



A GRAPHICAL OPTIMIZATION OF TAKE-OFF NOISE ABATEMENT PROCEDURES FOR SUBSONIC AIRCRAFT

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This paper describes a numerical approach to the simulation of noise contours generated during aircraft operations. Common features of many existing noise-contour programs make these procedures unsuitable for on-line piloted-simulator use. In fact, they usually require large computational tools and exhibit complex structure, so that they generally run quite slowly. The method proposed here is an attempt to overcome some of the above drawbacks. It works for arbitrarily complex take-off and landing paths, and reveals the influence of several quantities on the shape and size of the contours. Besides, the calculations are simple enough to be implemented on a handheld programmable calculator. The method runs fast, and quickly provides contour shape, evaluates area and analyzes main characteristics of the end. The method has been used to optimize noise abatement procedures for subsonic aircraft; for every take-off procedure the model can generate an isofootprint on the ground which helps the operator to choose the best take-off solution.

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1. INTRODUCTION

Noise impact on communities around an airport is a serious problem of aircraft operations. An increasing interest is being devoted to the monitoring and reduction of the acoustic disturbance, due to growing public concern towards the environment. A dedicated program has recently been undertaken at CIRA, with the main goal of updating current techniques for the acoustic monitoring of the airport area.

A key tool for the study of acoustic impact is the noise contour calculation: a number of computer codes have been developed over the last decade and have been widely used by industry and governments. These programs enable one to handle very general cases, which can include such aspects as, for example, ground attenuation, shielding, multiple flights, and various mixes of aircraft. The basic result is the generation of a noise contour, or footprint, which is currently the best technique to evaluate the noise impact due to a single operation, and assess possible reduction policies, including noise abatement procedures. In the

near future, international regulations on aircraft noise are likely to include these methods as major tools for community noise control.

2. THEORY

The theoretical approach described here is based on a closed-form solution of the noise contour problem, which is due to Stewart and Carson [1]. Noise contour area calculation and take-off cutback procedure analysis are introduced; the aircraft is assumed to be described by an isotropic source model, although more sophisticated representations should not increase dramatically the difficulties of calculation. To generate a single noise contour, the method proceeds along the following steps: the trajectory is divided into N straight-line segments; along the n th segment, the isotropic source which represents the aircraft generates a (prescribed value) constant noise-level surface, in the shape of a circular cylinder; the intersection of the cylinder with the ground plane gives rise to an elliptic curve, part of which will represent the contribution of the n th segment of trajectory to the total contour; the complete contour is generated by the envelope of contributions from all segments of the trajectory.

The analysis reveals the fundamental properties of the contours, and the influence of several quantities which affect its shape and size, the area and the characteristics of the end.

2.1. NOISE CONTOUR: GENERAL TAKE-OFF SOLUTION

In this section the noise impact for an aircraft flying a take-off trajectory is considered. A typical take-off three-dimensional trajectory consisting of straight-line segments is illustrated by the two plane views in Figure 1. There the trajectory point 1 is the start of the take-off roll, located a distance OT from the origin, trajectory point 2 is lift off, and the general n th segment is that lying between the n th and $(n+1)$ th way points, as indicated. Each segment is characterized by the following: (1) an angle ψ , positive counterclockwise from the x -axis to the projection of the segment on the xy -plane (which represent the ground plane); (2) an angle γ , positive above the horizontal plane, which measures the slope of the segment in the vertical plane which contains it; (3) a segment length s ; (4) a distance r from the aircraft, corresponding to the prescribed EPNL noise contour and specified thrust level.

The noise contour will be made up by contributions due to all segments, and the different pieces of the contour must be determined and suitably pieced together, so as to generate the complete contour [1]. For this purpose, consider the path of the aircraft along the n th segment in Figure 2. The aircraft, when represented by an isotropic spherical noise source moving on a straight-line segment, will generate a constant noise-level surface having the shape of a cylinder, as shown in Figure 2(a). The surface is assumed to be a cylinder of constant radius. This assumption is justified by the hypothesis that the aircraft has a constant acceleration along each segment. Now define the orthogonal coordinate system $\{u, v, w\}$, such that the u -axis is aligned with the n th segment and the origin is located at the intersection of the u -axis with the ground plane

