Journal of Sound and Vibration (1999) **222**(4), 547–563
Article No. jsvi.1998.2122, available online at http://www.idealibrary.com on IDEAL

WAVELET-BASED ANALYSIS OF THE NON-STATIONARY RESPONSE OF A SLIPPING FOUNDATION

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(*Received* 19 *March* 1998, *and in final form* 29 *October* 1998)

A wavelet-based stochastic formulation has been presented in this paper for the seismic analysis of a rigid block resting on a friction base. The ground motion has been modelled as a non-stationary process (both in amplitude and frequency) by using wavelets. The proposed formulation is based on replacing the non-linear system by an equivalent linear system with time-dependent properties. The expressions of the instantaneous damping, root mean square (r.m.s.) velocity response, and the power spectral density function (PSDF) of the velocity response have been obtained in terms of the input wavelet coefficients. For validation of the formulation, simulation based on twenty synthetically generated time-histories corresponding to an example ground motion process has been carried out. The effectiveness of the base-isolation system and the effect of the frequency non-stationarity on the non-linear response have also been studied in detail. It has been clearly shown how the frequency non-stationarity in the ground motion changes the non-linear response.

1. INTRODUCTION

Isolating structures from base excitations has emerged as a popular passive device to protect the structural systems from earthquake excitations. Among the available base-isolation techniques, the frictional base-isolation technique has been found to be reasonably effective and economical. The frictional base-isolation system operates on the basis of the slipping of the foundation during the moderate to high ground acceleration phases, and thus, limits the inertia forces exerted on the superstructure. The behaviour of such a system has been studied in a preliminary form by investigating the response of a rigid block which is subjected to base acceleration with dry or Coulomb's friction at the interface. Crandall *et al.* [1] obtained the root-mean-square (r.m.s.) value of the slip displacement by modelling the ground excitation as a white noise. Ahmadi [2] assumed the excitation to be a unit step modulated white-noise, and observed marginal differences between the stationary and non-stationary responses. The standard deviation of the slip displacement was obtained by applying equivalent linearization technique to solve the Fokker–Planck equation. Noguchi [3] estimated the slip displacement and acceleration transmission ratio in terms of the r.m.s. values for a linear mass-dashpot system subjected to filtered white-noise excitation.

The above mentioned studies have modelled the excitation process either as white-noise or as a filtered white-noise. The earthquake ground motion processes are however characterized by significant amplitude and frequency non-stationarity. By affecting the response considerably, the non-stationarity in an excitation process may actually influence the estimated parameters of the equivalent linear system. Even though the researchers in the past have emphasized more on the role of amplitude non-stationarity, the non-linear response of a dynamical system is also known to be sensitive to the frequency non-stationarity (e.g., see Yeh and Wen [4]). Recently, wavelet analysis has been developed as a very powerful tool to tackle frequency non-stationarities in earthquake ground motions. Wavelet transform provides information about the local frequency content in a process with both amplitude and frequency non-stationarities (Basu and Gupta [5]). By using this technique, Basu and Gupta [6, 7] have obtained the stochastic seismic responses of single-degree-of-freedom (SDOF) and classically damped multi-degree-of-freedom (MDOF) systems. Early contributions to the subject of wavelet analysis and its application to vibration analysis include Meyer [8], Daubechies [9], Chui [10], Stromberg [11], Grossman and Morlet [12], Battle [13], Lemarié [14], Mallat [15], and Newland [16–18].

This paper presents a formulation for the non-linear stochastic response of a friction base-isolated rigid block. The block is assumed to be subjected to a base acceleration process which is characterized by its wavelet co-efficients. The proposed formulation is validated through time history simulations. Further, the effectiveness of the base-isolation system and the effect of frequency non-stationarity on the non-linear response are investigated.

2. FORMULATION FOR STOCHASTIC RESPONSE

2.1. EQUATION OF MOTION

We consider a rigid block of mass, m, resting on a surface with friction coefficient, μ, and subjected to a non-stationary, zero-mean, locally Gaussian base acceleration process, $z(t)$. When the block slips, the relative displacement process, $x(t)$, can be described by

$$\ddot{x} + \mu \mathbf{g}\,\mathrm{sgn}(\dot{x}) = -z(t), \tag{1}$$

where $\mathbf{g}$ is the acceleration due to gravity; and $\mathrm{sgn}(\dot{x}) = 1$, for $\dot{x} \geqslant 0$, and -1 otherwise. During the sticking phase, the relative motion of the block with respect to the foundation is absent. Then, the relative acceleration is zero and the absolute acceleration is equal to the ground acceleration.

Let the input process, $z(t)$, be characterized by its wavelet coefficients, $E[W_\psi^2 z(a_j, b_i)]$ (see the Appendix for a brief discussion on wavelet transforms and time–frequency analysis). Using these wavelet coefficients, the instantaneous mean square values of the process can be written as (see Basu and Gupta [6, 7])

$$E[z^2(t)]|_{t=b_i} = K\sum_j \frac{E[W_\psi^2 z(a_j, b_i)]}{a_j} \tag{2}$$

to completely characterize the process, $z(t)$. The wavelet coefficients, $E[W_\psi^2 z(a_j, b_i)]$, are obtained by using a modified form of Littlewood–Paley (L–P) basis described by

$$\psi(t) = [1/\pi\sqrt{(\sigma-1)}](\sin\sigma\pi t - \sin\pi t)/t \tag{3}$$

This basis has been proposed by Basu and Gupta [5], and used earlier for the analysis of linear SDOF and MDOF systems (Basu and Gupta [6, 7]). Apart from a reasonably fast temporal decay (thus helping to capture the local temporal features), this basis offers the advantageous features like orthogonality and non-overlapping of the energy bands (Basu and Gupta [5–7]). The Fourier transform of this basis function may be expressed as

$$\hat{\psi}(\omega) = \left\{ \begin{array}{cc} 1/\sqrt{2(\sigma-1)\pi}, & \pi \leqslant |\omega| \leqslant \sigma\pi \\ 0 & \text{otherwise} \end{array} \right\}, \tag{4}$$

where σ is the scalar factor used in discretizing a (see Appendix for details).

