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LETTERS TO THE EDITOR

THE MODELLING OF A FLEXIBLE BEAM WITH PIEZOELECTRIC PLATES FOR ACTIVE VIBRATION CONTROL

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1. INTRODUCTION

Piezoelectric materials can transform mechanical energy into electrical energy and *vice versa*. Indeed they strain when an electrical field is applied across them and, on the contrary, produce a voltage under stress. The first property allows one to use these devices as actuators, while the latter makes them well suited as sensors. These properties together with their small size and weight allow one to use the piezoelectric materials to construct smart devices that can be used, for example, for active control of vibrations.

The design of a high performance controller requires integrated modelling of the structure, of the sensors and of the actuators. The problem has been considered by several authors. In reference [1] a static model of a piezoelectric device is coupled with the dynamic model of the structure. The influence of the electric network connected to the piezoelectric plate in a passive scheme has been investigated in reference [2]. The capability of a single piece of piezoelectric material to concurrently operate as sensor and actuator has been shown in reference [3]. The model of distributed piezoelectric polymer has been obtained in terms of a partial differential equation in reference [4], and by using a FEM approach in reference [5]. A more general formulation of the coupling effects between structure and piezoelectric devices is presented in reference [6]. In that paper the general equation of an elastic body with piezoelectric material of arbitrary geometry and arbitrary electrode arrangement is proposed. However that approach is quite difficult to use, especially because it requires one to construct the overall model as a whole.

The main aim of this paper is to present a simplified model of the interactions between the piezoelectric plates when they are applied on a beam-like structure. In particular, the proposed approach allows the designer to construct the overall integrated model by means of straightforward manipulations of standard form

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of the mechanical model, namely the typical form obtained via FEM approaches.

The proposed approach has been used by the authors to design active vibration controllers for ATR42 turbofan aircraft (see references [7] and [8]).

2. BEAM MODEL

In this section a finite element model is obtained for a beam-like structure: i.e., a structure with a dimension greater than the other two. Because of the three-dimensional nature of this structure, a 3D finite element approach should be used. This approach, however, generates high-dimensional models. To overcome this difficulty the authors, in reference [9], developed a simplified approach based on ‘‘beam theory’’ and on finite element methods.

By introducing a set of reference axes $Oxyz$ as shown in Figure 1, where O is the mass center of the cross-section at $z = 0$, x and y are supposed parallel to the principal axis of the structure; then the motion of each cross-section can be evaluated by superposition of a vertical deflection u, a lateral deflection v, and a rotation angle θ.

In order to obtain a finite dimensional model, the flexible beam is divided into n elements, and it is assumed that the shape of each of them is described, at each time instant, by the corresponding static elastic line.

In the case of decoupled motions one has the dynamic model

$$\begin{pmatrix} \mathbf{M}_u & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_v & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_\theta \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{q}}_u \\ \ddot{\mathbf{q}}_v \\ \ddot{\mathbf{q}}_\theta \end{pmatrix} + \begin{pmatrix} \mathbf{K}_u & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_v & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_\theta \end{pmatrix} \begin{pmatrix} \mathbf{q}_u \\ \mathbf{q}_v \\ \ddot{\mathbf{q}}_\theta \end{pmatrix} = \begin{pmatrix} \mathbf{f}_u \\ \mathbf{f}_v \\ \mathbf{c}_\theta \end{pmatrix}, \tag{1}$$