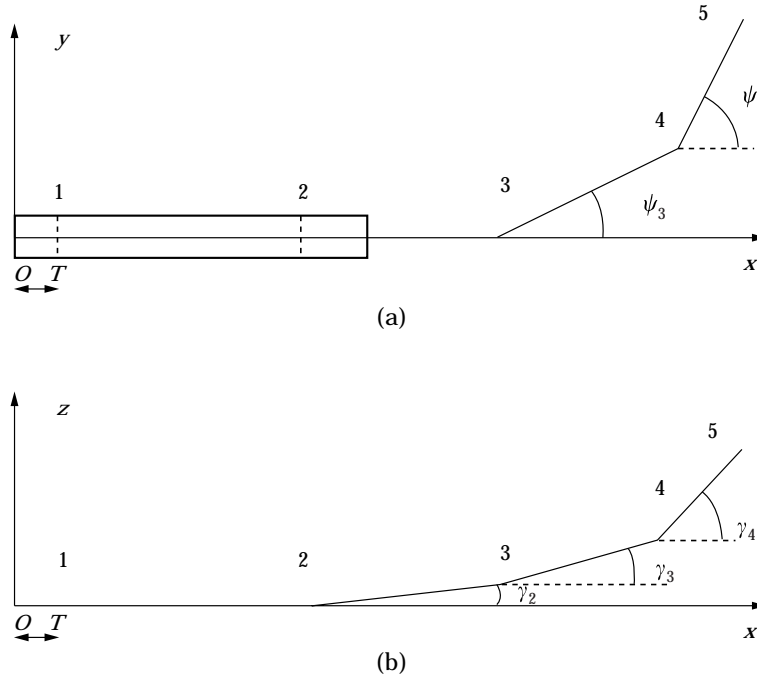


Figure 1. Top and side views — (a) and (b), respectively — of a typical take-off trajectory.

(i.e., the xy -plane, in the co-ordinate system $\{x, y, z\}$ shown in Figure 1, and the v -axis lies in the ground plane, as shown in Figure 2(b). Another orthogonal co-ordinate system, $\{x', y', z'\}$, is defined here, having the x' -axis as the projection of the u -axis on the ground plane, while the y' -axis coincides with the v -axis.

The equation for a general quadric is

$$U^T A' U = r^2, \quad (1)$$

where $U^T = (u, v, w)^T$. For a circular cylinder having the axis aligned with the u -axis, one has

$$A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

If one defines $X = (x', y', z')^T$, the transformation between two vectors X and U can be written as

$$X = \Omega U, \quad (2)$$

where Ω is an orthogonal matrix given by

$$\Omega = \begin{bmatrix} \cos \gamma & 0 & -\sin \gamma \\ 0 & 1 & 0 \\ \sin \gamma & 0 & \cos \gamma \end{bmatrix}. \quad (3)$$