2.2. WAVELET-BASED STOCHASTIC LINEARIZATION

On wavelet transforming both sides of equation (1), and on integrating $W_\psi \ddot{x}(a, b)$ by parts to give $(\partial/\partial b) W_\psi \dot{x}(a, b)$ (see Basu and Gupta [5] for details), one gets

$$(\partial/\partial b) W_\psi\, \dot{x}(a, b) + \mu\mathbf{g} W_\psi\, h(a, b) = -W_\psi\, z(a, b) \tag{5}$$

where $W_\psi h(a, b)$ denotes the wavelet transform of $\mathrm{sgn}(\dot{x})$. For linearization of equation (1), one considers an alternative equation with viscous damping, c, as

$$\ddot{x} + c\dot{x} = -z(t). \tag{6}$$

This can be written in wavelet domain as

$$(\partial/\partial b) W_\psi\, \dot{x}(a, b) + c W_\psi\, \dot{x}(a, b) = -W_\psi\, z(a, b). \tag{7}$$

On squaring the error, ε, between equations (5) and (7), one obtains

$$\varepsilon^2 = [\mu\mathbf{g} W_\psi\, h(a, b) - c W_\psi\, \dot{x}(a, b)]^2. \tag{8}$$

The error in the instantaneous response energy at a time instant, $t = b$, is now obtained by integrating ε^2 over a with a norm of $1/a^2$. Taking expectation of this error and minimizing with respect to c gives

$$\frac{\partial}{\partial c} E\left\{ \int \frac{1}{a^2} [\mu \mathbf{g} W_\psi \, h(a, b) - c W_\psi \, \dot{x}(a, b)]^2 \, \mathrm{d}a \right\} = 0. \tag{9}$$

On exchanging the integral operator with the expectation and differential operators, and on discretizing the integral (see equations (A.5) and (A.6) in the Appendix), one obtains

$$c = \mu \mathbf{g} \sum_j \frac{1}{a_j} E[W_\psi \, h(a_j, b_i) W_\psi \, \dot{x}(a_j, b_i)] \Big/ \sum_j \frac{1}{a_j} E[W_\psi^2 \, \dot{x}(a_j, b_i)]. \tag{10}$$

It may be mentioned here that compared to direct linearization in the time domain, the above (wavelet-based) linearization has been preferred since the input process is assumed to be characterized by its wavelet co-efficients. Equation (10) can be simplified by considering the following expressions

$$\sum_i \sum_j \frac{K}{a_j} E[W_{ij} \, \dot{x} W_{ij} \, h] = \sum_i E[\dot{x} \text{ sgn } \dot{x}] = \sum_i E[|\dot{x}|] \tag{11}$$

and

$$\sum_i \sum_j \frac{K}{a_j} E[W_{ij}^2 \, \dot{x}] = \sum_i E[\dot{x}^2]. \tag{12}$$

A deterministic version of equations (11) and (12) for continuous wavelet transform may be seen in Daubechies [9]. In these expressions, $W_{ij}(\cdot)$ denotes $W_\psi(\cdot)(a_j, b_i)$ for the notational convenience. Since the coefficients, $W_{ij}(\cdot)$, contribute locally at the instant, $t = b_i$, equations (11) and (12) lead to the following instantaneous relationships:

$$E[\dot{x}^2]\,|_{t=b_i} = \sum_j \frac{K}{a_j} E[W_{ij}^2 \, \dot{x}], \qquad E[|\dot{x}|]\,|_{t=b_i} = \sum_j \frac{K}{a_j} E[W_{ij} \, \dot{x} W_{ij} \, h]. \tag{13, 14}$$

Assuming the process $|\dot{x}|$ to be locally Gaussian at each instant, one has

$$E[|\dot{x}|] = \sqrt{2/\pi} \sqrt{E[\dot{x}^2]}, \tag{15}$$

and thus equations (13) and (14) lead to

$$\sum_j \frac{K}{a_j} E[W_{ij} \, \dot{x} W_{ij} \, h] = \sqrt{\frac{2}{\pi}} \sqrt{\sum_j \frac{K}{a_j} E[W_{ij}^2 \, \dot{x}]}. \tag{16}$$

On substituting this equation in equation (10), one obtains

$$c_i = \mu \mathbf{g} \sqrt{\frac{2}{\pi}} \left(1 \Big/ \sqrt{\sum_j \frac{K}{a_j} E[W_{ij}^2 \, \dot{x}]} \right) \tag{17}$$

as the expression of viscous damping, c, in the equivalent system at time instant,

