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ANALYSIS OF VIBRATING RECTANGULAR ANISOTROPIC PLATES WITH FREE-EDGE HOLES

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1. INTRODUCTION

The structural designer is frequently confronted with the need of providing convenient passage for utility ducts through openings in slabs and beams. This situation leads to a more complicated response from a structural dynamics viewpoint.

A very thorough study has been performed by Abdalla and Kennedy in the case of prestressed concrete beams [l]. The determination of the fundamental frequency of transverse vibration of simply supported isotropic and orthotropic rectangular plates with rectangular and circular holes has been tackled by the authors using analytical and numerical techniques [2–4].

The present study is concerned with the determination of the first four frequency coefficients in the case of fully anisotropic rectangular plates with equal aspect ratio rectangular holes (Figure 1) using an analytical approach whereby the displacement amplitude is expressed in terms of beam functions. This means using a truncated double Fourier series which is the exact solution of the vibrating, solid rectangular plate simply supported at its four edges. The frequency determinant is generated using the Rayleigh–Ritz method.

2. ANALYTICAL SOLUTION

Following previous studies and using Lekhnitskii's well established notation [5], one expresses the governing functional in the form

$$J[W] = U - T, \tag{1}$$

where U and T are the potential and kinetic energies, given respectively by

$$U = \frac{1}{2}\int_{A_p} [D_{11} W_{xx}^2 + 2D_{12} W_{xx} W_{yy} + D_{22} W_{yy}^2 + 4D_{66} W_{xy}^2 + 4(D_{16} W_{xx} + D_{26} W_{yy}) W_{xy}] \, \mathrm{d}x \, \mathrm{d}y \tag{2}$$

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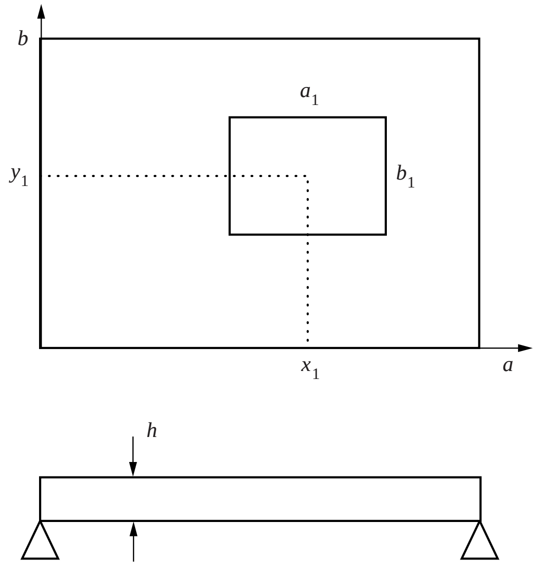

Figure 1. Vibrating system under consideration.

and

$$T = \frac{1}{2}\int_{A_p} \rho h \omega^2 W^2 \, \mathrm{d}x \, \mathrm{d}y, \tag{3}$$

where A_p is the plate area excluding the hole. That is to say, the energy functional in expression (1) is integrated over the physical doubly connected domain and so it is the difference between the virgin structural element and the hole energy functionals.

The displacement amplitude is now approximated using beam functions, given by the expression

$$W(x, y) \cong W_a(x, y) = \sum_{n=1}^{N}\sum_{m=1}^{M} A_{nm} \sin(nx/a) \, \sin(mx/b), \tag{4}$$

where each co-ordinate function $[\sin(nx/a) \; \sin(mx/b)]$ constitutes the exact eigenfunction in the case of an orthotropic, simply supported rectangular plate simply connected, and clearly, does not satisfy the governing, natural boundary conditions at the cutout. However, the procedure is a legitimate one when employing the Rayleigh–Ritz method: the vibrating beam functions constitute a complete set of trial functions and the minimization process of the functional (1) guarantees that, as the number of co-ordinate functions used approaches infinity, the generalized force type boundary conditions and the governing partial differential equation of motion will be exactly satisfied.

Substituting the approximating function (4) into equation (1) and minimizing $J[W]$ with respect to the A_{nm}'s results in a homogeneus, linear system of equations in the A_{nm}'s. The non-triviality condition yields a secular determinant whose roots constitute the frequency coeffcients $\Omega_i = \sqrt{\rho h / D_{11}}\omega_i a^2$.

3. NUMERICAL RESULTS

The first four frequency coefficients $\Omega_i = \sqrt{\rho h / D_{11}}\omega_i a^2$ have been determined for simply supported anisotropic rectangular plates of uniform thickness, with equal aspect ratio cutouts placed in different positions, and for $D_{12}/D_{11} = 0{\cdot}5$, $D_{22}/D_{11} = 0{\cdot}5$, $D_{66}/D_{11} = 0{\cdot}5$, $D_{16}/D_{11} = 1/3$, $D_{26}/D_{11} = 1/3$.