Thus, equation (1), when transformed by equation (2), gives

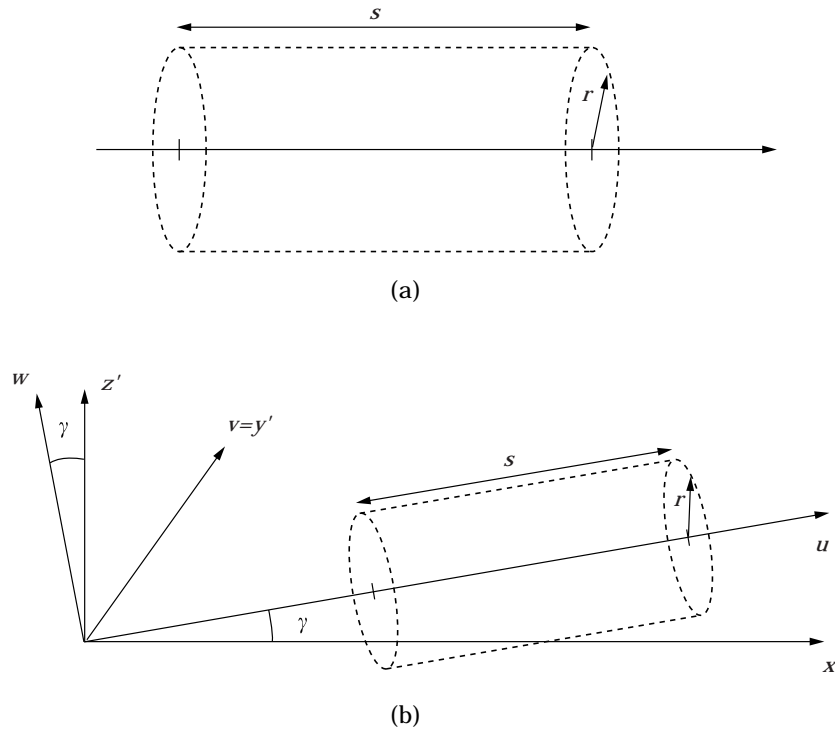


Figure 2. (a) The constant noise-level cylinder described by an isotropic source moving along a straight-line segment of trajectory of length s . (b) Relationship between the local co-ordinate systems $\{u, v, w\}$ and $\{x', y', z'\}$ defined for the analysis of the segment.

$$X^T \Omega A' \Omega^T X = r^2, \tag{4}$$

where

$$\Omega A' \Omega^T = \begin{bmatrix} \sin^2 \gamma & 0 & -\sin \gamma \cos \gamma \\ 0 & 1 & 0 \\ \sin \gamma \cos \gamma & 0 & \cos^2 \gamma \end{bmatrix}. \tag{5}$$

Equation (4) is a quadric equation for the three-dimensional noise contour for a given noise level in terms of the x', y', z' co-ordinate system.

The intersection of this surface with the ground plane will give the noise contour. A plane is represented by

$$M^T X = \kappa, \tag{6}$$

where $M = (m_1, m_2, m_3)^T$, and κ is a constant. For the horizontal ground-level plane,

$$M^T = (0, 0, m_3) \quad \text{and} \quad \kappa = 0.$$

Since X is orthogonal to M , it is given by

$$X = (x', y', 0)^T.$$

The contour can now be obtained by solving the system of equations (4) and (6). The matrix (5) can be written as

$$\Omega A' \Omega^T = \begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

with

$$A = \begin{bmatrix} \sin^2 \gamma & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -\sin \gamma \cos \gamma \\ 0 \end{bmatrix}, \quad C = [\sin \gamma \cos \gamma \quad 0], \quad D = [\cos^2 \gamma],$$

and substituting these matrices in equation (4), one obtains

$$X'^T A X' = r^2, \quad (7)$$

where

$$X' = \begin{bmatrix} x' \\ y' \end{bmatrix}.$$

Equation (7) represents a conic section. Since the diagonal elements in matrix (3) are greater than zero, equation (7) can be recognized to be an ellipse. Note that only two parameters, γ and r , are required to define the contour.

It should be pointed out that the form of equation (7) applies to every segment, with different values of r and γ , in general, and each segment contributes only a portion of its conic section to the total contour. The method to draw this portion, and to piece the different portions together, will be discussed in the following.

The values of r can be determined from the basic noise-source data, which are typically represented by parametric diagrams related to the thrust level, as illustrated in Figure 3. The value of r is determined directly from these data, for any prescribed EPNL contour and for a given value of thrust. This latter value could be either estimated, or determined from flight data or, alternatively, from a control generator.

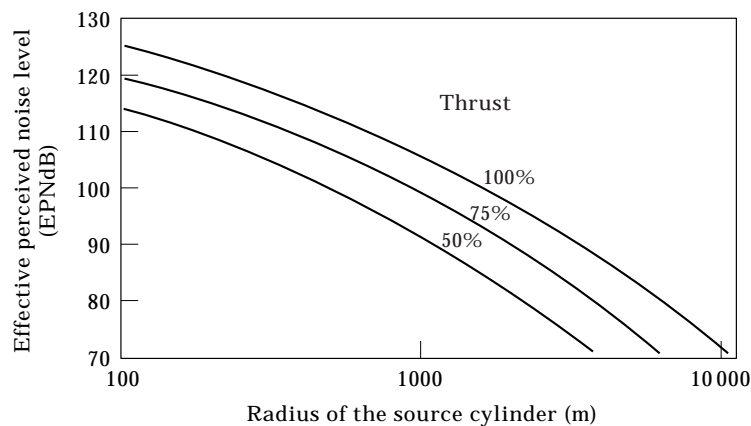


Figure 3. Sample diagram for the determination of the values of r to be used in equation (7).

