_{x1}\mathbf{U},\\
\varepsilon^N_{\xi\xi} &= \tfrac{1}{2}(\mathbf{G}_{x2}\mathbf{U})^{\mathrm{T}}(\mathbf{G}_{x2}\mathbf{U}),\\
\varepsilon^L_{\eta\eta} &= (1/F)\mathbf{Z}[\mathbf{G}_{\theta 1} + B\mathbf{H}(\mathbf{U}^0)^{\mathrm{T}}\mathbf{G}^{\mathrm{T}}_{\theta 2}\mathbf{G}_{\theta 2}]\mathbf{U} = (1/F)\mathbf{Z}\bar{\mathbf{G}}_{\theta 1}\mathbf{U},\\
\varepsilon^N_{\eta\eta} &= \tfrac{1}{2}(\mathbf{G}_{\theta 2}\mathbf{U})^{\mathrm{T}}(\mathbf{G}_{\theta 2}\mathbf{U}),\\
\gamma^L_{\xi\eta} &= (1/F)\mathbf{Z}[\mathbf{G}_{x\theta} + B\mathbf{H}(\mathbf{U}^0)^{\mathrm{T}}(\mathbf{G}^{\mathrm{T}}_{x2}\mathbf{G}_{\theta 2} + \mathbf{G}^{\mathrm{T}}_{\theta 2}\mathbf{G}_{x2})]\mathbf{U} = (1/F)\mathbf{Z}\bar{\mathbf{G}}_{x\theta}\mathbf{U},\\
\gamma^N_{\xi\eta} &= (\mathbf{G}_{x2}\mathbf{U})^{\mathrm{T}}(\mathbf{G}_{\theta 2}\mathbf{U}), \quad \gamma_{\xi\zeta} = 0, \quad \gamma_{\eta\zeta} = 0, \quad \varepsilon_{\zeta\zeta} = 0,
\end{aligned} \tag{7}$$

$\mathbf{U}_0$ is the initial deformation matrix due to centrifugal forces, and matrix $\mathbf{H}$ is defined as

$$\mathbf{H}^{\mathrm{T}} = [1 \quad p \quad 0]. \tag{8}$$

3. PROCEDURE ANALYZING FREE VIBRATION

For a thin panel, changes in centrifugal forces in the thickness direction can be neglected. Hence, deformation the position vector $\mathbf{r}$ of an arbitrary point in the rotating twisted cylindrical thin panel is given by

$$\begin{aligned}
\mathbf{r} &\doteq \left(\begin{bmatrix} x_0 \\ r\sin\theta \\ r\cos\theta - e \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} u \\ v \\ w \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 1 & 0 & -ek\sin\theta/B \\ k(e - r\cos\theta) & \cos\theta & \sin\theta/B \\ kr\sin\theta & -\sin\theta & \cos\theta \end{bmatrix}^{\mathrm{T}} \right) \begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \\ \mathbf{i}_3 \end{bmatrix}\\
&= \left(\begin{bmatrix} \bar{f}_1 \\ \bar{f}_2 \\ \bar{f}_3 \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} u \\ v \\ w \end{bmatrix}^{\mathrm{T}} \mathbf{E}^{\mathrm{T}} \right) \begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \\ \mathbf{i}_3 \end{bmatrix}.
\end{aligned} \tag{9}$$

By application of the second partial derivative of the position vector $\mathbf{r}$ with respect to time t_{ime}, the D'Alembert force vector $\mathbf{F}$ for per unit volume of the panel is given by the following:

$$\mathbf{F} = -\rho\partial^2\mathbf{r}/\partial t_{ime}^2 = \mathbf{F}_{in} + \mathbf{F}_{cen} + \mathbf{F}_{sup} + \mathbf{F}_{cor}, \tag{10}$$

where $\mathbf{F}_{in}$ is an inertia force vector, $\mathbf{F}_{cen}$ is a centrifugal force vector, $\mathbf{F}_{sup}$ is a supplementary centrifugal force vector and $\mathbf{F}_{cor}$ is a Coriolis force vector. In the case of neglecting the effect of $\mathbf{F}_{cor}$, the virtual work of $\mathbf{F}$ due to virtual displacement vector $\delta\mathbf{D}_{is}$ is

$$\mathbf{F}\delta\mathbf{D}_{is} = \mathbf{F}_{in}\delta\mathbf{D}_{is} + \mathbf{F}_{cen}\delta\mathbf{D}_{is}\mathbf{F}_{sup}\delta\mathbf{D}_{is},$$

$$\mathbf{F}_{in}\delta\mathbf{D}_{is} = -\omega^2\rho\begin{bmatrix} B^2u + k^2(r - e\cos\theta)^2u + k(e\cos\theta - r)v \\ k(e\cos\theta - r)u + v \\ w \end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\delta u\\ \delta v\\ \delta w\end{bmatrix},$$

