Journal of Sound and Vibration (1999) **223**(4), 529–541
Article No. jsvi.1998.2161, available online at http://www.idealibrary.com on IDEAL®

THE APPLICATION OF CORRELATION DIMENSION IN GEARBOX CONDITION MONITORING

J. D. JIANG, J. CHEN
The State Key Laboratory of Vibration, Shock & Noise, Shanghai Jiao Tong University, Shanghai 200030, *People's Republic of China*

AND

L. S. QU
Research Institute of Machinery Diagnostics & Cybernetics, Xi'an Jiao Tong University, Xi'an 710049, *People's Republic of China*

(*Received* 18 *May* 1998, *and in final form* 28 *November* 1998)

This paper reports a study of correlation dimension in gearbox condition monitoring. In contrast to other fault diagnosis methods, such as Fourier spectrum analysis, time–frequency analysis, etc., the correlation dimension can provide some intrinsic information of an underlying dynamic system reconstructed from measured scalar time series. A three-stage analysis procedure using correlation dimension is presented. Some important influencing factors relating directly to the computational precision of correlation dimension are discussed. Industrial gearbox vibration signals measured from different operating conditions are analyzed using the above method. Results show that the correlation dimension is able to identify clearly a gearbox-operating condition with fatigue crack or broken tooth compared with the normal condition.

1. INTRODUCTION

A gear transmission converts a rotary input motion into another rotary motion at a different frequency. In an ideal mechanism the relationship between input and output depends only on geometry or kinematics of the meshing gears. However, in real industrial gearboxes, non-linear departures from the ideal mechanism, such as gaps, play and friction, clause complex dynamic behavior of the meshing mechanism. Though there have been many efforts in gearbox faults diagnosis [1–3], the understanding of non-linear vibration or noise in gearboxes has been a neglected subject. Modern developments in non-linear dynamics have provided new tools to deal with this hitherto unsolved problem.

A number of papers have appeared which treat the possibility of non-linear and chaotic vibrations in gear kinematics mechanism. Complex dynamic compacts in gear transmission systems have been given by Pfeiffer and co-

0022–460X/99/240529 + 13 $30.00/0

workers, who have pioneered in the application of the Poincaré map technique to predict noise in automotive and other gear transmission systems [4, 5]. Comparin and Singh have investigated various non-linear vibrations of gear systems including chaotic dynamics [6].

The development of non-linear dynamics theory has brought new methodologies to identify and forecast complex non-linear vibration behavior. One of the important concepts is the correlation dimension [7–9]. Calculating correlation dimension can be used in that it: (1) gives an indication as to how many state variables are influencing the process output, (2) can be used to reject a null hypothesis in which the system is random, (3) can be used as a descriptive statistic and (4) may indicate that some short term prediction is possible [10].

Correlation dimension has been widely used as a powerful tool for interpreting irregular signals in electrical, mechanical, biological and other engineering domains. Researchers have also explored the application of the correlation dimension to machinery condition monitoring and the fault diagnosis field. Logan and Mathew introduced the correlation dimension for the diagnosis of rolling element bearing defects [11]. Promising results show that the correlation dimension can classify three major rolling element bearing faults: outer race fault, inner race fault and roller fault.

A gearbox in normal condition generates considerably different time series to one with a cracked or broken tooth due to the different dynamic mechanisms. The correlation dimension may be able to distinguish such differences. In this paper, the authors intend to introduce a new technique for extracting quantitative parameters from raw time series of vibration acceleration data with the correlation dimension. Results show that there are distinctive differences in the correlation dimension obtained from a normal gearbox and those with cracked or broken tooth.

The structure of this paper is as follows: first, the basic non-linear time series analysis method based on correlation dimension is introduced; then the experimental investigation and discussion are presented; lastly, the analysis results, discussion and conclusions are drawn.

2. BASIC THEORY

The non-linear dynamic or chaos theory has concentrated on characterizing irregular, broadband signals, which are generic in non-linear dynamical systems, and extracting physically interesting and useful information from such signals [8, 9]. The development of the chaos theory has brought some new tools such as the correlation dimension, the Liyapunov exponent, and so on, to interpret observations of physical systems where the time trace of the measured quantities is irregular. The attractor dimension has been the most widely studied invariant quantity for dynamical systems. The concept of a phase space or state space dimension is the number of quantities needed to identify the state of a dynamic system. In the case of ordinary differential equations, the state space dimension corresponds to the number of equations. In the case of partial differential equations or the Navier–Stokes system, the state space dimension is infinite.