An example of special interest is a flight path consisting of only one segment at a constant flight path angle (as indicated in Figure 2). The contour in this case is given by equation (7) as

$$y' = (r^2 - x'^2 \sin^2 \gamma)^{1/2}.$$

For given r and γ , this equation allow y' to be plotted as a function of x' . The maximum values of the contour co-ordinates are

$$x'_{max} = r / \sin \gamma, \quad y'_{max} = \pm r.$$

Now it will be shown how the total contour is obtained by properly combining the contributions of the various segments [2]. The pattern is illustrated in Figure 4. Recall that the contour, in the co-ordinate system $\{x', y'\}$, is given by equation (7). This equation must be transformed to the co-ordinate system $\{x, y\}$, which is common to all segments. This is done by applying rotation and translation transformations. As far as the rotation is concerned, the transformation is

$$X' = A_n X'' \tag{8}$$

where the vector X'' is defined by the co-ordinate system shown in Figure 4, and A_n is an orthogonal matrix given by

$$A_n = \begin{bmatrix} \cos \psi_n & \sin \psi_n \\ -\sin \psi_n & \cos \psi_n \end{bmatrix},$$

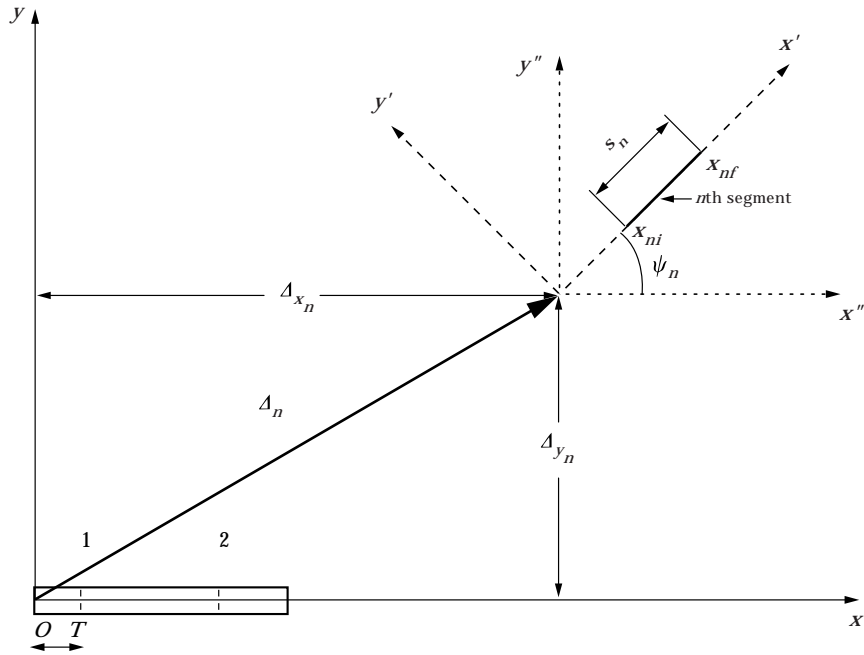


Figure 4. Relationship between the general co-ordinate system $\{x, y, z\}$ and the local co-ordinate systems $\{x', y', z'\}$ and $\{x'', y'', z''\}$.

according to the discussion following Figure 1. Thus, combining equation (7)—written for A_n , that is with $\sin^2 \gamma_n$ and r_n —and equation (8), one obtains

$$X''^T A_n^T A_n A_n X'' = r_n^2. \quad (9)$$

For the translation, the transformation is

$$X = X'' + \Delta_n, \quad (10)$$

where

$$\Delta_n = \begin{bmatrix} \Delta_{x_n} \\ \Delta_{y_n} \end{bmatrix}. \quad (11)$$

Combining equations (9) and (10) one obtains

$$(X - \Delta_n)^T A_n^T A_n A_n (X - \Delta_n) = r_n^2. \quad (12)$$

This is the equation of the contour generated by an aircraft flying along the extended n th segment in terms of the basic co-ordinate system $\{x, y\}$. Note that this equation applies to each segment, but the parameters for each segment may be different. Also note that in addition to the parameters γ and r , previously discussed with equation (7), there are now two more quantities involved: that is, Δ_n and A_n .