where $\mathbf{q}_u = (u_1, \alpha_1, \ldots, u_{n+1}, \alpha_{n+1})^{\mathrm{T}}$ is the vector of vertical lagrangian coordinates, i.e., displacement u_i and slopes α_i in the x direction at abscissae z_i of the spatial discretization ($z_1 = O$, $z_{n+1} = L$, with L being the length of the beam), $\mathbf{f}_u = (f_{u_1}, c_{u_1}, \ldots, f_{v_{n+1}}, c_{u_{n+1}})^{\mathrm{T}}$ is the vector of the vertical external generalized forces acting at the $n+1$ nodes and $\mathbf{M}_u$, $\mathbf{K}_u$ are the mass and stiffness matrices of the vertical motions [10]. Similar notations are used for lateral motions with $\mathbf{q}_v = (v_1, \beta_1, \ldots, v_{n+1}, \beta_{n+1})^{\mathrm{T}}$ and $\mathbf{f}_v = (f_{u_1}, c_{v_1}, \ldots, f_{v_{n+1}}, c_{v_{n+1}})^{\mathrm{T}}$. As regards the torsional motions, $\mathbf{q}_\theta = (\theta_1, \chi_1, \ldots, \theta_{n+1}, \chi_{n+1})^{\mathrm{T}}$ is the vector of twist angles θ_i and rate of variation of twist angles χ_i of the cross-section at $n+1$ nodes, and $\mathbf{c}_\theta = (c_{\theta_1}, 0, \ldots, c_{\theta_{n+1}}, 0)^{\mathrm{T}}$ the vector of the torsion moments (for further details see reference [9]).

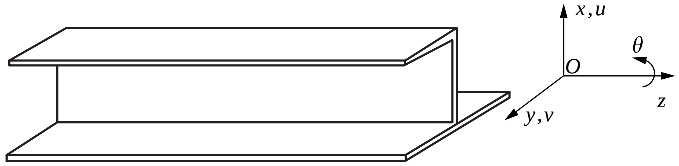

Figure 1. Beam co-ordinate system and displacement conventions.

It is worth noting that, in the case of a constrained beam, the dynamic model can be obtained from equation (1) by eliminating the clamped Lagrangian co-ordinates, the corresponding generalized forces and the corresponding rows and columns of the mass and stiffness matrices.

In order to obtain a more general model, one must remove the assumption of decoupled motions. Coupling phenomena can be determined by external constraints or external forces acting in other directions than the inertia axes. Moreover, coupling phenomena arise when the mass center G does not coincide with the shear center C, so that a transversal load produces a torsional moment and *vice versa*. These phenomena may be taken into account modifying the stiffness matrix [9] to become

$$\mathbf{K} = \begin{pmatrix} \mathbf{K}_u & 0 & y_c\mathbf{K}_u \\ 0 & \mathbf{K}_v & x_c\mathbf{K}_v \\ y_c\mathbf{K}_u & x_c\mathbf{K}_v & \mathbf{K}_\theta + x_c^2\mathbf{K}_u + y_c^2\mathbf{K}_v \end{pmatrix}, \tag{2}$$

where $C = (x_c, y_c)$ is the position of the shear center with respect to the center of mass. Then the dynamic model of the beam becomes

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{B}_f\mathbf{f}, \tag{3}$$

where $\mathbf{q} = [\mathbf{q}_u^{\mathrm{T}}, \mathbf{q}_v^{\mathrm{T}}, \mathbf{q}_\theta^{\mathrm{T}}]$ is the vector of Lagrangian coordinates, $\mathbf{M}$ and $\mathbf{K}$ the mass and stiffness matrix respectively, $\mathbf{f}$ the vector of actual generalized forces, and $\mathbf{B}_f$ a matrix relating the external forces to the significant forces $\mathbf{f}_u$, $\mathbf{f}_v$, $\mathbf{c}_\theta$.

Model (3) does not take into account internal damping. An easy way to assign to each mode a given damping coefficient ξ_i, is to consider a damping factor of the form

$$\mathbf{C} = \mathbf{M}\mathbf{V}\,\mathrm{diag}(2\xi_1\omega_1, \ldots, 2\xi_\nu\omega_\nu)\mathbf{V}^{-1}, \tag{4}$$

where $\mathbf{V}$ is the matrix of the eigenvectors of $\mathbf{M}^{-1}\mathbf{K}$, ν is the number of Lagrangian variables, and ω_i, $i = 1, \ldots, \nu$ are the natural frequencies of the system. Then the dynamic model of the mechanical system with damping may be put in the form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{B}_f\mathbf{f}. \tag{5}$$

3. PIEZOELECTRIC MATERIALS

In this section the piezoelectric material relations are summarized [6]. In particular, one focuses the attention on a piezoelectric plate with the poling direction orthogonal to the plate (see Figure 2).