However, in dissipative dynamical systems it is frequently the case that the system behavior is attracted to a low dimensional state space. If the attractor is strange, the state space dimension becomes a non-integer value called the fractal dimension.

There are several different dimensions in use now, of which the most widely used one is the correlation dimension. Before calculating the correlation dimension, it is necessary to reconstruct the attractor state space from the measured raw time series.

2.1. STATE SPACE RECONSTRUCTION

The system state space, which plays an important role in dynamics theory, is a mathematical space with orthogonal co-ordinate directions representing each of the variables needed to specify the instantaneous state of the dynamical system [12]. However, in many practical situations it is difficult to measure all variables of a system. It is usually the case that only one variable scalar time series can be observed and it seems difficult to infer any information about the underlying system from the measured time series. In this instance, the time-delay embedding theory [13, 14] is available to reconstruct the state space of the underlying system from a measured raw time series $\{x(i),\ i = 1, 2, \ldots, N\}$. The main idea of the time-delay embedding theory is that it is unnecessary to know the derivatives to form a co-ordinate system in which one can capture the structure of orbits; one can use directly the lagged variable to construct the state space. Upon choosing the time lag τ and the dimension of the space m, the state space vectors can be reconstructed as:

$$\mathbf{X}(i) = [x(i), x(i+\tau), \ldots, x(i+(m-1)\tau)], \quad i = 1, 2, \ldots, N-(m-1)\tau \qquad (1)$$

where $X(i)$ is the reconstructed state space vector, $x(i)$ is the scalar time series, m is the embedding dimension of the reconstructed state space, and N is the length of data sequence.

Two important parameters, lag τ and embedding dimension m, must be determined before reconstructing the state space. The fundamental theory of reconstruction, introduced by Takens [13], gives no restrictions on the selection of τ, while for m it states the sufficient (but not necessary) condition $m \geqslant 2d + 1$, where d is the fractal dimension of the underlying attractor. For limited noisy observations, the selection of the parameters τ and m is important for the quality of reconstruction.

If the value of lag τ is too small, the co-ordinates $x(i)$ and $x(i + \tau)$ will be so close to each other in numerical value that one cannot distinguish between them. From any practical point of view they have not provided two independent co-ordinates. Similarly, if the selected value of τ is too large, $x(i)$ and $x(i + \tau)$ will be completely independent of each other in a statistical sense. Among several suggested methods, a simple and reasonable choice is to let the value of τ correspond to the first zero of the auto-correlation function of the time series $x(i)$, i.e., let $R(\tau) = 0$, where $R(\tau)$ is the auto-correlation function of time series $x(i)$ [15].

The standard way to choose embedding dimension m is the method of "saturation" with the correlation dimension. Therefore, the appropriate embedding dimension m^* can be assessed by computing the correlation dimension D_2 for $m = 1, 2, \ldots$ until the variation of D_2 ceases.

2.2. CORRELATION DIMENSION

The basic correlation dimension algorithm is now discussed—the GP algorithm—which was introduced by Grassberger and Procacia in 1983 [7]. A natural procedure is to start with an initial τ and m, then compute the correlation dimension until the embedding dimension m is sufficiently large. Firstly one defines the correlation integral $C(r)$, a statistic that measured the number of pairs of points on the attractor having a separation distance less than some value r,

$$C(r) = \frac{2}{N(N-1)} \sum_{i \neq j}^{N} \Theta(r - |X(i) - X(j)|), \tag{2}$$

where $C(r)$ is the correlation integral, r is the distance between two points in the state space, N is the length of data sequence, $|\cdot|$ denotes the Euclid form, $\mathbf{X}(i)$ and $\mathbf{X}(j)$ are state space vectors, $\Theta(x)$ is the Heaviside function defined as $\Theta(x) = 0$ for $x < 0$ and $\Theta(x) = 1$ for $x \geqslant 0$.

The correlation dimension is defined as follows:

$$D_2 = \lim_{r \to 0} (\log C(r) / \log r), \tag{3}$$

where D_2 is the correlation dimension, $C(r)$ is the correlation integral versus specific hyperspherical radius r.

For deterministic systems, the correlation integral behaves in the power law as $C(r) \approx r^{\nu}$. The exponent ν should be estimated from the slope of the graph of log $C(r)$ against log r over a linear region, which gives the numerical estimation of correlation dimension D_2.