Now some of the quantities required in equation (12) will be determined. First, the matrix $A_n^T A_n A_n$ can easily be seen to have the form

$$A_n^T A_n A_n = \begin{bmatrix} a_n & b_n \\ b_n & d_n \end{bmatrix},$$

where

$$a_n = 1 - \cos^2 \psi_n \cos^2 \gamma_n, \quad b_n = -\sin \psi_n \cos \psi_n \cos^2 \gamma_n, \quad d_n = 1 - \sin^2 \psi_n \cos^2 \gamma_n.$$

The vector Δ_n is determined as follows: n is the segment between the n th and $(n+1)$ th way points, s_n is the length of the projection of the segment on the ground plane, while l_n is the unit vector in its direction,

$$l_n = \begin{bmatrix} \cos \psi_n \\ \sin \psi_n \end{bmatrix},$$

where ψ_n is measured positive counterclockwise from the x'' -axis to the segment, as discussed above following Figure 1. Now, from Figure 4, it can be shown that for the n th segment

$$\Delta_n = \sum_{k=1}^{n-1} l_k s_k - l_n (\tan \gamma_n)^{-1} \sum_{k=1}^{n-1} s_k \tan \gamma_k + \begin{bmatrix} OT \\ 0 \end{bmatrix}.$$

The only variable is now r , which has been previously discussed. Equation (12) can be put in a form more suitable for computation. In fact, by using

$$X - \Delta_n = \begin{bmatrix} x - \Delta_{x_n} \\ y - \Delta_{y_n} \end{bmatrix},$$

equation (12) is easily shown to reduce to

$$a_n(x - \Delta_{x_n})^2 + 2b_n(x - \Delta_{x_n})(y - \Delta_{y_n}) + d_n(y - \Delta_{y_n}) = r_n^2. \quad (13)$$

This is the equation for the contour in an usual computational form. For any given choice of x , the corresponding value of y is readily obtained as the solution of a quadric equation in the unknown $y - \Delta_{y_n}$. It should be noted that, for a given choice of x , the parameters in equation (13) should correspond to the segment closest to x . The appropriate segment can be found by examining the geometry of the configuration. Otherwise it can be calculated as follows. Determine the beginning, x_{ni} , and the end, x_{nf} , of the n th segment by

$$x_{ni} = OT + \sum_{k=1}^{n-1} s_k \cos \psi_k, \quad x_{nf} = OT + \sum_{k=1}^n s_k \cos \psi_k.$$

Then the n th segment to use must be the one such that

$$x_{ni} < x < x_{nf}.$$

Of course, the values of x should not exceed x_{max} , the value of which will be discussed later.

2.2. CONTOUR AREA

The contour area will be composed of contributions due to all the different segments of the flight path [3]. The first segment, which is on the ground, gives rise to a contour portion of area

$$S_1 = 2r_1s_1 + \pi r_1^2/2.$$

The area of the contour portion due to the n th segment (with edges denoted by LL and UL) is given, in terms of co-ordinate x' , by

$$S_n|_{LL}^{UL} = \left[x'(r_n^2 - x'^2 \sin^2 \gamma_n)^{1/2} + \frac{r_n^2}{\sin \gamma_n} \sin^{-1} \frac{x' \sin \gamma_n}{R_n} \right]_{LL}^{UL}.$$

This equation applies to every segment, and the quantities r and γ_n are known for each of these segments. The limits, however, need some careful consideration.

2.3. CONTOUR END

The end of the contour is of special interest, and it can be readily determined from the equations already given. According to results from the previous section, on the last segment (for which $n = N$), one has

$$x'_{max} = L_N = \frac{r_N}{\sin \gamma_N}, \quad y'_{max} = 0.$$

Then, using the transformations (8) and (10), one has

$$X_{max} = X''_{max} + \Delta_n = A_n^T X'_{max} + \Delta_n:$$

that is,

$$\begin{bmatrix} x_{max} \\ y_{max} \end{bmatrix} = \begin{bmatrix} \cos \psi_n & -\sin \psi_n \\ \sin \psi_n & \cos \psi_n \end{bmatrix} \begin{bmatrix} x'_{max} \\ 0 \end{bmatrix} + \begin{bmatrix} \Delta x_n \\ \Delta y_n \end{bmatrix}.$$

3. VALIDATION OF THE METHOD

In this section two validation tests of the noise contours method are performed. The closed form solution proposed by Stewart and Carson, has been modified and adapted to study new noise abatement take-off procedures. A fast method to generate different take-off procedures with different cutback points has been introduced. A contour end method has been projected with the aim of calculating the exact contour border line on the ground. A new graphical subroutine has been realized to optimize noise contours shape. The end of the noise contour is correlated with the distance from the aircraft to the ground. When the intersection between the cylindrical surface and the ground does not have an analytical solution, the end of the contour has been closed.

