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A NOTE ON TRANSVERSE VIBRATIONS OF COMPOSITE, CIRCULAR MEMBRANES WITH A CENTRAL, POINT SUPPORT

P. A. A. LAURA, C. A. ROSSIT AND S. LA MALFA,
Institute of Applied Mechanics (CONICET) and Department of Engineering, Universidad Nacional del Sur, 8000 *Bahía Blanca, Argentina*

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1. INTRODUCTION

In a recent, interesting study Wang [1] proved that the fundamental frequency of a circular membrane with a central, pinpoint support is the same one as in the case of the circular membrane without central support. It is a well known fact the fundamental frequency coefficient is then

$$\Omega_{0_1} = \sqrt{\frac{\rho}{S}}\,\omega_{01} a = \beta_0, \tag{1}$$

where β_0 is the first root of Bessel's function of the first kind and order zero ($\beta_0 = 2{\cdot}4048256$).

Additional numerical and experimental studies [2–3] showed that the property also holds for higher axisymmetric modes of vibration. It is the purpose of this note to show that in the case of vibrating, composite circular membranes (Figure 1) with a central pinpoint support the natural frequencies corresponding to axisymmetric modes are also the same as the ones of vibrating, composite circular membranes without a central support. A numerical example is provided based on the situation where the internal radius "a" of a non-homogeneous annular membrane approaches zero; see Figure 2 [4].

2. NUMERICAL EXAMPLE

The exact eigenvalues of the configuration shown in Figure 2 vibrating axisymmetrically have been obtained in reference [4].

The solutions of the Helmholtz equations for each subdomain are

$$U_1 = A_n J_0\left(\frac{\omega}{\alpha_1} r\right) + B Y_0\left(\frac{\omega}{\alpha_1} r\right), \quad U_2 = C_n J_0\left(\frac{\omega}{\alpha_2} r\right) + D_n Y_0\left(\frac{\omega}{\alpha_2} r\right), \tag{2}$$

where $\alpha_1 = \sqrt{S/\rho_1}$ and $\alpha_2 = \sqrt{S/\rho_2}$.

The determinantal equation is generated using the boundary conditions at $r = a,b$ and the compatibility or continuity conditions at $r = c$.

Values of $\Omega_{0i} = \sqrt{(\rho_2/S)}\omega_{0i} b$ for several values of ρ_2/ρ_1, a/b and c/b were obtained in reference [4].

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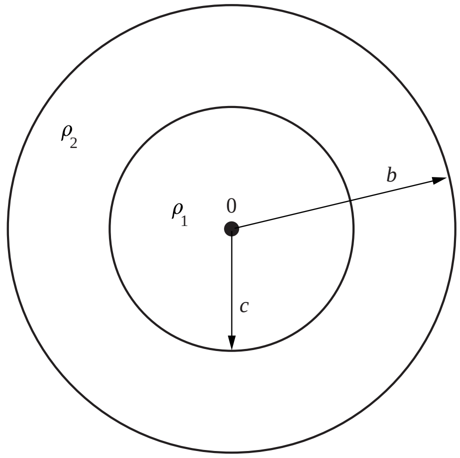

Figure 1. Circular, composite membrane with a central, pinpoint support.

The configuration corresponding to $\rho_2/\rho_1 = 10$ and $c/b = 0{\cdot}5$ was selected for the present numerical investigation. The parameter a/b was varied between 10^{-1} and 10^{-1000} and the first five eigenvalues Ω_{0i} were computed using *Mathematica* [5].

The numerical results are depicted in Table 1 which contains, in the last column at the right, the eigenvalues corresponding to a solid, composite membrane. One can immediately notice the fact that for $a/b = 10^{-1000}$, which resembles closely the case of a composite circular membrane with a central support, the eigenvalues are extremely close to those of a solid composite membrane without central support, the differences being of the order of 0·03%.

As a concluding remark it is of interest to point out that the present results and those obtained in previous studies [2, 3] are of interest in unsteady temperature distribution problems governed by Fourier's equation in the case of a very small hole at the center of the configuration. If the temperature $T(r, t)$ is

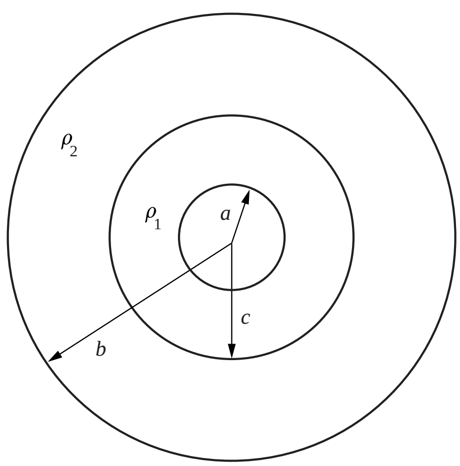

Figure 2. Circular, annular, composite membrane fixed at r = a, b.

TABLE 1

Values of Ω_{0i} in the case of a circular, annular, composite membrane as a/b decreases ($\rho_2/\rho_1 = 10$; $c/b = 0{\cdot}5$; see Figure 2)

a/b	10^{-1}	10^{-2}	10^{-10}	10^{-100}	10^{-1000}	Solid composite membrane
Ω_{01}	1·412682	1·091625	0·925961	0·896790	0·894005	0·893697
Ω_{02}	3·600570	2·871333	2·595209	2·556758	2·553237	2·552849
Ω_{03}	5·577940	4·695969	4·378908	4·340602	4·337173	4·336796
Ω_{04}	6·825371	6·087418	5·863862	5·838141	5·835858	5·835612
Ω_{05}	8·749776	7·284063	6·961916	6·931624	6·928997	6·928709

equal to zero at the boundaries for $t > 0$ one will obtain essentially the same eigenvalues as for the solid "disk". The eigenfunctions will resemble closely those corresponding to the solid "disk" also but with a sudden drop at $r = a \rightarrow 0$.

Certainly the thermoelastic situation will be considerably different and stress concentration will be developed by the presence of the very small hole. These considerations are certainly of interest to mechanical designers and analysts.

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