$$\mathbf{F}_{cen}\delta\mathbf{D}_{is} = -\Omega^2\rho\begin{bmatrix}\bar{f}_1\\ \bar{f}_2\\ \bar{f}_3\end{bmatrix}^{\mathrm{T}}\mathbf{CE}\begin{bmatrix}\delta u\\ \delta v\\ \delta w\end{bmatrix} = -\Omega^2\rho\begin{bmatrix}f_1\\ f_2\\ f_3\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\delta u\\ \delta v\\ \delta w\end{bmatrix},$$

$$\mathbf{F}_{sup}\delta\mathbf{D}_{is} = -\Omega^2\rho\begin{bmatrix}u\\ v\\ w\end{bmatrix}^{\mathrm{T}}\mathbf{E}^{\mathrm{T}}\mathbf{CE}\begin{bmatrix}\delta u\\ \delta v\\ \delta w\end{bmatrix}$$

$$= -\Omega^2\rho\begin{bmatrix}u\\ v\\ w\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} h_{11} & h_{12} & h_{13}\\ & h_{22} & h_{23}\\ \text{Sym.} & & h_{33}\end{bmatrix}\begin{bmatrix}\delta u\\ \delta v\\ \delta w\end{bmatrix},$$

$$\delta\mathbf{D}_{is} = \begin{bmatrix}\delta u\\ \delta v\\ \delta w\end{bmatrix}^{\mathrm{T}}\mathbf{E}^{\mathrm{T}}\begin{bmatrix}\mathbf{i}_1\\ \mathbf{i}_2\\ \mathbf{i}_3\end{bmatrix}, \tag{11}$$

where ρ is the density of the material, ω is the angular frequency of vibration and matrix $\mathbf{C}$ is a transfer matrix between the co-ordinate systems given by

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0\\ 0 & \cos^2(kx+\phi) & -\sin(kx+\phi)\cos(kx+\phi)\\ 0 & -\sin(kx+\phi)\cos(kx+\phi) & \cos^2(kx+\phi)\end{bmatrix}. \tag{12}$$

f_i and h_{ij} (i, j = 1, 2, 3) are given in the Appendix.

3.1. DEFORMATION DUE TO CENTRIFUGAL FORCE

To analyze the vibrations of rotating panels, the deformations caused by centrifugal forces must be known. For the steady state deformation analysis, the steady equilibrium equation of a rotating twisted cylindrical thin panel is given by the principle of virtual work,

$$\iiint_m (\sigma_{\xi\xi}\delta\varepsilon_{\xi\xi} + \sigma_{\eta\eta}\delta\varepsilon_{\eta\eta} + \tau_{\xi\eta}\delta\gamma_{\xi\eta})F\,\mathrm{d}x\,r\,\mathrm{d}\theta\,\mathrm{d}z - \iiint_m (\mathbf{F}_{cen} + \mathbf{F}_{sup})\delta\mathbf{D}_{is}F\,\mathrm{d}x\,r\,\mathrm{d}\theta\,\mathrm{d}z = 0. \quad (13)$$

By substituting equation (6) into equation (13), integrating with respect to z and neglecting the terms having the factor z^i where i is greater than 2, equation (13) becomes

$$\iint_A \delta\mathbf{U}^{\mathrm{T}}\mathbf{Q}\mathbf{U}\,\mathrm{d}x\,r\,\mathrm{d}\theta - \iint_A \mathbf{F}_{cen}\delta\mathbf{D}_{is}Bt\,\mathrm{d}x\,r\,\mathrm{d}\theta - \iint_A \mathbf{F}_{sup}\delta\mathbf{D}_{is}Bt\,\mathrm{d}x\,r\,\mathrm{d}\theta = -\iint_A \delta\mathbf{U}^{\mathrm{T}}\boldsymbol{\mathcal{R}}\,\mathrm{d}x\,r\,\mathrm{d}\theta, \quad (14)$$