$t = b_i$. This implies that it is necessary to consider time-varying damping in the equivalent system to linearize the non-linear system. One can simplify equation (17) further by describing the coefficients, $E[W_{ij}^2 \dot{x}]$, in terms of the input coefficients, $E[W_{ij}^2 z]$. For this, one formulates the stochastic input–output relationship in the wavelet domain. On substituting the following discretized wavelet expansion of $\dot{x}$,

$$\dot{x} = \sum_i \sum_j \frac{K\Delta b}{a_j} W_{ij} \dot{x} \psi_{a_j, b_i}(t), \tag{18}$$

and on taking Fourier transform of both sides of equation (6), one obtains

$$\sum_i \sum_j \frac{1}{a_j} W_{ij} \dot{x} \hat{\psi}_{a_j, b_i}(\omega) = \sum_i \sum_j \frac{1}{a_j} W_{ij} z Q(\omega) \hat{\psi}_{a_j, b_i}(\omega), \tag{19}$$

where

$$Q(\omega) = 1/(c_i - \mathrm{i}\omega) \tag{20}$$

is the frequency domain transfer function for the relative velocity response of the rigid block. Since the L–P basis function used for the analysis has the property that the dilates of the function have their Fourier transforms supported over mutually non-overlapping intervals, the response in one particular band is unaffected by the input in the other band. This property of the wavelet basis function further simplifes equation (19) to

$$\sum_i \frac{1}{a_j} W_{ij} \dot{x} \hat{\psi}_{a_j, b_i}(\omega) = \sum_i \frac{1}{a_j} W_{ij} z Q(\omega) \hat{\psi}_{a_j, b_i}(\omega). \tag{21}$$

On taking square of the amplitude on both sides of equation (21), integrating over ω, and on neglecting the cross-terms associated with the coefficient, $E[W_{ij} z W_{kj} f]$, $i \neq k$ (see Basu and Gupta [6, 7]), one obtains

$$\sum_i E[W_{ij}^2 \dot{x}] = \sum_i E[W_{ij}^2 z] \int_{-\infty}^{\infty} |\, Q(\omega) \,|^2 |\, \hat{\psi}_{a_j, b_i}(\omega) \,|^2 \, \mathrm{d}\omega. \tag{22}$$

Now, on using the time localization property of the wavelets, i.e., the property of the wavelet coefficients, $W_{ij} \dot{x}$, to temporally contribute to the response at the time instant, $t = b_i$, and its close neighbourhood only, one can write the following instantaneous input–output relationship

$$E[W_{ij}^2 \dot{x}] = E[W_{ij}^2 z] \int_{-\infty}^{\infty} |\, Q(\omega) \,|^2 \,|\, \hat{\psi}_{a_j, b_i}(\omega) \,|^2 \, \mathrm{d}\omega. \tag{23}$$

In view of this, equation (17) may be written as

$$c_i = \mu \mathbf{g} \sqrt{2} \Bigg/ \sqrt{\sum_j \frac{K}{(\sigma - 1) c_i} \tan^{-1}[(\sigma - 1)\pi c_i a_j / (c_i^2 a_j^2 + \sigma \pi^2)] E[W_{ij}^2 z]}. \tag{24}$$

This transcendental algebraic equation is to be solved iteratively to obtain the instantaneous value of c_i. This calculation can however be simplified further by using the following approximation,

$$\tan^{-1}(\sigma-1)\pi c_i a_j/(c_i^2 a_j^2+\sigma\pi^2)\approx(\sigma-1)\pi c_i a_j/(c_i^2 a_j^2+\sigma\pi^2) \tag{25}$$

to give

$$c_i^2\sum_j K\frac{\pi a_j}{c_i^2 a_j^2+\sigma\pi^2}E[W_{ij}^2 z]=2\mu^2\mathbf{g}^2. \tag{26}$$

The approximation in equation (25) holds good, since the maximum value of the kernel (i.e., $(\sigma-1)/2\sqrt{\sigma}$ as obtained for $c_i a_j=\pi\sqrt{\sigma})$ on the left side of equation (25) is much less than unity. Further, the term, $a_j/(c_i^2 a_j^2+\sigma\pi^2)$, in equation (26) varies with a very mild slope for variation in the expected range of values of $c_i a_j$. Thus, it can be treated as a constant independent of j. If the left side of equation (26) receives maximum contribution from $E[(W_{ij}^2 z)]$ for $j=m$, this constant may be assumed to be equal to $a_m/(c_i^2 a_m^2+\sigma\pi^2)$. Then, equation (26) gives the instantaneous damping of the equivalent system as

$$c_i=\mu\mathbf{g}\sqrt{2\pi\sigma}\bigg/\sqrt{K\sum_j E[W_{ij}^2 z]a_m-2a_m^2\mu^2\mathbf{g}^2/\pi} \tag{27}$$

directly in terms of the wavelet coefficients of the input process. This may now be used to characterize the response quantity of interest pertaining to the given non-linear system. For example, on substituting equation (27) into equation (17), we get the expression of instantaneous mean-square velocity response as

$$\sigma_i^2=E[\dot{x}^2]|_{t=b_i}=\frac{[K\sum_j E[W_{ij}^2 z]a_m-2a_m^2\mu^2\mathbf{g}^2/\pi]}{\pi^2\sigma}. \tag{28}$$

3. SPECTRAL MOMENTS

The statistics of a non-stationary response process may be estimated by obtaining the moments of the instantaneous PSDF of the process. To obtain the expression for the instantaneous PSDF of velocity response, one considers that the different energy bands in equation (23) are overlapping and that σ is close to unity. Thus, it is possible to write (see Basu and Gupta [6, 7])

$$S_{\dot{x}}|_{t=b_i}=\sum_j\frac{K}{a_j}[|\hat{\psi}(a_j\omega)|^2/(\omega^2+c_i^2)]E[W_{ij}^2 z], \tag{29}$$

as the expression for instantaneous PSDF. On taking the sth order moment, this equation gives

$$m_s|_{t=b_i}=\sum_j K'E[W_{ij}^2 z]\int_{\pi/a_j}^{\sigma\pi/a_j}\frac{\omega^s}{\omega^2+c_i^2}\,\mathrm{d}\omega, \tag{30}$$