In Figure 5(a) the two take-off trajectories used for the validation are shown. Validation tests have been performed for two different cases: a standard take-off procedure and a noise abatement procedure. Figure 5(b) shows the 80 EPNdB contour for the standard trajectory (no. 1). Each take-off segment contributes to the total noise contour with a portion of its conic section. Figure 5(c), shows the 80 EPNdB noise contour related to the noise abatement take-off procedures (no. 2). It can be seen that the thrust reduction along the flight path produced a deep reduction on the contour area.

4. SIMULATION ON THE AIRPORT “L. DA VINCI” DI ROMA

In this section an application of the noise-contours method proposed is illustrated. A typical current conventional jet transport aircraft, such as MD80, has been considered. To optimize a N.A.P. one can simulate a number of different take-off procedures from a reference runway. The results proposed here, refer to a simulation performed at the airport of Rome, “Leonardo Da Vinci” [4].

The simulation is performed on a general take-off procedure used on runway no. 2 (25-07) of the airport. The aim of this simulation is the definition of a new noise abatement take-off procedure. Ten take-off profiles with different cutback points have been considered. In Figure 6 the standard take-off profile and the other take-off profiles used in the simulation are illustrated. The configuration and results are illustrated in Figure 7, where ten different 80 EPNdB contours (footprints) are shown. The footprint labelled 1 refers to the standard full power