In the absence of electrical field (e.g., short circuit), the mechanical characteristics are defined by

$$\boldsymbol{\sigma} = \mathbf{C}^E\boldsymbol{\delta}, \tag{6}$$

where $\boldsymbol{\delta} = (\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx})^{\mathrm{T}}$ is the vector of material strains and $\boldsymbol{\sigma} =$

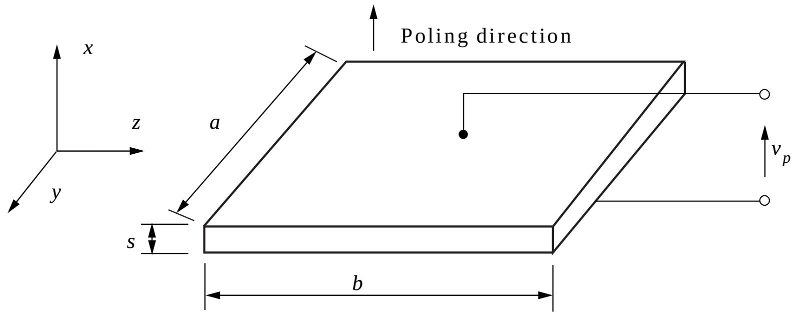

Figure 2. Piezoelectric plate with two thin film electrode surfaces.

$(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx})^{\mathrm{T}}$ is the vector of material stress. In the absence of strain (e.g., clamped element), the electrical characteristics are defined by

$$\mathbf{d} = \boldsymbol{\varepsilon}^S \mathbf{e}, \tag{7}$$

where $\mathbf{d} = (d_x, d_y, d_z)^{\mathrm{T}}$ is the charge displacement vector, $\mathbf{e} = (e_x, e_y, e_z)^{\mathrm{T}}$ is the electrical field vector.

The piezoelectric characteristics, in the absence of electrical field, are defined by

$$\mathbf{d} = \mathbf{P}\boldsymbol{\delta}, \tag{8}$$

and, in the absence of strain, by

$$\boldsymbol{\sigma} = -\mathbf{P}^{\mathrm{T}}\mathbf{e}. \tag{9}$$

Finally, the global constitutive relations of a piezoelectric element can be written as

$$\begin{pmatrix} \mathbf{d} \\ \boldsymbol{\sigma} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\varepsilon}^S & \mathbf{P} \\ -\mathbf{P}^{\mathrm{T}} & \mathbf{C}^E \end{pmatrix} \begin{pmatrix} \mathbf{e} \\ \boldsymbol{\delta} \end{pmatrix}. \tag{10}$$

As far as the matrices $\boldsymbol{\varepsilon}^S$, $\mathbf{P}$, $\mathbf{C}^E$ are concerned, if the poling direction coincides with the x-axis, equation (10) can be written as

$$\begin{bmatrix} d_y \\ d_z \\ d_x \\ \hline \sigma_y \\ \sigma_z \\ \sigma_x \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \left[\begin{array}{ccc|cccccc} \varepsilon_1^S & 0 & 0 & 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & \varepsilon_1^S & 0 & 0 & 0 & 0 & e_{15} & 0 & 0 \\ 0 & 0 & \varepsilon_3^S & e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \\ \hline 0 & 0 & -e_{31} & c_{11}^E & c_{12}^E & c_{13}^E & 0 & 0 & 0 \\ 0 & 0 & -e_{31} & c_{12}^E & c_{11}^E & c_{13}^E & 0 & 0 & 0 \\ 0 & 0 & -e_{33} & c_{13}^E & c_{13}^E & c_{33}^E & 0 & 0 & 0 \\ 0 & -e_{15} & 0 & 0 & 0 & 0 & c_{44}^E & 0 & 0 \\ 0e_{15} & 0 & 0 & 0 & 0 & 0 & 0 & c_{55}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{55}^E \end{array}\right] \begin{bmatrix} e_y \\ e_z \\ e_x \\ \hline \varepsilon_y \\ \varepsilon_z \\ \varepsilon_x \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix}. \tag{11}$$

The superscript S means that the values are measured at constant strain and the superscript E means that the values are measured with short-circuited electrodes.