with $K'=K/(\sigma-1)\pi$. On evaluating the integral in equation (30), the zeroth and the first instantaneous moments are obtained as

$$m_0\,|_{t=b_i} = \sum_j K' E[W_{ij}^2 z] I_{0,j}(c_i\,,\, a_j) \tag{31}$$

and

$$m_1\,|_{t=b_i} = \sum_j K' E[W_{ij}^2 z] I_{1,j}(c_i\,,\, a_j) \tag{32}$$

respectively, with

$$I_{0,j} = (1/c_i)\tan^{-1}[(\sigma - 1)\pi a_j\, c_i/(a_j^2 c_i^2 + \sigma\pi^2)] \tag{33}$$

and

$$I_{1,j} = \tfrac{1}{2}\ln[(\sigma^2\pi^2 + c_i^2 a_j^2)/(\pi^2 + c_i^2 a_j^2)]. \tag{34}$$

Further, the higher moments of the instantaneous PSDF can be obtained by using the following recursive relationship,

$$m_s\,|_{t=b_i} = \sum_j K' E[W_{ij}^2 z]\left\{\frac{1}{s}\left(\frac{\pi}{a_j}\right)^{s-1}(\sigma^{s-1} - 1) - c_i^2 I_{s-2,j}\right\}. \tag{35}$$

The moments of the instantaneous response PSDF can be used to obtain several response statistics of interest. For example, the instantaneous rate of crossings, Ω_i, and bandwidth parameter, λ_i, are respectively obtained as

$$\Omega_i = (1/2\pi)\sqrt{m_2\,|_{t=b_i}/m_0\,|_{t=b_i}}, \qquad \lambda_i = \sqrt{1 - m_1^2\,|_{t=b_i}/m_0\,|_{t=b_i}\, m_2\,|_{t=b_i}}. \tag{36, 37}$$

Further, the largest peak statistics in the process, $\dot{x}(t)$, with duration, T, can be calculated based on the probability that the process, $|\,\dot{x}(t)\,|$, remains below the level, $\dot{x}$, during the time interval, (0, T). This probability is given as (Vanmarcke [19])

$$P_T(\dot{x}) = \exp\left[-\int_0^T \alpha(t)\,\mathrm{d}t\right] = \exp\left[-\sum_i \alpha_i\,\Delta b\right], \tag{38}$$

where

$$\alpha_i = 2\Omega_i\{[1 - \exp(-\sqrt{\pi/2}\lambda_{ie}\,\dot{x}/\sigma_i)]/[1 - \exp(-\dot{x}^2/2\sigma_i^2)]\}\mathrm{e}^{-\dot{x}^2/2\sigma_i^2} \tag{39}$$

is the rate parameter with $\lambda_{ie} = \lambda_i^{1+b}$ and $b = 0{\cdot}2$.

4. VALIDATION OF PROPOSED FORMULATION

The proposed formulation has been validated by generating an ensemble of twenty accelerograms by using the SYNACC program (Wong and Trifunac [20]), as in Basu and Gupta [6, 7], for the recorded ground motion at Pacoima dam site during the 1971 San Fernando earthquake. These time histories are

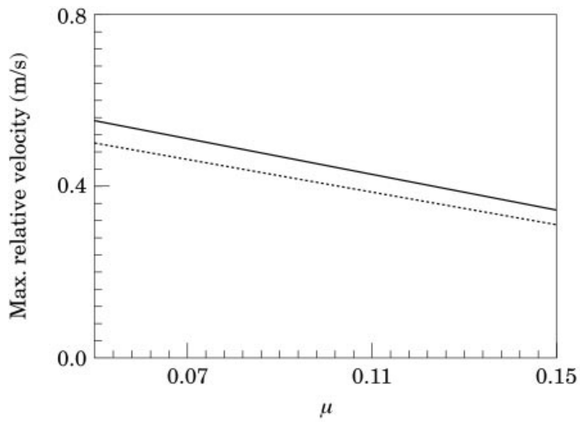

Figure 1. Comparison of the expected maximum velocity estimates from simulation and wavelet-based approach for different μ values. Key: —, simulation; $\cdots\cdot$, wavelet.

then used to obtain the wavelet coefficients (see equation (A.1)) with $i = 1$–2047, $j = -17$–4, $\sigma = 2^{1/4}$, and $\Delta b = 0{\cdot}02$. On averaging over the ensemble, these coefficients squared, i.e., $E[W_\psi^2 z(a_j, b_i)]$, characterize the stochastic input process.

Figure 1 shows the expected maximum relative velocity response as calculated from the (proposed) wavelet formulation as well as the time-history simulations for μ varying from 0·05–0·15. These results compare quite well, and as expected, it is observed that with the increase in the value of μ, the maximum velocity decreases. This indicates that with an increasing μ, the duration of the sticking phase increases with more transmission of vibration to the superstructure. Similar observations are made in Figure 2 for the r.m.s. velocity response. To have a more detailed comparison, the instantaneous r.m.s. velocity response for a typical value of $\mu = 0{\cdot}1$ has been calculated from the wavelet formulation and

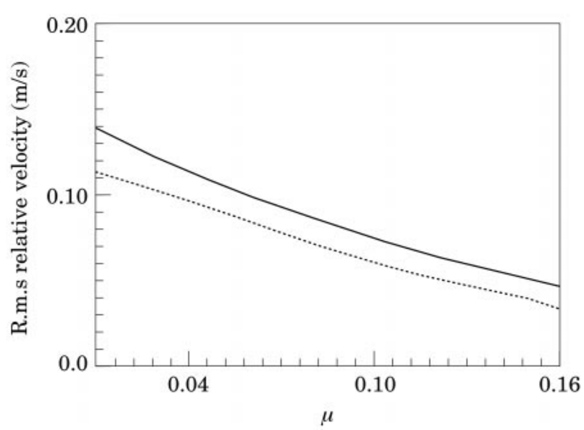

Figure 2. Comparison of the r.m.s. velocity estimates from simulation and wavelet-based approach for different μ values. Key: —, simulation; $\cdots\cdot$, wavelet.