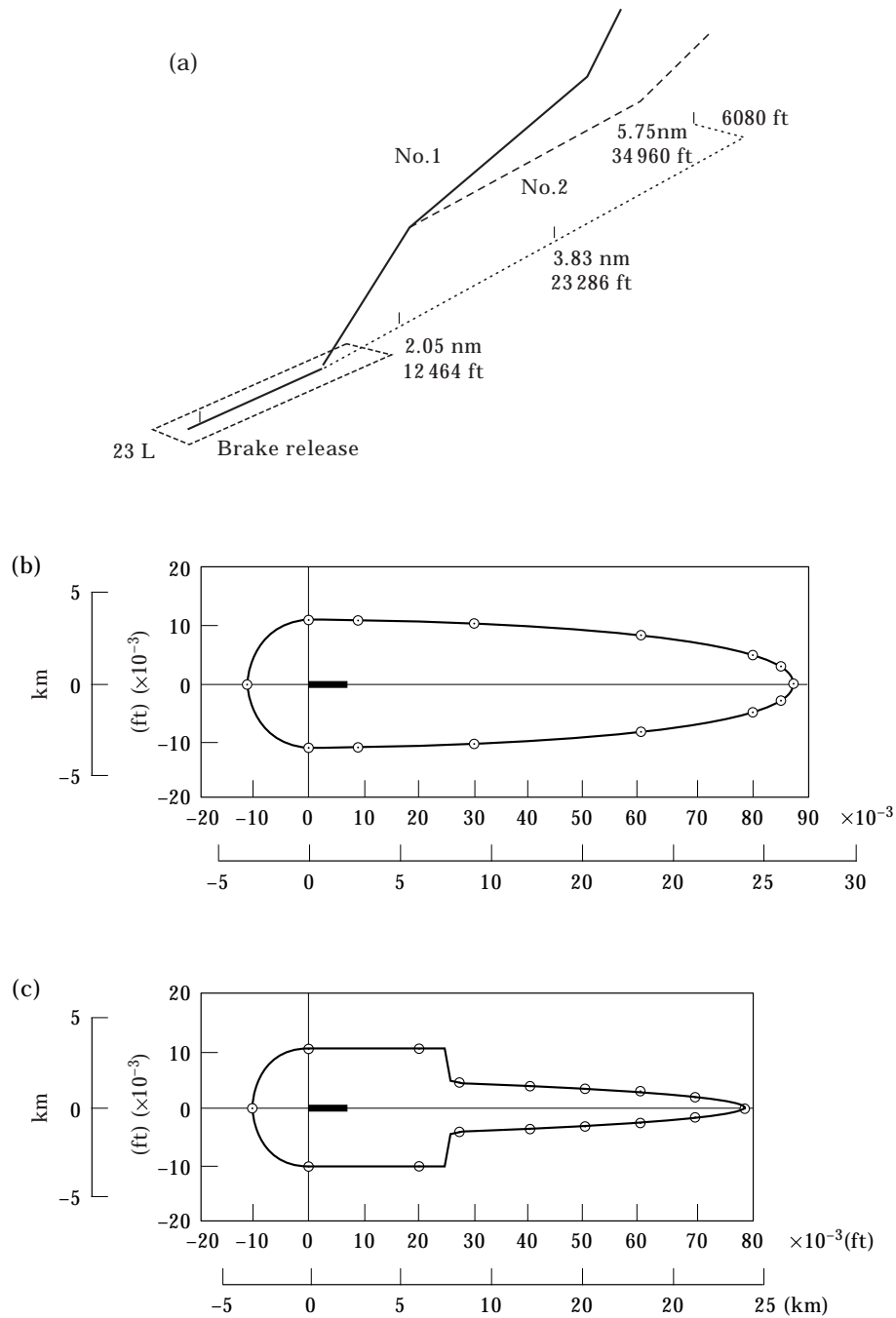


Figure 5. The two take-off procedures used in the validation: no. 1 the standard procedure, no. 2 noise abatement procedure; (b) noise contour for the standard take-off procedure; (c) noise contour for the noise abatement take-off procedure.

take-off procedure performed on runway no. 2 of the airport, and it will be considered as the reference footprint. If a general procedure with a point of cutback is considered, it is easily seen that the area of the new footprint is

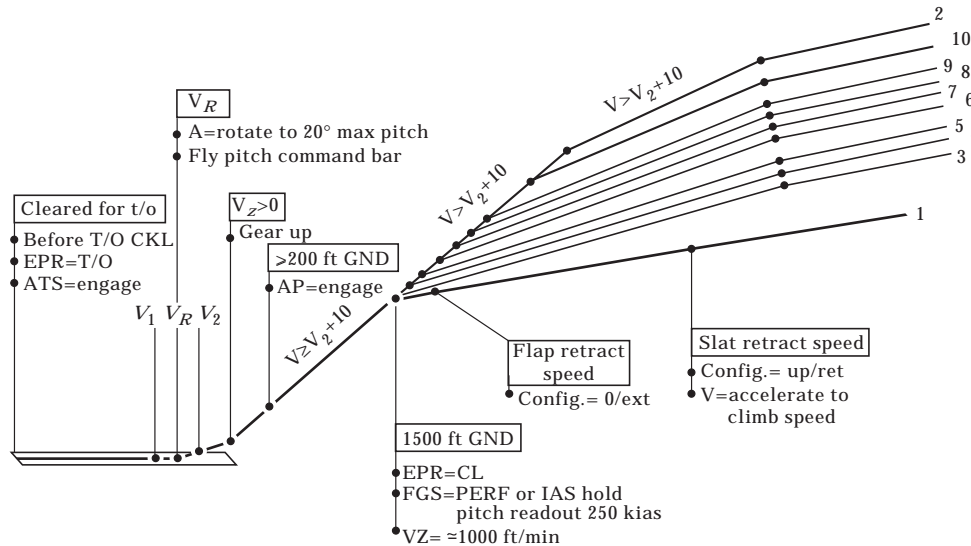


Figure 6. Ten different take-off profiles used for the simulation on the airport of Rome.

smaller than the reference one (see, for example, footprints labelled 4 and 5). Then, the primary result of the cutback is the reduction of contour area, which means a global reduction of noise impact in the surrounding area of the airport. An important result can be observed looking at the other footprints: in fact, considering take-off profiles with points of cutback at increasing altitude, the corresponding contours exhibit progressively smaller area, as shown in Figure 6. Usually, a noise abatement procedure gives good results if the point of cutback lies somewhere between 800 and 1000 m.

A particular aspect can be observed by focusing on the footprint labelled 2, which refers to a take-off procedure with no cutback and a 70% of maximum thrust. The contour area is larger than in footprints obtained with cutback profiles, but this particular configuration makes this solution the best choice for noise control in this area of the airport: in fact, the footprint labelled 2 is the thinnest one, and most of its area extends on to the sea. This means that the best noise abatement procedure has to be chosen looking at the area as well as the shape of the contour.