Let one consider the thin piezoelectric plate shown in Figure 2 and let s be its thickness. Moreover by assuming that, in the presence of an electric voltage $v_p(t)$ applied via two thin film electrode surfaces, the electrical field is constant within the plate volume and equal to $v_p(t)/s$. Under these hypotheses, for each planar, infinitesimally small element of the plate one can write

$$\sigma_x = c_{33}^E \varepsilon_x + c_{13}^E \varepsilon_y + c_{13}^E \varepsilon_z - e_{33} v_p/s, \tag{12a}$$

$$\sigma_y = c_{13}^E \varepsilon_x + c_{11}^E \varepsilon_y + c_{12}^E \varepsilon_z - e_{31} v_p/s, \tag{12b}$$

$$\sigma_z = c_{13}^E \varepsilon_x + c_{12}^E \varepsilon_y + c_{11}^E \varepsilon_z - e_{31} v_p/s, \tag{12c}$$

$$\tau_{yz} = c_{44}^E \gamma_{yz}, \quad \tau_{zx} = c_{55}^E \gamma_{zx}, \quad \tau_{xy} = c_{55}^E \gamma_{xy}, \tag{12d–f}$$

$$d_x = e_{31}(\varepsilon_y + \varepsilon_z) + e_{33}\varepsilon_x + \varepsilon_3^S v_p/s. \tag{12g}$$

When the piezoelectric plate is perfectly bounded on the mechanical structure, the strain equals to the corresponding one of the structure at the interface.

Upon assuming that there are no external forces acting on the piezoelectric plate, the mechanical interaction between the piezoelectric plate and the structure essentially consists of a uniformly distributed force acting at the contour, orthogonal to the contour itself [1] and having a value per unit of contour length given by $p = -e_{31} v_p$; while the charge displacement vector is

$$d_x = (e_{31} - e_{33} c_{13}^E / c_{33}^E)(\varepsilon_y + \varepsilon_z) + (\varepsilon_3^S + e_{33}/c_{33}^E) v_p/s. \tag{13}$$

The current i_p drained by the piezoelectric plate is equal to

$$i_p = \int\int_S \frac{\mathrm{d}\, d_x}{\mathrm{d}t}\, \mathrm{d}S = \pi_p \int\int_S \frac{\mathrm{d}}{\mathrm{d}t}(\varepsilon_y + \varepsilon_z)\, \mathrm{d}S + C_p \frac{\mathrm{d}\, v_p}{\mathrm{d}t}, \tag{14}$$

where S indicates the piezoelectric area,

$$\pi_p = e_{31} - e_{33} c_{13}^E / c_{33}^E \tag{15}$$

and C_p is the piezoelectric capacitance

$$C_p = (\varepsilon_3^S + e_{33}/c_{33}^E)(S/s). \tag{16}$$

In most applications the term e_{33}/c_{33}^E is negligible.

4. INTEGRATED MODEL

The integrated mechanical/piezoelectric model should take into account the dynamics of the mechanical structure, the dynamics of the piezoelectric plates and the dynamics of the external electrical circuits. However, in spite of use specialized procedure [6, 5], one can obtain a simplified model integrating the previous described models. For the sake of notational simplicity, a structure with only one piezoelectric plate will be considered.

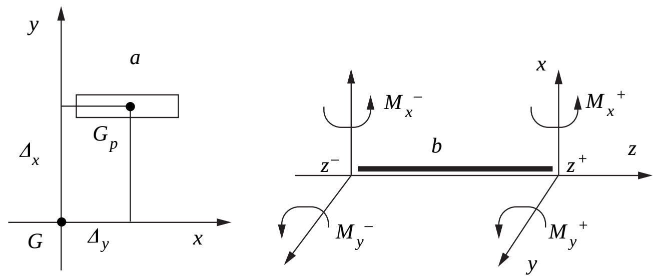

Figure 3. Action exerted by piezoelectric plate on a beam structure.