in which

$$\begin{aligned}
\mathbf{Q} = {} & \bar{\mathbf{G}}_{x1}^{\mathrm{T}}\mathbf{D}_1(\bar{\mathbf{G}}_{x1} + \nu\bar{\mathbf{G}}_{\theta1}) + \bar{\mathbf{G}}_{\theta1}^{\mathrm{T}}\mathbf{D}_1(\nu\bar{\mathbf{G}}_{x1} + \bar{\mathbf{G}}_{\theta1}) + [(1-\nu)/2]\bar{\mathbf{G}}_{x\theta}^{\mathrm{T}}\mathbf{D}_1\bar{\mathbf{G}}_{x\theta} \\
& + N_{\xi\xi}^0\mathbf{G}_{x2}^{\mathrm{T}}\mathbf{G}_{x2} + N_{\eta\eta}^0\mathbf{G}_{\theta2}^{\mathrm{T}}\mathbf{G}_{\theta2} + N_{\xi\eta}^0(\mathbf{G}_{x2}^{\mathrm{T}}\mathbf{G}_{\theta2} + \mathbf{G}_{\theta2}^{\mathrm{T}}\mathbf{G}_{x2}),
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{\mathcal{R}} = {} & \{\mathbf{G}_{x2}^{\mathrm{T}}\mathbf{G}_{x2}\mathbf{U}^0\mathbf{D}_2(\bar{\mathbf{G}}_{x1} + \nu\bar{\mathbf{G}}_{\theta1}) + \mathbf{G}_{\theta2}^{\mathrm{T}}\mathbf{G}_{\theta2}\mathbf{U}^0\mathbf{D}_2(\nu\bar{\mathbf{G}}_{x1} + \bar{\mathbf{G}}_{\theta1}) \\
& + [(1-\nu)/2](\mathbf{G}_{x2}^{\mathrm{T}}\mathbf{G}_{\theta2} + \mathbf{G}_{\theta2}^{\mathrm{T}}\mathbf{G}_{x2})\mathbf{U}^0\mathbf{D}_2\bar{\mathbf{G}}_{x\theta} \\
& + \tfrac{1}{2}\bar{\mathbf{G}}_{x1}^{\mathrm{T}}\mathbf{D}_2^{\mathrm{T}}(\mathbf{U}^0)^{\mathrm{T}}(\mathbf{G}_{x2}^{\mathrm{T}}\mathbf{G}_{x2} + \nu\mathbf{G}_{\theta2}^{\mathrm{T}}\mathbf{G}_{\theta2}) \\
& + \tfrac{1}{2}\bar{\mathbf{G}}_{\theta1}^{\mathrm{T}}\mathbf{D}_2^{\mathrm{T}}(\mathbf{U}^0)^{\mathrm{T}}(\nu\mathbf{G}_{x2}^{\mathrm{T}}\mathbf{G}_{x2} + \mathbf{G}_{\theta2}^{\mathrm{T}}\mathbf{G}_{\theta2}) + [(1-\nu)/2]\bar{\mathbf{G}}_{x\theta}^{\mathrm{T}}\mathbf{D}_2^{\mathrm{T}}(\mathbf{U}^0)^{\mathrm{T}}\mathbf{G}_{x2}^{\mathrm{T}}\mathbf{G}_{\theta2} \\
& + (BH/2)[\mathbf{G}_{x2}^{\mathrm{T}}\mathbf{G}_{x2}\mathbf{U}^0(\mathbf{U}^0)^{\mathrm{T}}(\mathbf{G}_{x2}^{\mathrm{T}}\mathbf{G}_{x2} + \nu\mathbf{G}_{\theta2}^{\mathrm{T}}\mathbf{G}_{\theta2}) \\
& + \mathbf{G}_{\theta2}^{\mathrm{T}}\mathbf{G}_{\theta2}\mathbf{U}^0(\mathbf{U}^0)^{\mathrm{T}}(\nu\mathbf{G}_{x2}^{\mathrm{T}}\mathbf{G}_{x2} + \mathbf{G}_{\theta2}^{\mathrm{T}}\mathbf{G}_{\theta2}) \\
& + (1-\nu)(\mathbf{G}_{x2}^{\mathrm{T}}\mathbf{G}_{\theta2} + \mathbf{G}_{\theta2}^{\mathrm{T}}\mathbf{G}_{x2})\mathbf{U}^0(\mathbf{U}^0)^{\mathrm{T}}\mathbf{G}_{x2}^{\mathrm{T}}\mathbf{G}_{\theta2}]\}\mathbf{U}^0 \\
& + \bar{\mathbf{G}}_{x1}^{\mathrm{T}}\mathbf{M}_{\xi\xi}^0 + \bar{\mathbf{G}}_{\theta1}^{\mathrm{T}}\mathbf{M}_{\eta\eta}^0 + \bar{\mathbf{G}}_{x\theta}^{\mathrm{T}}\mathbf{M}_{\xi\eta}^0,
\end{aligned}$$

$$N_{\xi\xi}^0 = \int_{-t/2}^{t/2}\sigma_{\xi\xi}^0 F\,\mathrm{d}z, \quad N_{\eta\eta}^0 = \int_{-t/2}^{t/2}\sigma_{\eta\eta}^0 F\,\mathrm{d}z, \quad N_{\xi\eta}^0 = \int_{-t/2}^{t/2}\tau_{\xi\eta}^0 F\,\mathrm{d}z,$$