3. INFLUENCES OF SAMPLE SIZE AND NOISE LEVEL ON THE CORRELATION DIMENSION

Including the lag and embedding dimension, size of data set and noise levels are two important factors that influence the computational precision significantly. Here one discusses in depth how the two factors influence the computational precision of the correlation dimension.

3.1. INFLUENCE OF SAMPLE SIZE

In general, a great amount of data N is required to compute the correlation dimension D_2. In fact, there is no consistent agreement on how many data can meet the computation needs. Here, some of the existing viewpoints are listed.

Theiler states $N \approx \Theta^D$. "Experience indicates that Θ should be of the order of 10, but experience also indicates the need for more experience." [16].

Smith *et al.* claims that to keep errors below 5% one must have $N > 42^M$, where M is the largest integer less than the set's dimension [17].

Ruelle argues that if one estimates a dimension, a data set of least size $10^{D/2}$ is required [18].

In order to study the influence of data set size on the computational precision of the correlation dimension, the correlation dimensions of simulated chaos signals are calculated using the GP algorithm for different data set sizes. The simulated chaos signals are integrated from the Lorentz attractor, the Ikeda attractor and the Hénnon attractor. The numerical integration method of the Lorenz system is fourth and fifth order Runge–Kutta formulas with a desired accuracy of 10^{-3}. The definitions of attractors are:

1. *Lorenz attractor*

$$\dot{x} = \sigma(y - z), \quad \dot{y} = (\rho - z)x - y, \quad \dot{z} = xy - \beta z,$$

where

$$\sigma = 10, \quad \rho = 28, \quad \beta = 8/3 \tag{4}$$

2. *Ikeda attractor*

$$Z_{n+1} = P + BZ_n \, \mathrm{e}^{\mathrm{i}K - \mathrm{i}\alpha/(1+|Z_n|^2)},$$

where

$$P = 0{\cdot}92, \quad B = 0{\cdot}9, \quad K = 0{\cdot}4, \quad \alpha = 6{\cdot}0; \tag{5}$$

3. *Hénnon attractor*

$$x_{n+1} = 1 - \alpha x_n^2 + y_n, \quad y_{n+1} = \beta x_n,$$

where

$$\alpha = 1{\cdot}4, \quad \beta = 0{\cdot}3. \tag{6}$$

Since correlation dimensions of all the above attractors are less than 3 (the attractor dimensions of Lorenz, Ikeda, Hénnon are 2·02, 1·80 and 1·26 respectively), one takes the embedding dimension as 5. Correlation dimensions of signals with different sample sizes (256, 512, 1024 and 4096) are computed using the GP algorithm.

Figure 1 demonstrates the log–log plots of correlation integral $C(r)$ against normalized distance r. From the plots it is obvious that when the sample sizes of signals increase, the correlation integral plots become smoother. Gradually, the separate regions with different slopes merge into a uniform scaling region with a single slope. Therefore, more precise computational results will be obtained as the sample size increases. The computational results are shown in Table 1 in detail. From the table one can see that when the sample size $N = 512$, all related error is less than 5%; while the sample size $N = 4096$, all related error is less than 1%.

From the above experiments one can summarize that the computational precision increases as the sample size increases. For the low dimensional system

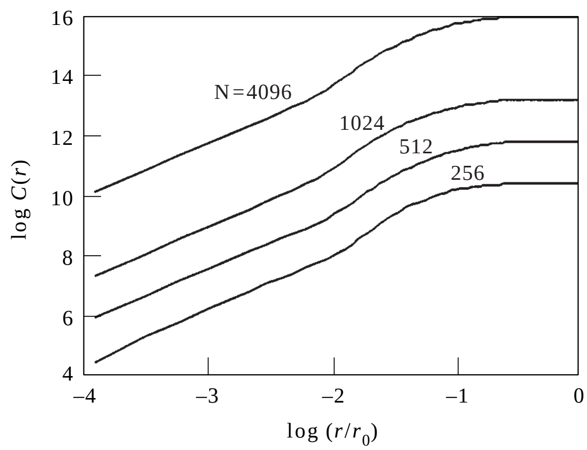

Figure 1. Log–log plots of correlation integral $C(r)$ versus normalized distance r. It is shown that the "smoothness" of the curves depends on the data length N.

($D_2 < 3$), a small sample size with the order of 500 can meet computational accuracy requirements of related error less than 5%. It indicates that the opinion of Theiler (1990) is closest to our findings.