The mechanical part of the piezoelectric dynamics can be simply included into the mechanical model of the structure by modifying the mechanical parameters (mass, stiffness, etc.) of the *elements* on which the piezoelectric plate is bounded. Henceforth one refers to equation (5) as the mechanical model of structure and bonded piezoelectric plate.

The forces exerted by the piezoelectric plate on the structure give rise to some torque moments acting on the elastic line and to a couple of axial forces of magnitude $F = ae_{31}v_p$, that can be generally neglected. In particular, for a piezoelectric plate bonded with a side parallel to the beam axis (see Figure 3), the torques will be

$$M_x^+ = ae_{31}\Delta_x v_p, \quad M_y^+ = -ae_{31}\Delta_y v_p, \tag{17}$$

$$M_x^- = -ae_{31}\Delta_x v_p, \quad M_y^- = ae_{31}\Delta_y v_p, \tag{18}$$

where a is the length of the piezoelectric plate in the direction orthogonal to the beam axis, and Δ_x and Δ_y are the distances of the center of mass of the piezoelectric plate from the principal axis of inertia of the structure. The moments are applied at the knots in the correspondence of the piezoelectric plate extreme. It is useful to impose that z^- and z^+ coincide with knots of the spatial discretization. In this case, by means of straightforward calculations, the torques can be taken into account in the vector of generalized forces $\mathbf{f}$ of (5) by writing

$$\mathbf{f} = \mathbf{f}_p^{\mathrm{T}} v_p(t), \tag{19}$$

where $\mathbf{f}_p$ can be constructed by means of the scheme

$$\begin{aligned} \mathbf{q} = \quad & (\ldots,\ \alpha^-, \ldots,\ \alpha^+, \ldots,\ \beta^-, \ldots,\ \beta^+, \ldots)^{\mathrm{T}}, \\ & \qquad \downarrow \qquad\quad \downarrow \qquad\quad \downarrow \qquad\quad \downarrow \\ \mathbf{f}_p = e_{31}a & (\ldots,\ -\Delta_x, \ldots,\ \Delta_x, \ldots,\ \Delta_y, \ldots,\ -\Delta_y, \ldots)^{\mathrm{T}}, \end{aligned} \tag{20}$$

where α^- is the slope at the z^- extreme of the piezoelectric plate (see Figure 3), and so on.

As far as the electric dynamics are concerned, the current drained by the piezoelectric plate is given in equation (14) as a function of some components of the strain vector ε. To express this current as a function of the Lagrangian variables, it is convenient to rewrite equation (14) as

$$i_p = (\mathrm{d}/\mathrm{d}t)Q \tag{21}$$

where

$$Q = \iint_S d_x \, \mathrm{d}S = \pi_p \iint_S (\varepsilon_y + \varepsilon_z) \, \mathrm{d}S + C_p v_p. \tag{22}$$

To evaluate the integral on the right side of equation (22), one notes that

$$\iint_S (\varepsilon_y + \varepsilon_z) \, \mathrm{d}S = \int_{\Delta_y - a/2}^{\Delta_y + a/2} \left(\int_{z^-}^{z^+} (\varepsilon_y + \varepsilon_z) \, \mathrm{d}z \right) \mathrm{d}y, \tag{23}$$

where z^- and z^+ are the abscissae of the piezoelectric plate extreme. The term $\varepsilon_y + \varepsilon_z$ related to the ith element of the discretized beam can be approximated, by using the elastic line equation, as

$$\begin{aligned} \varepsilon_{yi} + \varepsilon_{zi} \approx \frac{m-1}{m} \Bigg\{ & \left[(u_{i+1} - u_i) \left(\frac{12}{l_i^3} \xi - \frac{6}{l_i^2} \right) + \alpha_i \left(\frac{4}{l_i} - \frac{6}{l_i^2} \xi \right) + \alpha_{i+1} \left(\frac{2}{l_i} - \frac{6}{l_i^2} \xi \right) \right] x \\ & + \left[(v_{i+1} - v_i) \left(\frac{12}{l_i^3} \xi - \frac{6}{l_i^2} \right) + \beta_i \left(\frac{4}{l_i} - \frac{6}{l_i^2} \xi \right) + \beta_{i+1} \left(\frac{2}{l_i} - \frac{6}{l_i^2} \xi \right) \right] y \Bigg\}, \end{aligned} \tag{24}$$

where m is the Poisson's ratio of the beam, l_i is the length of the ith element and $\xi \in [0, \, l_i]$.

