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FUNDAMENTAL FREQUENCY OF TRANSVERSE VIBRATION OF SIMPLY SUPPORTED AND CLAMPED REGULAR POLYGONAL PLATES WITH A CENTRAL, PINPOINT SUPPORT

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1. Introduction

The problem of vibrations of plates of complicated boundary shape is of interest in several areas of applied sciences and technology as in the case of printed circuit boards in electronic equipment subjected to severe dynamic environments [1]. Placing an additional internal support is sometimes required in order to increment its fundamental frequency; see Figure 1. The present study tackles the title problem using the conformal mapping method to transform the given shape in the z-plane onto a unit circle in the ζ-plane [2]. The transformed functional is then solved by means of the optimized Rayleigh–Ritz method [3].

2. Approximate solution

As it is well known the problem of small amplitude, transverse vibrations of thin elastic plates is governed by the functional

$$J(W) = D \int\int_P [(W_{x^2} + W_{y^2})^2 - 2(1-\upsilon)(W_{x^2} W_{y^2} - W_{xy}^2)]\, \mathrm{d}x\, \mathrm{d}y - \rho h \omega^2 \int\int_P W^2\, \mathrm{d}x\, \mathrm{d}y. \tag{1}$$

Regular, polygonal shapes in the z-plane are transformed onto unit circles in the ζ-plane by means of the Schwarz–Christoffel transformation yielding

$$z = x + \mathrm{i}y = f(\zeta) = A_s a_p \int_0^{\zeta} \frac{\mathrm{d}\zeta}{(1+\zeta^s)^{1/s}} = a_p A_s F(\zeta), \tag{2}$$

where s is the degree of the polygon, A_s is the coefficient [2], a_p is the apothem of the polygon, $\zeta = \xi + \mathrm{i}\eta = r\,\mathrm{e}^{\mathrm{i}\theta}$.

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Substituting equation (2) in equation (1) results in

$$J(W) = D \iint_C \left[\frac{1 + \upsilon(W_{\xi^2} + W_{\eta^2})^2}{|\, t'(\zeta)\,|^4} + \{(1-\upsilon)/2\} \,|\, (W_{\xi^2} - W_{\eta^2} - 2\mathrm{i} W_{\xi\eta}) f'(\zeta) - 2(W_\xi - \mathrm{i} W_\eta) f''(\zeta)\,|^2 / |\, f'(\zeta)\,|^6 \right] \times |\, f'(\zeta)\,|^2 \,\mathrm{d}\xi\,\mathrm{d}\eta - \rho h \omega^2 \iint_C W^2 \,|\, f'(\zeta)\,|^2 \,\mathrm{d}\xi\,\mathrm{d}\eta. \tag{3}$$

Finding an exact solution of equation (3) will be at best, an exceedingly difficult task. In view of this fact W will be replaced by an approximate functional relation $W_a(r)$ which, as a first order approximation, will not take into account the azimuthal dependence of the fundamental mode shape. In view of this equation (3) becomes, after some manipulations,

$$\frac{A_s^2 a_p^2}{D} J(W_a) = \iint_C \left[\frac{1+\upsilon}{2} \frac{(W''_a - W'_a/r)^2}{|\, F'(\zeta)\,|^2} + \frac{1-\upsilon}{2} \frac{|\,(W''_a - W'_a/r)\,\mathrm{e}^{-\mathrm{i}2\theta} F'(\zeta) - 2W'_a\,\mathrm{e}^{-\mathrm{i}\theta} F''(\zeta)\,|^2}{|\, F'(\zeta)\,|^4} \right] r\,\mathrm{d}r\,\mathrm{d}\theta - \frac{A_s^4}{16 tg^4 \pi/s} \Omega^2 \iint_C W^2 \,|\, F'(\zeta)\,|^2 \, r\,\mathrm{d}r\,\mathrm{d}\theta, \tag{4}$$

where $\Omega^2 = (\rho h a^4/D)\omega^2$, and a is the side of the polygon and $tg\ \pi/s = \tan \pi/s$.

The following expressions have been used for $W_a(r)$.

$$\textit{Simply supported edge}: \quad W_a = \sum_{j=1}^{3} C_j (r^2 - r^{p+j-1}). \tag{5}$$

$$\textit{Clamped edge}: \quad W_a = \sum_{j=1}^{3} C_j (r - r^{p+j-1})^2. \tag{6}$$

3. NUMERICAL RESULTS

In order to test the adequateness of the proposed approach it was first applied to a circular plate of radius a yielding $\Omega_1 = \sqrt{\rho h/D}\,\omega_1 a^2 = 14{\cdot}81$ in the case of a simply supported plate and 22·83 when the outer boundary is clamped, for $\nu = 0{\cdot}30$. Both results are in excellent agreement with the eigenvalues quoted in Leissa's classical monograph [4]. Table 1 depicts values of the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho h/D}\,\omega_1 a^2$ for simply supported and clamped plates of regular polygonal shape with a central, pinpoint support.

TABLE 1

Fundamental frequency coefficients, $\Omega_1 = \sqrt{\rho h/D}\omega_1 a^2$ *of regular polygonal plates with a central, pinpoint support*

	Square	Pentagon	Hexagon	Heptagon	Octagon
Simply supported	53·545	29·674	19·070	13·571	10·146
Clamped	80·773	44·451	28·764	20·311	15·183

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