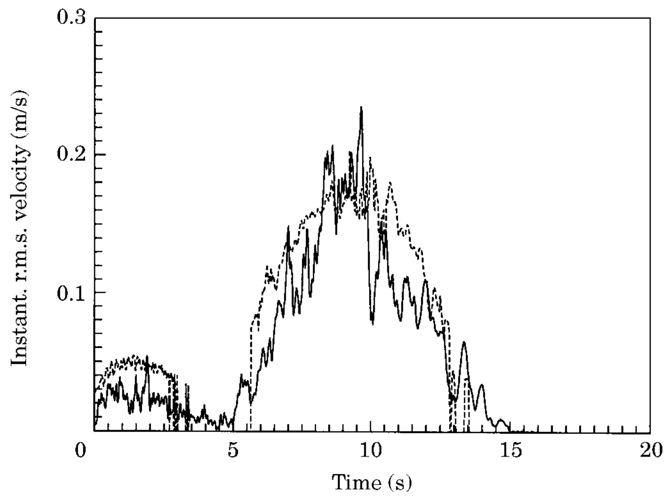

Figure 3. Comparison of the instantaneous r.m.s. velocity estimates from simulation and wavelet-based approach for $\mu = 0{\cdot}1$. Key: —, simulation; $\cdots\cdot$, wavelet.

the time-history analyses. The values plotted in Figure 3 show a very good agreement, particularly with respect to the sticking and slipping phases.

5. EFFECTIVENESS OF FRICTION BASE-ISOLATION SYSTEM

In this section, the proposed formulation is used to study the effectiveness of the friction base-isolation system. This is done in terms of the mean-square absolute acceleration of the block versus the mean-square ground acceleration.

It may be observed from equation (27) that the damping, c_i, of the equivalent system theoretically becomes infinite when the wavelet coefficients, $E[W_{ij}^2 z]$, of the input acceleration satisfy the condition that

$$K \sum_j E[W_{ij}^2 z] \geqslant 2a_m \mu^2 \mathbf{g}^2/\pi. \tag{40}$$

This implies that the system is in its sticking phase at the instant, $t = b_i$. Thus, for the total duration, T, of the excitation, the percentage sticking time, η_{stk}, can be obtained as

$$\eta_{\text{stk}} = \frac{100}{T} \sum_i U\left(-K \sum_j E[W_{ij}^2 z] + \frac{2a_m \mu^2 \mathbf{g}^2}{\pi}\right) \Delta b \tag{41}$$

where $U(\cdot)$ denotes the unit-step function. A value of η_{stk} close to 100% implies fixed-base behaviour of the structural system. When the system sticks to the foundation, the relative motion of the block with respect to the ground is absent. In that case, the absolute acceleration of the block is equal to the ground acceleration. When the base acceleration exceeds the value, $\mu\mathbf{g}$, the system starts slipping and the modulus of the absolute acceleration becomes equal to $\mu\mathbf{g}$. Now, on using the information on the relative durations of the sticking and slipping phases, it is possible to calculate the temporally integrated mean-square value, A^2, of the absolute acceleration of the block over the entire duration of the excitation. This can be expressed as

$$A^2 = \sum_i E[(\ddot{x} + z)^2]\,|_{t=b_i}\, \Delta b$$

$$= \sum_i \left\{ \sum_j \frac{K}{a_j} E[W_{ij}^2 z] Z_i(\mu) + [1 - Z_i(\mu)]\mu^2 \mathbf{g}^2 \right\} \Delta b \qquad (42)$$

with

$$Z_i(\mu) = U\left(-K \sum_j E[W_{ij}^2 z] + 2a_m\, \mu^2 \mathbf{g}^2/\pi \right). \qquad (43)$$

By taking a ratio of this with the temporally integrated value of the mean-square ground acceleration, one gets a measure of the effectiveness of the base-isolation. By calling the square-root of this ratio as the acceleration transmission ratio, r, it may be expressed as

$$r = \sqrt{\sum_i \left\{ \sum_j \frac{K}{a_j} E[W_{ij}^2 z] Z_i(\mu) + [1 - Z_i(\mu)]\mu^2 \mathbf{g}^2 \right\} \Bigg/ \sum_i \sum_j \frac{K}{a_j} E[W_{ij}^2 z]}. \qquad (44)$$

A lower value of this ratio implies greater effectiveness of the base-isolation system.

The instantaneous absolute acceleration responses as calculated from the proposed formulation are plotted in Figures 4 and 5 respectively for $\mu = 0{\cdot}1$ and 0·2. The results of time-history simulations using the ensemble as in section 4 are

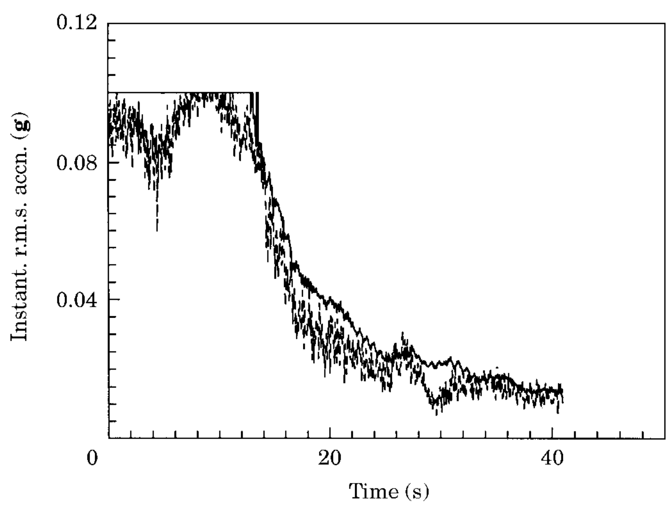

Figure 4. Comparison of the instantaneous r.m.s. acceleration estimates from simulation and wavelet-based approach for $\mu = 0{\cdot}1$. Key: —, simulation; · · · ·, wavelet.