$$\mathbf{M}_{\xi\xi}^0 = \int_{-t/2}^{t/2}\sigma_{\xi\xi}^0\mathbf{Z}^{\mathrm{T}}\,\mathrm{d}z, \quad \mathbf{M}_{\eta\eta}^0 = \int_{-t/2}^{t/2}\sigma_{\eta\eta}^0\mathbf{Z}^{\mathrm{T}}\,\mathrm{d}z, \quad \mathbf{M}_{\xi\eta}^0 = \int_{-t/2}^{t/2}\tau_{\xi\eta}^0\mathbf{Z}^{\mathrm{T}}\,\mathrm{d}z,$$

$$\mathbf{D}_1 = \frac{E}{1-\nu^2}\int_{-t/2}^{t/2}\frac{1}{F}\mathbf{Z}^{\mathrm{T}}\mathbf{Z}\,\mathrm{d}z, \quad \mathbf{D}_2 = \frac{E}{1-\nu^2}\int_{-t/2}^{t/2}\mathbf{Z}\,\mathrm{d}z, \quad H = \frac{Et}{1-\nu^2}, \quad (15)$$

where E is Young's modulus and ν is Poisson's ratio.

In order to investigate the parameters which influence the vibrations of the panels conveniently, the following non-dimensional variables are introduced:

$$\bar{x} = \frac{x}{l}, \quad \bar{u} = \frac{u}{l}, \quad \bar{v} = \frac{v}{l}, \quad \bar{w} = \frac{w}{l}, \quad \bar{r} = \frac{r}{l}, \quad \bar{e} = \frac{e}{r}, \quad \bar{k} = kl, \quad \bar{x}_0 = \frac{x_0}{l}, \quad \bar{\Omega} = \frac{\Omega}{\omega_0}, \tag{16}$$

where ω_0 is the fundamental angular frequency of the non-rotating panel.

The Rayleigh–Ritz method is used to present a numerical procedure, and so the non-dimensional displacement functions $\bar{u}$, $\bar{v}$ and $\bar{w}$ are assumed to be two dimensional algebraic polynomials with respect to $\bar{x}$ and θ, which should satisfy the geometrical boundary conditions at $\bar{x} = 0$, given by the following:

$$\bar{u} = \sum_{i=1}^{N_{\bar{u}}} \sum_{j=0}^{M_{\bar{u}}} a_{ij} \bar{x}^i \theta^j, \quad \bar{v} = \sum_{\kappa=1}^{N_{\bar{v}}} \sum_{l=0}^{M_{\bar{v}}} b_{\kappa l} \bar{x}^\kappa \theta^l, \quad \bar{w} = \sum_{m=2}^{N_{\bar{w}}} \sum_{n=0}^{M_{\bar{w}}} c_{mn} \bar{x}^m \theta^n, \tag{17}$$

where a_{ij}, $b_{\kappa l}$ and c_{mn} are unknown coefficients, and N_i, M_j ($i, j = \bar{u}, \bar{v}, \bar{w}$) are the coefficients related to the number of terms in the displacement functions, namely, $N_{\bar{u}} \times (M_{\bar{u}} + 1)$, $N_{\bar{v}} \times (M_{\bar{v}} + 1)$ and $(N_{\bar{w}} - 1) \times (M_{\bar{w}} + 1)$ terms in $\bar{u}$, $\bar{v}$ and $\bar{w}$ displacement functions respectively.

By substituting equations (16) and (17) into equation (14) and integrating over the panel area, the equilibrium equation of rotating panel in respect to deformation is given by

$$[\mathbf{K} + \mathbf{K}(\mathbf{U}^0) + \mathbf{K}(\mathbf{N}^0) - \bar{\Omega}^2 \mathbf{K}(\mathbf{F}_{sup})]\mathbf{q} = \mathcal{F}(\bar{\Omega}^2) + \mathcal{F}_{un}, \tag{18}$$

where $\mathbf{K}$ is a stiffness matrix of the non-rotating panel, $\mathbf{K}(\mathbf{U}^0)$ and $\mathbf{K}(\mathbf{N}^0)$ are stiffness matrices due to rotation, which depend on the initial deformations $\mathbf{U}^0$ and initial stress resultants $\mathbf{N}^0$ respectively, $\mathbf{K}(\mathbf{F}_{sup})$ is a supplementary stiffness matrix, $\mathcal{F}(\bar{\Omega}^2)$ is a centrifugal force vector acting on the panel, and $\mathcal{F}_{un}$ is an imbalanced force vector obtained from the term on the left side of equation (14). $\mathbf{q}$ is a vector consisting of unknown coefficients a_{ij}, $b_{\kappa l}$ and c_{mn}.

For the non-linearity of equation (18), the initial deformations and the initial stress resultants corresponding to the centrifugal forces can be estimated by the interative method. First, assume $\mathbf{U}^0 = 0$ the $\mathbf{q}$ is evaluated by equation (18), and the initial deformations and the initial stress resultants can be obtained from equations (17) and (15) respectively. Second, the matrices in equation (18) are calculated through the obtained initial deformations and initial stress resultants, and $\mathbf{q}$ is modified by equation (18) again. The second calculating routine is iterated until the vector $\mathbf{q}$ converges.