3.2. INFLUENCE OF NOISE LEVEL

In practical application the measured time series are unavoidably contaminated by noise. Noise or random system fills its state space uniformly because there is no relationship between the state space vector $\mathbf{X}(t)$ and $\mathbf{X}(t+\tau)$. Therefore, the correlation integral $C(r)$ is proportional to the hyperspherical radius r and the correlation dimension is proportional to the embedding

TABLE 1

Computational results of correlation dimensions with different sample size

Signal type	True value: D_2	Results: $D_2 \pm \sigma$	Related error (%)	Signal length
Lorenz	2·02	1·834 ± 0·235	9·4	256
		2·089 ± 0·149	3·4	512
		2·070 ± 0·054	2·5	1024
		2·034 ± 0·019	0·7	4096
Ikeda	1·80	1·789 ± 0·018	0·6	256
		1·745 ± 0·013	3·1	512
		1·778 ± 0·004	1·2	1024
		1·797 ± 0·017	0·17	4096
Hénnon	1·26	1·211 ± 0·013	3·7	256
		1·255 ± 0·024	0·4	512
		1·287 ± 0·020	2·1	1024
		1·258 ± 0·014	0·16	4096

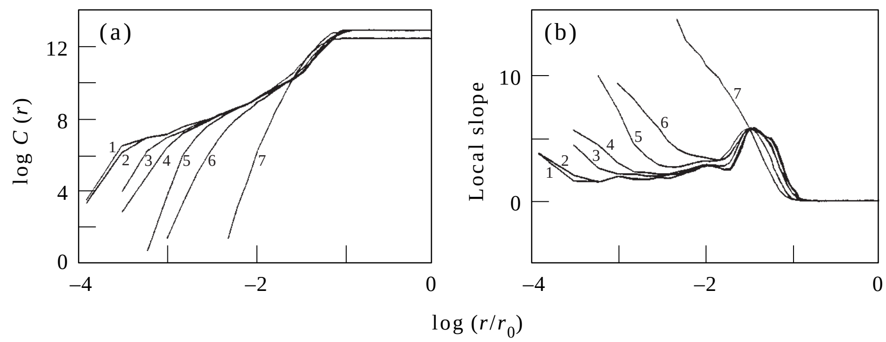

Figure 2. The correlation integrals of a Lorenz signal imposed with different level white noises. Curve 1 corresponds to a pure chaotic signal, curves 2, 3, 4, 5, 6 and 7 correspond to mixed signals for which $SNR = 1000$, 100, 50, 20, 10 and 1 respectively. (a) Log–log plot of $C(r)$ versus r; (b) local slope of the correlation integral.

dimension [11]. When a chaotic signal is superimposed by noise, the correlation integral exhibits similar features as displayed in Figure 2.

From the graph one can see that the scaling region becomes narrower as the signal–noise ratio is reduced. If the noise level is low ($SNR > 20$, where SNR is defined as energy ratio of signal to noise), there is a sufficiently broad platform in the correlation integral as shown in Figure 2(b) to estimate the correlation dimension reliably. If the noise level is rather high, the platforms of the correlation integral disappear and we cannot induce any information about the deterministic system masked by the noise. Therefore, it is necessary to preprocess the data using noise reduction methods if non-linear time series analysis is used to analyze practical signals.

3.3. NOISE REDUCTION

As is known, the observations of physical systems are usually contaminated by "noise". In the absence of prior knowledge about the contamination, it is plausible to assume the system state space is rather high dimensionally and the noise will fill in low dimensional state space more or less uniformly [11].

Let the necessary embedding dimension of the dynamic system be d_N and one works in d_E dimensional state space, where $d_E > d_N$. In a heuristic sense, $d_E - d_N$ dimensions of state space are filled with noise alone. The observations are considered to be composed of signal $x(n)$ and noise $c(n)$: $y(n) = x(n) + c(n)$, and are embedded in d_E dimensional state space. Then construct $d_E \times d_E$ dimensional sample covariance matrix:

$$\mathbf{Cov} = \frac{1}{N}\sum_{n=1}^{N}[y(n) - y_{av}(n)][y(n) - y_{av}(n)]^{\mathrm{T}}, \tag{7}$$

where

$$y_{av} = \frac{1}{N}\sum_{n=1}^{N} y(n).$$

The first d_N eigenvalues of the covariance matrix almost arise from the signal (slightly contaminated); the left $d_E - d_N$ eigenvalues arise from the noise. If the contamination is white noise or quite high dimensionally, it will fill in $d_E - d_N$ dimensional state space uniformly. In this case, it is possible to find a "noise floor" that arises from noise by investigating the eigenvalue of the covariance matrix. By removing those extra eigenvalues, a great amount of noise is reduced. This method is based on singular value decomposition and is called singular spectrum analysis (SSA) [20].