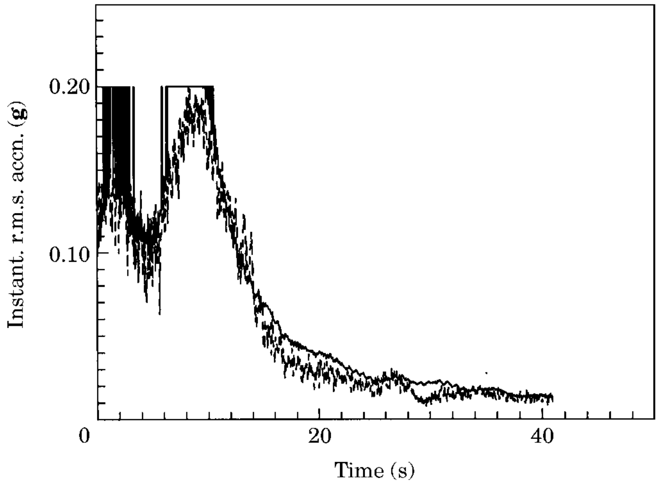

Figure 5. Comparison of the instantaneous r.m.s. acceleration estimates from simulation and wavelet-based approach for $\mu = 0{\cdot}2$. Key: —, simulation; $\cdots\cdot$, wavelet.

also shown in these figures. Both figures show a very good agreement in the two sets of results. The acceleration transmission ratio, r, as computed from the proposed formulation is also in good agreement with the simulation results as shown in Figure 6. It is also seen in this figure that the transmission of vibration may be reduced by 60% to 20% for μ ranging from 0·05–0·15.

In the friction base-isolation systems, the slip displacement is also an important quantity as this has to be restricted within a reasonable level from a functional point of view. It is however not possible to have the stochastic estimates of the slip displacements by using the present formulation as this is based on a zero mean, locally Gaussian response assumption. A typical realization of the time history of slip displacement as obtained for a sample of the generated ensemble (see section 5) is shown in Figure 7. This clearly indicates that the response is non-zero mean and non-Gaussian. Thus, the proposed

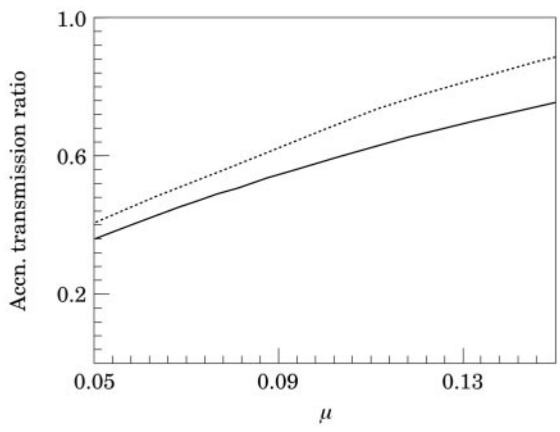

Figure 6. Comparison of the acceleration transmission ratio, r, from simulation and wavelet-based approach for different μ values. Key: —, simulation; $\cdots\cdot$, wavelet.

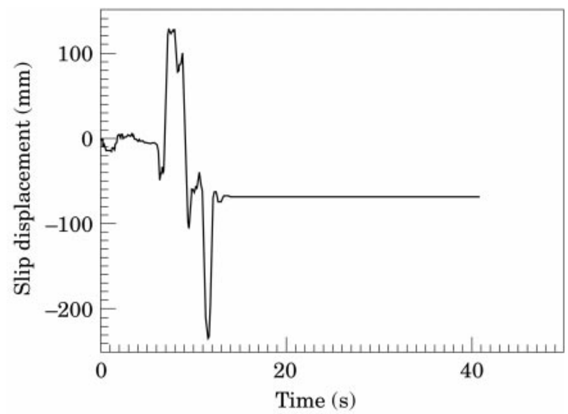

Figure 7. A typical slip displacement time history for $\mu = 0{\cdot}1$.

formulation needs to be extended to the non-zero mean and non-Gaussian response processes for the wavelet-based stochastic estimation of the slip displacement. It may be mentioned that the time-history simulations using the same ensemble show in Figure 8 that the average maximum slip displacements may be of the order of a few centimeters only, for the friction coefficients above 0·15.