3.2. FREE VIBRATION ANALYSIS

After the determination of the steady initial deformations $\mathbf{U}^0$ and the initial stress resultants $\mathbf{N}^0$, considering the virtual work done by the vibration inertia force and supplementary centrifugal force, and by using the principle of virtual work, the vibration equation for rotating the panel is presented based on the Rayleigh–Ritz method, namely,

$$[\mathbf{K} + \mathbf{K}(\mathbf{U}^0) + \mathbf{K}(\mathbf{N}^0) - \bar{\Omega}^2\mathbf{K}(\mathbf{F}_{sup}) - \lambda^2\mathbf{M}]\mathbf{q} = 0, \tag{19}$$

where $\mathbf{M}$ is a mass matrix, $\mathbf{q}$ is an eigenvalue vector, λ is a non-dimensional frequency parameter defined as

$$\lambda^2 = \rho t\omega^2 l^4/D, \tag{20}$$

and $D = Et^3/\{12(1-\nu^2)\}$ is flexural rigidity.

Equation (19) is a standard eigenvalue equation, and the eigenvalues and corresponding eigenvectors can be evaluated through general methods.

4. RESULTS AND DISCUSSIONS

In this paper, the x-axis is considered as the twisting center axis which passes through the center of gravity of each cross section. Thus e is given by

$$e = r\sin(\Omega_1/2)/\Omega_1/2. \tag{21}$$

The Gauss–Legendre integration method having 12 integration points is used, which ensures that the frequency parameters converge.

4.1. CONVERGENCE OF FREQUENCY PARAMETERS

The convergence property of the frequency parameters about the number of terms in the displacement functions is studied for a model having the following parameters: an aspect ratio $b/l = 0{\cdot}5$, a thickness ratio $b/t = 20$, $\Omega_1 = 60°$, $\bar{k} = 60°$, $\phi = 0°$, Poisson's ratio $\nu = 0{\cdot}3$, a radius of the rotating disc $\bar{x}_0 = 0$ and

TABLE 1

Convergence of λ versus the number of terms in the displacements ($b/l = 0{\cdot}5$, $b/t = 20$, $\Omega_1 = 60°$, $\bar{k} = 60°$, $\phi = 0°$, $\bar{x}_0 = 0$, $\bar{\Omega} = 0{\cdot}5$)

$N_{\bar{u}}$, $M_{\bar{u}}$	6, 7	7, 7	7,8
$N_{\bar{v}}$, $M_{\bar{v}}$	6, 7	7, 7	7, 8
$N_{\bar{w}}$, $M_{\bar{w}}$	7, 8	8, 8	8, 9
	Terms		
No.	48/48/54	56/56/63	63/63/70
1	6·5632	6·5719	6·5716
2	19·986	19·997	19·996
3	36·425	36·391	36·391
4	54·998	54·781	54·780
5	76·615	76·389	76·389
6	91·938	91·715	91·714
7	107·16	106·45	106·45
8	110·87	109·47	109·46
9	136·81	135·73	135·73
10	166·79	161·62	161·62

TABLE 2

Comparison with the frequencies (Hz) *of the blades* [7] ($a = b = 30{\cdot}5$ cm, $t = 0{\cdot}305$ cm, $r = 61{\cdot}0$ cm, $\bar{x}_0 = 0$, $\bar{\Omega} = 1{\cdot}0$)

ϕ		0°		45°		90°	
Mode		Leissa	Present	Leissa	Present	Leissa	Present
Anti.	1	150·98	150·72	110·23	124·20	105·10	90·403
	2	418·24	418·97	404·90	409·79	403·48	400·38
	3	623·35	572·55	592·37	549·42	589·06	525·76
	4	829·04	817·23	819·63	809·27	818·58	797·42
Sym.	1	170·28	173·01	150·22	158·98	147·84	142·07
	2	301·94	296·58	266·56	274·49	262·69	250·42
	3	463·48	459·78	448·12	448·82	446·48	438·42
	4	789·09	800·38	780·63	795·71	779·76	783·36

$\lambda_0 = 5{\cdot}6373$ which is the fundamental angular frequency of the non-rotating panel. The variations in the first ten frequency parameters λ for the different number of terms in the displacement functions are shown in Table 1 when the rotating speed $\bar{\Omega} = 0{\cdot}5$. It can be seen that the first ten frequency parameters λ converge using the displacement polynomial functions $\bar{u}$, $\bar{v}$ and $\bar{w}$ with 63, 63, and 70 terms respectively.