Tables 1–3 depict values of Ω_i for $b/a = 2/3$, 1 and 3/2, respectively, when the cutout takes three positions. The analytical determinations have been made with $M = N = 20$ and 30. Good rate of convergence was observed when increasing N and M.

4. CONCLUSIONS

As a general conclusion one may say that the fact that the use of a double Fourier series yields results, which seem to be very accurate as the size of the determinantal equation is enlarged, is quite interesting from an academic

TABLE 1

Frequency coefficients Ω_i (i = 1, 2, 3, 4) *determined in the case of a simply supported rectangular anisotropic plate* ($D_{12}/D_{11} = 0{\cdot}5$, $D_{22}/D_{11} = 0{\cdot}5$, $D_{66}/D_{11} = 0{\cdot}5$, $D_{16}/D_{11} = 1/3$, $D_{26}/D_{11} = 1/3$) *with a rectangular hole with free edge in the case* $b/a = 2/3$ (*see Figure* 1)

a_1/a	x_1/a	y_1/b	Number of terms	Ω_1	Ω_2	Ω_3	Ω_4
	1/4	1/4	400	29·00	57·89	78·82	98·16
			900	28·89	57·74	78·32	97·97
0·10	1/4	1/2	400	28·90	57·55	79·41	98·06
			900	28·76	57·27	79·18	97·79
	1/2	1/2	400	28·74	57·96	79·66	97·42
			900	28·55	57·82	79·39	97·09
	1/4	1/4	400	28·86	57·73	78·22	97·98
			900	28·73	57·57	77·78	97·75
0·15	1/4	1/2	400	28·65	57·28	78·72	97·60
			900	28·51	57·03	78·36	97·28
	1/2	1/2	400	28·35	57·73	78·68	97·09
			900	28·17	57·54	78·07	96·82
	1/4	1/4	400	28·65	57·49	78·47	97·58
			900	28·50	57·33	78·04	97·30
0·20	1/4	1/2	400	28·44	57·50	77·32	96·81
			900	28·29	57·26	76·84	96·44
	1/2	1/2	400	28·17	57·16	75·87	97·25
			900	27·98	56·92	75·03	97·00

TABLE 2

Idem Table 1, *for* $b/a = 1$ (*see Figure* 1)

a_1/a	x_1/a	y_1/b	Number of terms	Ω_1	Ω_2	Ω_3	Ω_4
	1/4	1/4	400	19·38	39·20	51·75	65·61
			900	19·31	39·10	51·42	65·48
0·10	1/4	1/2	400	19·32	39·05	51·69	65·59
			900	19·22	38·94	51·42	65·43
	1/2	1/2	400	19·23	39·26	52·26	65·28
			900	19·10	39·16	52·09	65·06
	1/4	1/4	400	19·30	39·08	51·41	65·44
			900	19·21	38·97	51·13	65·30
0·15	1/4	1/2	400	19·15	38·78	51·19	65·25
			900	19·05	38·65	50·95	65·08
	1/2	1/2	400	18·99	39·10	51·67	65·15
			900	18·88	38·98	51·28	64·98
	1/4	1/4	400	19·15	38·86	51·61	65·16
			900	19·06	38·75	51·34	64·98
0·20	1/4	1/2	400	18·97	38·41	51·12	64·68
			900	18·87	38·25	50·88	64·48
	1/2	1/2	400	18·90	38·72	49·94	65·35
			900	18·78	38·56	49·42	65·18

TABLE 3

Idem Table 1, *for* $b/a = 3/2$ (*see Figure* 1)

a_1/a	x_1/a	y_1/a	Number of terms	Ω_1	Ω_2	Ω_3	Ω_4
	1/4	1/4	400	14·47	25·02	39·43	44·09
			900	14·42	24·92	39·33	43·88
0·10	1/4	1/2	400	14·41	24·94	39·45	43·86
			900	14·34	24·88	39·29	43·65
	1/2	1/2	400	14·30	25·07	39·40	44·63
			900	14·21	25·01	39·23	44·47
	1/4	1/4	400	14·36	24·91	39·17	43·72
			900	14·30	24·78	39·06	43·55
0·15	1/4	1/2	400	14·22	24·76	39·24	43·31
			900	14·13	24·67	39·05	43·13
	1/2	1/2	400	14·02	25·01	39·46	43·83
			900	13·93	24·93	39·32	43·29
	1/4	1/4	400	14·19	24·72	38·82	43·82
			900	14·12	24·59	38·69	43·66
0·20	1/4	1/2	400	13·96	24·47	38·95	43·24
			900	13·87	24·36	38·79	43·05
	1/2	1/2	400	13·84	24·88	39·94	41·29
			900	13·75	24·77	39·82	40·63

viewpoint considering the fact that, individually, each co-ordinate function does not satisfy the boundary conditions at the edge of the hole. However, as the size of the determinant approaches infinity, the natural boundary conditions at the hole edges tend to be satisfied. The mathematical model is quite realistic, within the realm of the classical theory of vibrating plates.

In the case of a plate with clamped edges one can follow the useful approach developed in reference [6].

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