The inner integral of equation (23) can be rewritten as

$$\sum_{i=p^-}^{p^+} \int_0^{l_i} (\varepsilon_{yi} + \varepsilon_{zi}) \, \mathrm{d}\xi, \tag{25}$$

where p^- and p^+ are the indices of the piezoelectric plate abscissae extreme. By cumbersome calculations the integral expression (25) can be rewritten as

$$(\alpha^+ - \alpha^-)\Delta_x + (\beta^- - \beta^+)y, \tag{26}$$

and then one has

$$\iint_S (\varepsilon_y + \varepsilon_z) \, \mathrm{d}S = (\alpha^+ - \alpha^-)\Delta_x a + (\beta^- - \beta^+)\Delta_y a, \tag{27}$$

so that the current drained by the piezoelectric plate may be expressed as

$$i_p = \pi_p[(m-1)/m][(\dot{\alpha}^+ - \dot{\alpha}^-)a\Delta_x + (\dot{\beta}^- - \dot{\beta}^+)a\Delta_y] + C_p \dot{v}_p. \tag{28}$$

Equation (28) may be rewritten in the more compact form

$$i_p = \mathbf{h}^{\mathrm{T}} \dot{\mathbf{q}} + C_p \dot{v}_p, \tag{29}$$

where

$$\mathbf{h} = [(m-1)/m](\pi_p/e_{31})\mathbf{f}_p. \tag{30}$$

Remark 1. It is interesting to note that i_p depends on the Lagrangian coordinates $\mathbf{q}$ through the same vector $\mathbf{f}_p$ that apears in equation (19); thus the well known passivity property of static colocated feedback is guaranteed.

The electric dynamics of the piezoelectric plate and of the external electric circuits may be included into the mechanical model by considering the equivalent scheme shown in Figure 4, in which $\mathcal{P}$ is an "ideal" piezoelectric element which absorbs the current $i_p = \mathbf{h}^{\mathrm{T}}\dot{\mathbf{q}}$ and exerts a force given by equation (19) on the mechanical structure,

$$(\mathrm{d}/\mathrm{d}t)i = -(R/L)i - (1/L)v_p + (1/L)v_0, \tag{31}$$

$$(\mathrm{d}/\mathrm{d}t)v_p = (1/C_p)i - (1/R_pC_p)v_p - (\mathbf{h}^{\mathrm{T}}/C_p)\dot{\mathbf{q}}. \tag{32}$$

$v_0(t)$ is the internal voltage of the voltage source, $v_p(t)$ is the voltage at the piezoelectric electrodes, $i(t)$ is the total current drained by the piezoelectric plate, R and L are the electrical resistance and inductance of the voltage source and of the electrode leads, C_p is the capacitance of the piezoelectric plate and R_p is a resistance taking into account the dielectric energy dissipated into the piezoelectric plate.

Remark 2. The model of a piezoelectric plate used as sensor is obtained from the previous equations by setting $v_0 = 0$; i.e., short-circuiting the piezoelectric electrodes.

The whole dynamic model in the presence of one piezoelectric sensor and one piezoelectric actuator is

$$\begin{pmatrix} \mathbf{M} & \mathbf{0} & \mathbf{0} \\ \mathbf{h}_a^{\mathrm{T}}/C_{pa} & 1 & 0 \\ \mathbf{0} & 0 & 1 \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{q}} \\ \ddot{v}_a \\ \ddot{i}_s \end{pmatrix} + \begin{pmatrix} \mathbf{C} & \mathbf{0} & \mathbf{f}_{ps}L_s \\ \mathbf{h}_a^{\mathrm{T}}R_a/C_{pa}L_a & a'_a & 0 \\ -\mathbf{h}_s^{\mathrm{T}}/L_sC_{ps} & 0 & a'_s \end{pmatrix} \begin{pmatrix} \dot{\mathbf{q}} \\ \dot{v}_a \\ \dot{i}_s \end{pmatrix}$$
$$+ \begin{pmatrix} \mathbf{K} & -\mathbf{f}_{pa} & R_s\mathbf{f}_{ps} \\ \mathbf{0} & a''_a & 0 \\ \mathbf{0} & 0 & a''_s \end{pmatrix} \begin{pmatrix} q \\ v_a \\ i_s \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{B}_f \\ 1/L_aC_{pa} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v_0 \\ \mathbf{f} \end{pmatrix}, \tag{33}$$

where the subscript s refers to piezoelectric sensor quantities and the subscript a to actuator ones, and