4.2. COMPARISON WITH PREVIOUS RESULTS

To verify the practicability of the numerical procedure presented in this paper, first, a shallow shell model with a rectangular planform for a blade [7] is analysed by the present method. The length a and the width b of the planform are 30·5 cm, the radius r of curvature in the chordwise direction is 61·0 cm, the thickness t of the blade is 0·305 cm and the properties of the material are: $\nu = 0{\cdot}3$, $E = 2{\cdot}1 \times 10^6$ kgf/cm^2 and $\rho = 7{\cdot}85 \times 10^{-6}$ kgf s^2/cm^4. The comparison is shown in Table 2 which gives the first four antisymmetric and four symmetric frequencies (Hz) in the case of the radius of the rotating disc $\bar{x}_0 = 0$, the rotating speed $\bar{\Omega} = 1{\cdot}0$ and the fundamental angular frequency ω_0 of the non-rotating panel. For the case of the setting angle $\phi = 0°$, the two results agree well except for the third antisymmetric frequency. When the setting angle $\phi = 45°$ and 90°, there are differences in the first and third antisymmetric frequencies of the two results respectively. But they have the same tendency to decrease with an increasing setting angle. It is possible that the differences are caused by using the shallow shell theory, approximate initial stress resultants [7] and the different number of terms in the displacement functions in two methods.

Second, a rotating twisted plate having an aspect ratio $b/l = 0{\cdot}5$, a thickness ratio $b/t = 20$, twist angle $\bar{k} = 60°$ and the radius of the rotating disc $\bar{x}_0 = 0$ is studied by the present method for a different setting angle ϕ (0°, 45°, 90°). The comparison between the present results and the previous results [8] are shown by Table 3. It can be seen there is good agreement between the two results.

TABLE 3

Comparison with frequency parameter λ for the rotating twisted plate [8] ($b/l = 0{\cdot}5$, $b/t = 20$, $\bar{k} = 60°$, $\bar{x}_0 = 0$)

ϕ	0°						45°				90°			
$\bar{\Omega}$	0		0·4		1·0		0·4		1·0		0·4		1·0	
Mode	Tsuiji	Present	Tsuiji	Present	Tsuiji	Present	Tsuiji	Present	Tsuiji	Present	Tsuiji	Present	Tsuiji	Present
F_1	3·409	3·408	3·661	3·658	4·757	4·816	3·496	3·494	3·915	3·916	3·428	3·427	3·491	3·487
F_2	15·02	15·02	15·21	15·20	16·16	16·26	15·21	15·21	16·17	16·17	15·22	15·21	16·20	16·20
T_1	30·15	30·15	30·08	30·07	29·73	30·26	30·09	30·08	29·72	29·76	30·16	30·16	30·18	30·17
P_1	49·81	49·86	50·01	50·06	51·03	51·25	50·02	50·07	51·10	51·16	50·02	50·07	51·09	51·14
F_3	76·63	76·68	76·58	76·63	76·31	76·51	76·60	76·66	76·45	76·52	76·68	76·73	76·95	77·00
T_2	79·86	79·87	79·71	79·72	78·95	79·31	79·78	79·79	79·37	79·44	79·87	79·88	79·96	79·97
F_4	94·38	94·40	94·39	94·39	94·40	95·01	94·39	94·40	94·43	94·47	94·42	94·43	94·60	94·60
T_3	119·8	119·7	119·6	119·5	118·6	118·9	119·7	119·6	119·2	119·3	119·8	119·8	120·1	120·1

TABLE 4

Effects of curvature and twist on frequency parameter λ of rotating panels ($b/l = 0{\cdot}5$, $b/t = 20$, $\phi = 0°$, $\bar{x}_0 = 0$)

Ω_1	30°						60°						90°					
$\bar{k}$	0°		30°		60°		0°		30°		60°		0°		30°		60°	
$\bar{\Omega}$	0·4	0·8	0·4	0·8	0·4	0·8	0·4	0·8	0·4	0·8	0·4	0·8	0·4	0·8	0·4	0·8	0·4	0·8
1	5·7579	6·2984	4·9939	5·5923	4·2965	4·8728	9·3565	9·6696	7·5048	7·9286	5·7668	6·2134	13·056	13·253	10·137	10·440	7·6166	7·9731
2	15·016	15·393	18·293	18·447	16·780	17·230	15·221	15·560	18·449	18·554	19·138	19·393	15·506	15·789	19·032	19·094	20·531	20·710
3	31·602	32·141	27·518	27·964	31·488	31·351	48·965	49·260	38·826	39·199	36·174	36·144	52·618	52·986	47·363	47·619	42·766	42·770
4	48·874	49·324	53·108	53·361	51·725	52·177	50·354	50·770	54·269	54·514	54·001	54·311	57·961	57·918	56·445	56·595	55·896	56·145
5	57·519	57·471	63·670	63·372	75·659	75·390	57·674	57·628	66·734	66·488	76·336	76·140	65·478	65·658	72·608	72·561	78·103	78·055
6	77·737	78·307	78·485	78·848	87·824	87·821	90·088	90·201	90·281	90·353	91·280	91·338	87·397	87·490	88·321	88·312	89·833	89·882
7	92·830	92·972	93·156	93·224	94·243	94·268	99·543	100·06	99·673	99·847	106·10	105·91	107·19	101·63	104·63	104·91	106·13	106·12
8	94·530	95·100	99·998	100·07	109·50	109·40	108·04	108·40	108·63	108·98	108·88	109·19	120·32	120·54	123·62	123·65	127·16	127·30
9	125·26	125·51	127·05	127·15	131·32	131·37	126·78	126·99	129·86	129·90	135·47	135·39	136·54	136·83	138·75	138·96	142·04	142·15
10	140·26	140·89	140·07	140·59	144·05	144·15	166·92	167·51	162·91	163·35	161·06	161·17	181·10	181·61	171·73	172·34	161·63	161·89