6. EFFECT OF FREQUENCY NON-STATIONARITY

As mentioned in the introductory remarks, conventional modelling of the input (non-stationary) process as an amplitude-modulated process may not be acceptable for estimating the response of non-linear systems. To demonstrate this in the present case, one considers the ground acceleration process as characterized in section 4 through the wavelet coefficient functions, $E[W_{ij}^2 z]$, and alternatively model this as an amplitude-modulated process such that the new (modified) process has the same temporal non-stationary characteristics as the

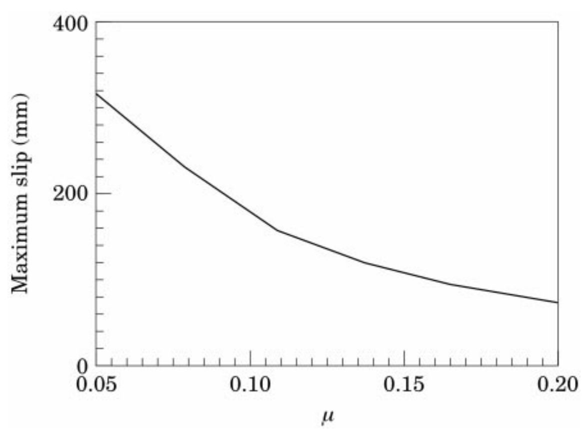

Figure 8. Variation of expected maximum slip displacement with μ.

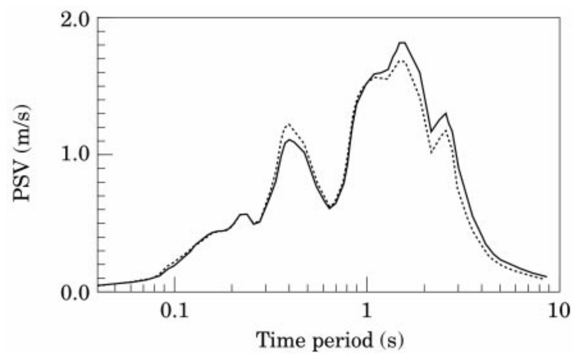

Figure 9. Comparison of 5% damping pseudo spectral velocity spectra for modified and unmodified processes. Key: —, modified; · · · ·, unmodified.

old (unmodifed) process in terms of instantaneous mean-square values. For the amplitude-modulated modelling, the coefficient functions, $E[W_{ij}^2 z]$, are modified in two stages. In the first stage, those are modified to $E[W_{ij}'^2 z]$ for different values of j in such a way that

$$E[W_{ij}'^2 z] = \frac{\Delta b}{T} \sum_m E[\tilde{W}_{mj}^2 z] \quad \forall i. \tag{45}$$

This condition ensures that the modified process has the same shape of the energy spectrum at all time instants and that this shape is same as that of the average energy spectrum of the unmodified process. Further, the co-efficients, $E[W_{ij}'^2 z]$ are modified by multiplying a constant factor to $E[\tilde{W}_{ij}^2 z]$, for all j values, such that

$$\sum_j \frac{K}{a_j} E[W_{ij}'^2 z] = \sum_j \frac{K}{a_j} E[\tilde{W}_{ij}^2 z]. \tag{46}$$

Through this condition, the (instantaneous) energy spectrum of the modified

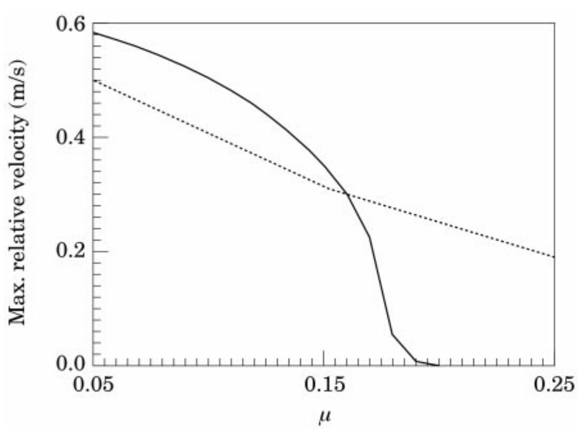

Figure 10. Comparison of expected maximum velocity for modified and unmodified processes in case of different μ values. Key: —, modified; · · · ·, unmodified.

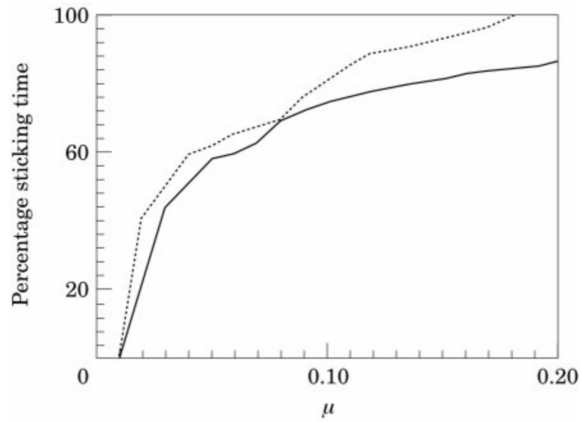

Figure 11. Comparison of η_{stk} for modified and unmodified processes in case of different μ values. Key: —, modified; $\cdots\cdot$, unmodified.

process is scaled up/down such that the instantaneous mean square values of both modified and original processes are identical.

The modified and original processes which are characterized by the wavelet coefficient functionals, $E[\tilde{W}_{ij}^2 z]$ and $E[W_{ij}^2 z]$ respectively, are identical as regards the response of a linear system. To show this, the expected pseudo-spectral velocity response has been estimated for the two processes in case of a set of linear SDOF oscillators with 5% damping and natural periods varying between 0·04 s to 8·5 s. The stochastic formulation of Basu and Gupta [7] has been used for this purpose. The results for the two processes have been compared in Figure 9. It is seen that the responses for both the processes are very close, and thus the original and modified processes are almost identical. The two processes may however lead to widely different responses in case of the proposed formulation