5. CONCLUDING REMARKS

The simplicity of the noise analysis presented here enables one to obtain the noise contour, its area, and its end for arbitrarily complex flight paths for both take-off and landing. The method is simple and fast, and results can be obtained by means of a small programmable calculator.

It should be noted that the single event contour discussed here is the obvious choice for minimizing noise impact. The impact of multiple flights can be handled by an immediate extension of the single event results.

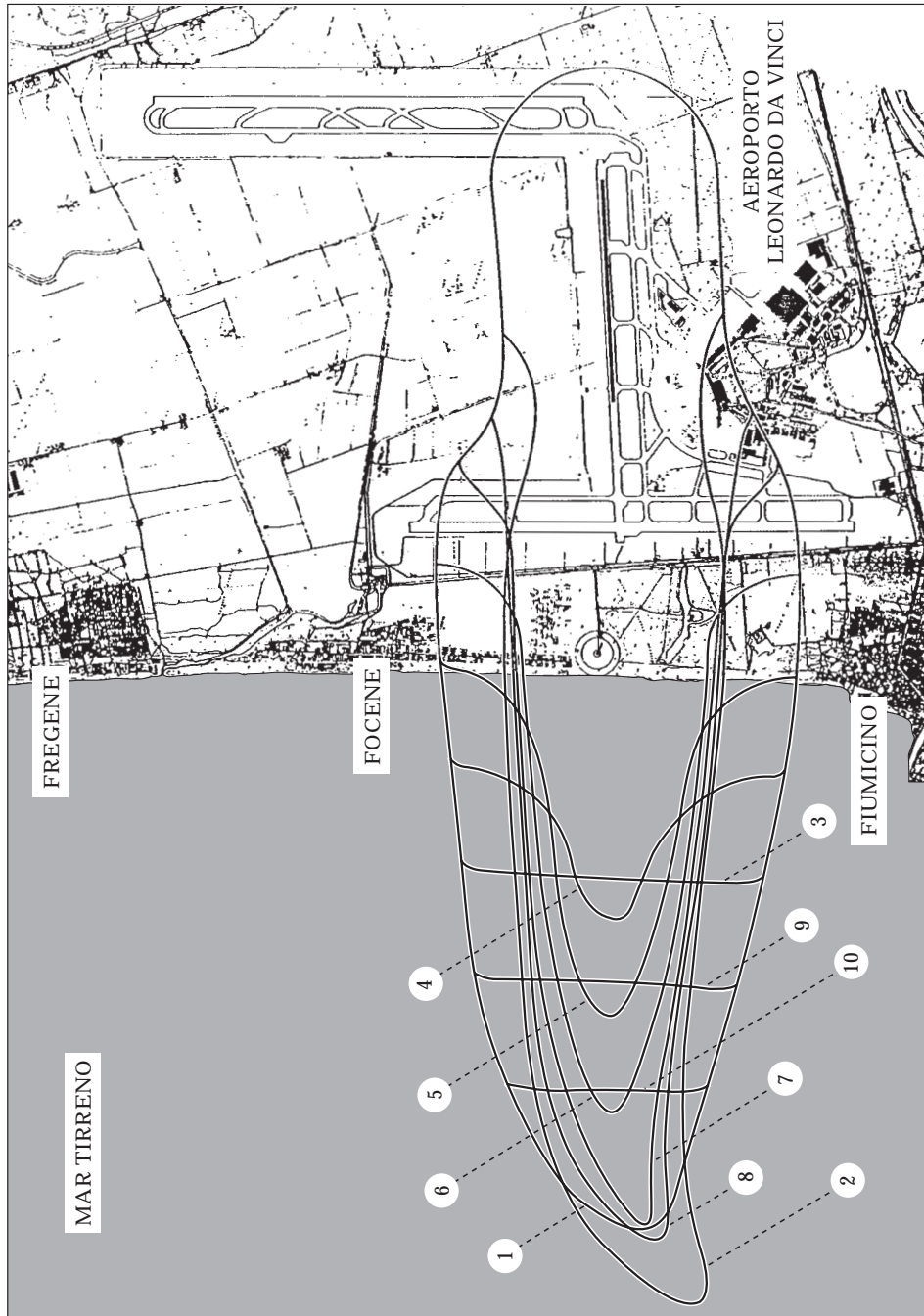


Figure 7. 80 EPNdB noise contours corresponding to different take-off profiles from runway no. 2 (25-07).

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