4.3. RESULTS FOR ROTATING TWISTED PANELS

Models having an aspect ratio $b/l = 0{\cdot}5$, a thickness ratio $b/t = 20$ and Poisson's ratio $\nu = 0{\cdot}3$ are investigated for the effects of twist angle, center angle, setting angle, radius of the rotating disc and rotating speed on the vibrations of the panels. For non-rotating panels with different twist angles and center angles, the fundamental frequencies are considerably different. In order to compare the influences of twist angle, center angle, setting angle, radius of the rotating disc and rotating speed on the frequency parameters for different rotating twisted panels, ω_0 is selected as the fundamental angular frequency of the non-rotating twisted cantilevered plate to analyze the vibrations of rotating twisted cylindrical thin panels.

The effects of center angle Ω_1 and twist angle $\bar{k}$ on the vibrations of rotating panels are investigated first. Table 4 shows the first ten frequency parameters of the panels with different center angles Ω_1 (30°, 60°, 90°) and different twist angles $\bar{k}$ (0°, 30°, 60°) at the rotating speed $\bar{\Omega} = 0{\cdot}4$ and $0{\cdot}8$.

In the case of a given twist angle $\bar{k}$, the fundamental frequency parameter λ increases with the increasing center angle. When the rotating speed $\bar{\Omega}$ increases, the fundamental frequency parameter λ increases and the ratio of the increase decreases with an increase in the center angle, which is 20% for the case of $\Omega_1 = 30°$, 10% for 60° and 6% for 90° with respect of the case of $\bar{k} = 60°$, respectively.

In the case of a given center angle Ω_1, for the panel with twist angle $\bar{k} = 0°$ and 30°, the first four frequency parameters show the increases as the rotating speed $\bar{\Omega}$ increases, but only the first two increase as the rotating speed increases for the case of $\bar{k} = 60°$. The fundamental frequency parameter λ decreases with an increase in the twist angle $\bar{k}$, and their variations are different as rotating speed $\bar{\Omega}$ increases. For an instance of $\Omega_1 = 30°$, the variation is 13% for the case of $\bar{k} = 0°$, 17% for 30° and 20% for 60° as the rotating speed changes from 0 to 0·8.

TABLE 5

Effect of setting angle ϕ on the frequency parameter λ ($b/l = 0{\cdot}5$, $b/t = 20$, $\Omega_1 = 30°$, $\bar{k} = 30°$, $\bar{x}_0 = 1{\cdot}0$)

ϕ		0°		45°		90°	
$\bar{\Omega}$	0·0	0·4	0·8	0·4	0·8	0·4	0·8
1	4·7744	5·2658	6·5311	5·1524	5·8749	5·0968	5·9141
2	18·271	18·329	18·827	18·260	18·127	18·371	18·676
3	27·370	27·725	28·800	27·709	28·950	27·704	28·686
4	53·007	53·248	53·953	53·238	53·739	53·284	54·010
5	63·881	63·579	63·354	63·554	63·328	63·765	63·684
6	78·338	78·695	79·673	78·687	79·686	78·734	79·815
7	93·157	93·147	93·365	93·118	93·300	93·173	93·262
8	100·03	100·09	100·69	100·06	100·52	100·26	101·02
9	127·05	127·10	127·52	127·09	127·47	127·16	127·54
10	139·89	140·35	141·68	140·36	141·82	140·38	141·77

TABLE 6

Effect of the radius $\bar{x}_0$ of the rotating disc on frequency parameter λ ($b/l = 0{\cdot}5$, $b/t = 20$, $\Omega_1 = 30°$, $\bar{k} = 30°$, $\phi = 45°$, $\bar{\Omega} = 0{\cdot}7$)