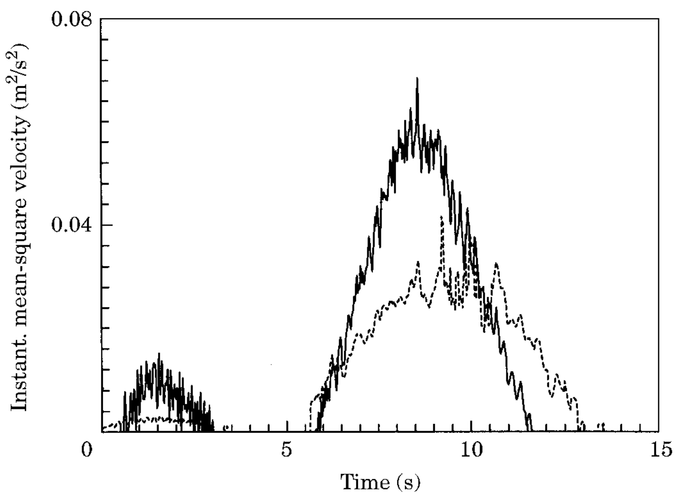

Figure 12. Comparison of instantaneous mean-square velocity for modified and unmodified processes in case of $\mu = 0{\cdot}1$. Key: —, modified; $\cdots\cdot$, unmodified.

for a non-linear system. Figure 10 shows this clearly in case of the expected (maximum) velocity response for a range of μ values. This effect of disparity in frequency non-stationarity is also supported by Figure 11 where the percentage sticking time, η_{stk}, is compared for the two processes. It may be seen that the modified process corresponds to the fixed-base response for $\mu > 0{\cdot}18$. On the other hand, the unmodified process corresponds to slipping for about 12–15% of the total excitation duration at these values of μ. A comparison of the instantaneous mean square velocity responses calculated for both the processes as shown in Figure 12 for $\mu = 0{\cdot}1$ also shows that the two 'identical' processes may lead to substantially different non-linear responses.

7. CONCLUSIONS

A wavelet-based stochastic linearization technique has been developed for obtaining the stochastic seismic response of a rigid block resting on a rough surface. This linearization technique involves the replacement of the non-linear system by a linear system with time-varying damping. The proposed formulation takes into account the effects of both amplitude and frequency non-stationarity, and accurately predicts the response statistics as seen in case of the considered example. It has also been shown how the proposed formulation can be used to test the effectiveness of a given friction base-isolation system for a ground motion process characterized through wavelet coefficients. Through another numerical study, it has been shown that the conventional use of frequency-independent modulating function to characterize the ground motion process may be inappropriate, particularly in case of the higher friction coefficients. This happens due to the sticking–slipping behaviour getting substantially altered in case of different frequency non-stationarity.

ACKNOWLEDGMENT

This research was supported by the Department of Atomic Energy, Government of India under grant number 11/16/94-G. The authors are grateful for this support.

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APPENDIX

A zero-mean, non-stationary process, $f(t)$, with locally Gaussian characteristics is considered. The wavelet transform, $W_\psi f(a, b)$, of $f(t)$ with respect to a wavelet basis, $\psi(t)$, and its inversion relationship are given respectively as follows (see, e.g., Daubechies [9])

$$W_\psi f(a, b) = \int_{-\infty}^{\infty} f(t)\psi_{a,b}(t)\,\mathrm{d}t, \quad a > 0 \tag{A.1}$$

and

$$f(t) = \frac{1}{2\pi C_\psi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{a^2}W_\psi f(a, b)\psi_{a,b}(t)\,\mathrm{d}a\,\mathrm{d}b \tag{A.2}$$

with

$$C_{\psi} = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} \, \mathrm{d}\omega < \infty. \tag{A.3}$$

In equations (A.1) and (A.2), $\psi_{a,b}(t)$ $(= |a|^{-1/2}\psi([t-b]/a))$ is the translated and dilated form of the wavelet function, $\psi(t)$, and in equation (A.3), $\hat{\psi}(\omega)$ is the Fourier transform of the basic wavelet function, $\psi(t)$, given by

$$\hat{\psi}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(t) \mathrm{e}^{-\mathrm{i}\omega t} \, \mathrm{d}t. \tag{A.4}$$

In the convolution integral of equation (A.1), the parameter, b, has the physical significance of localizing the basis function at $t = b$ and its neighbourhood. The parameter, a, captures the contribution of $f(t)$ to the frequency band corresponding to the domain of the Fourier transform of $\psi_{a,b}(t)$ (i.e., $\hat{\psi}(a\omega)\mathrm{e}^{\mathrm{i}\omega b}$). For numerical evaluation of the above integrals, a scheme similar to that by Alkemede [21] may be followed, and a and b may be discretized respectively at $a_j = \sigma^j$ and $b_j = (j-1)\Delta b$, where σ and Δb are the discretization parameters. The step changes at $a = a_j$ and $b = b_j$ respectively are now defined as

$$\Delta b_j = [(b_{j+1} - b_j) + (b_j - b_{j-1})]/2 = \Delta b \tag{A.5}$$

and

$$\Delta a_j = [(a_{j+1} - a_j) + (a_j - a_{j-1})]/2 = (a_j/2)(\sigma - 1/\sigma). \tag{A.6}$$

The discretized version of equation (A.2) is therefore obtained as

$$f(t) = \sum_i \sum_j (K\Delta b/a_j) W_{\psi} f(a_j, b_i) \psi_{a_j,b_i}(t) \tag{A.7}$$

with

$$K = (1/4\pi C_{\psi})(\sigma - 1/\sigma). \tag{A.8}$$

Also, the instantaneous mean-square value of the process, $f(t)$, is obtained as

$$E[f^2(t)]|_{t=b_i} = K \sum_j E[W_{\psi}^2 f(a_j, b_i)]/a_j. \tag{A.9}$$