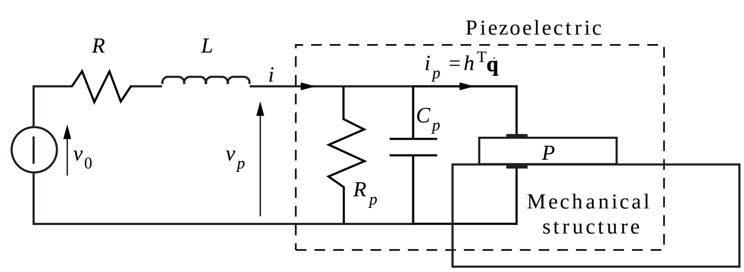

Figure 4. Equivalent scheme of a piezoelectric device with external electronics.

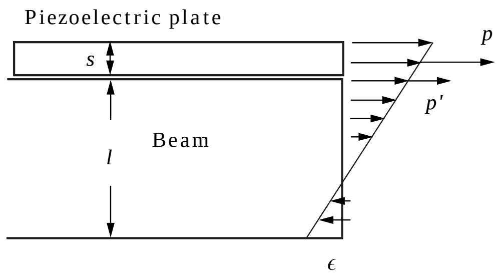

Figure 5. Reduction of the efficiency of a piezoelectric actuator due to finite thickness.

$$a'_a = R_a/L_a + 1/R_{pa}C_{pa}, \quad a''_a = R_a/L_aR_{pa}C_{pa} + 1/L_aC_{pa},$$
$$a'_s = R_s/L_s + 1/R_{ps}C_{ps}, \quad a''_s = R_s/L_sC_{ps}R_{ps} + 1/L_sC_{ps}.$$

It can be noted that, even if $\mathbf{h}_a = \mathbf{h}_s$ (colocated control), by choosing $v_0(t)$ proportional to $i_s(t)$, the passivity condition is no longer guaranteed, due to the presence of sensor and actuator dynamics in accordance with the well known result of reference [11].

Remark 3. Note that due to finite thickness of the piezoelectric plate and of the beam, only a fraction of the generated force,

$$p' = \eta p, \tag{34}$$

is transmitted to the beam (see Figure 5). Upon assuming linear strain and that the beam surface is equal to that of the bonded piezoelectric plate, an approximation of η is given in reference [9] as

$$\eta \approx 1/(1 + (4s/l)(s^E_{11} + s^E_{12})/\bar{s}_{11} + \bar{s}_{12}), \tag{35}$$

where s^E_{ij} are the entries of the inverse of the $\mathbf{C}^E$ matrix, and $\bar{s}_{ij}$ are analogue parameters for the beam. This phenomenon may be taken into account by modifying equations (17) to become

$$M^+_x = \bar{c}_x a e_{31} \Delta_x v_p, \quad M^+_y = -\bar{c}_y a e_{31} \Delta_y v_p, \tag{36}$$

$$M^-_x = -\bar{c}_x a e_{31} \Delta_x v_p, \quad M^-_y = -\bar{c}_y a e_{31} \Delta_y v_p, \tag{37}$$

where $\bar{c}_x$ and $\bar{c}_y$ are positive constants smaller than 1.

5. CONCLUSIONS

In this note, the authors have shown how to develop a simplified model of a beam-like structure with bonded piezoelectric plates. The model is obtained by integrating usual electrical and mechanical models. In particular the mechanical structure is modelled by means of a FEM approach, whereas electrical dynamics of the piezoelectric plates have been described via an RLC circuit.

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