	$\bar{x}_0$				
	0	0·5	1·0	1·5	2·0
1	5·0750	5·4748	5·7512	6·0059	6·7069
2	18·211	18·305	17·946	17·656	19·274
3	27·777	28·094	28·399	28·694	29·095
4	53·259	53·466	53·539	53·629	54·332
5	63·377	63·343	63·263	63·217	63·365
6	78·709	79·014	79·295	79·570	80·035
7	93·129	93·148	92·879	92·656	93·622
8	99·961	100·16	100·25	100·37	101·06
9	127·08	127·20	127·17	127·17	127·78
10	140·44	140·85	141·25	141·64	142·17

The effect of setting angle ϕ on the vibration of the panel with $\Omega_1 = 30°$, $\bar{k} = 30°$ and $\bar{x}_0 = 1{\cdot}0$ is shown in Table 5. The first frequency parameters increase and the other frequency parameters increase or decrease as rotating speed $\bar{\Omega}$ increases. It can be seen that the maximum variation of the fundamental frequency parameter appears when the setting angle $\phi = 0°$. Comparing the results between $\bar{\Omega} = 0$ and 0·8, the ratio of the increase is 36·8% for the case of $\phi = 0°$, 23% for 45°, and 24% for 90°, respectively.

Table 6 shows that the frequency parameters change as the radius of the rotating disc changes for the panel having $\Omega_1 = 30°$, $\bar{k} = 30°$ and $\phi = 45°$. It can be seen that the changes of the lower frequency parameters are larger than those of the higher ones with the radius of the rotating disc $\bar{x}_0$. When the radius $\bar{x}_0$ changes from 0·0 to 2·0, the variation is 32·2% for the fundamental frequency parameter, 9·2% for the second one and 4·7% for the third one, respectively. The other frequency parameters show increases or decreases with an increasing radius $\bar{x}_0$, but the variations are less than 2%.

5. CONCLUSIONS

In order to present a procedure for analyzing the vibrations of rotating blades, blades are treated as a model of curved and twisted cylindrical thin panels and the exact strain–displacement relations under considering initial deformation due to rotation are proposed. The numerical procedure is presented by applying the principle of virtual work and the Rayleigh–Ritz method assuming two dimensional algebric polynomials as the displacement functions. By comparison of the results obtained by the present procedure with previous data, it is shown that the present procedure is applicable to vibration analysis of rotating curved and twisted thin panels. Also, the present procedure is based on the general thin shell theory so that there are no limitations for twist and curvature of panels.

The effects of various parameters such as twist angle, center angle, setting angle, radius of rotating disc and rotating speed on the vibrations of rotating twisted cylindrical thin panels are investigated by this method. It is found that the fundamental frequency parameter for the rotating twisted panels investigated in this paper shows an increase as the rotating speed increases, and also that the parameters have little effects on the higher frequency parameters.

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APPENDIX

The coefficients f_i and h_{ij} $(i, j = 1, 2, 3)$ in equation (11) are given by the following,

$$f_1 = x_0 - (ek/B)\sin\theta\cos(kx+\phi)[-r\sin(kx+\phi-\theta) + e\sin(kx+\phi)],$$

$$f_2 = x_0k(e - r\sin\theta) + \cos\theta\cos(kx+\phi)[-r\sin(kx+\phi-\theta) + e\sin(kx+\phi)] + (1/B)\sin\theta\cos(kx+\phi)[r\cos(kx+\phi+\theta) - e\cos(kx+\phi)],$$

$$f_3 = x_0kr\sin\theta - \cos(kx+\phi)[r\sin\theta\sin(kx+\phi-\theta) + (r\cos\theta - e)\cos(kx+\phi+\theta)],$$

$$h_{11} = 1 + (e^2k^2/B^2)\sin^2\theta\cos^2(kx+\phi),$$

$$h_{12} = k(e - r\cos\theta) + (ek/4B)\sin 2\theta\sin 2(kx+\phi) - (ek/B^2)\sin^2\theta\cos^2(kx+\phi),$$

$$h_{13} = kr\sin\theta - (ek/B)\sin\theta\cos(kx+\phi)\cos(kx+\phi-\theta),$$

$$h_{22} = k^2(e - r\cos\theta)^2 + \cos^2\theta\cos^2(kx+\phi) - (1/2B)\sin 2\theta\sin 2(kx+\phi)$$

$$+ (1/B)\sin^2\theta\cos^2(kx+\phi),$$

$$h_{23} = k^2r(e - r\cos\theta)\sin\theta - \cos\theta\cos(kx+\phi)\sin(kx+\phi+\theta)$$
$$+ (1/B)\sin\theta\cos(kx+\phi)\cos(kx+\phi-\theta),$$

$$h_{33} = k^2r^2\sin^2\theta + \cos(kx+\phi)[\sin 2\theta\sin(kx+\phi) + \cos(kx+\phi)]. \tag{22}$$