4. EXPERIMENTAL INVESTIGATION

4.1. TESTING

The experimental set-up contains a two-stage reduction gearbox (6J90T) manufactured by Shannxi Vehicle Gear Manufacturing Plant. The gearbox transmission plot is shown in Figure 3. The gearbox translational vibration signals are measured externally on the gearbox bearing case using acceleration sensor YD42 and amplified by charge amplifier B&K 2626 to monitor the operating condition of the gearbox. The sampling frequency is 5 kHz.

The total testing time of the gear fatigue experiments is up to 180 h. During the testing process, the gearbox's running condition undergoes three different stages naturally. At first, the gearbox's running condition is normal. Then a crack in one tooth root arises and enlarges gradually. Lastly one tooth of the meshing gear is broken and the testing experiment is terminated. It is found that only one tooth of the meshing gear with 36 teeth is broken when the gearbox is stripped. For more details of the experiment one may refer to reference [19]. In the above paper the authors detected the occurrence of gear fatigue crack based on the amplitude and phase demodulation technique proposed by McFadden [1]. In this paper the correlation dimension is used to detect the gear fatigue crack.

Raw vibration signals measured from outside of the gearbox in different operating conditions are displayed in Figure 4. Obviously there are distinct

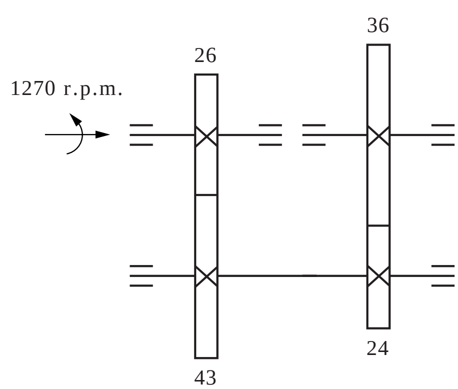

Figure 3. Gearbox transmission graph, with two pairs of meshing gears and the input rotating speed 1270 r.p.m. The teeth numbers of meshing gear pairs are denoted in the plot.

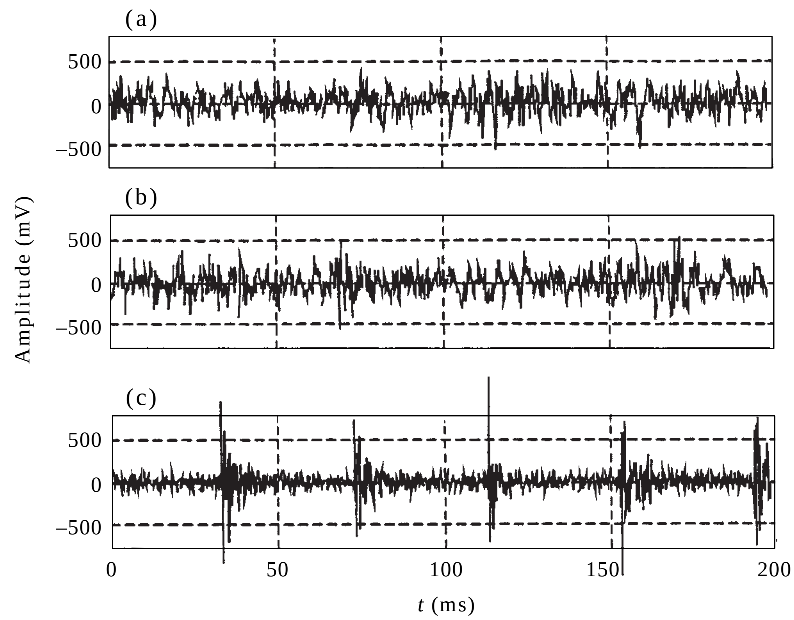

Figure 4. Gearbox vibration acceleration signals of different operating conditions: (a) normal, (b) with cracked tooth and (c) with broken tooth.

differences between the vibration signals of the gearbox with a broken tooth and those of the other two operating conditions, but there is only a slight difference between the vibration signals of the normal condition and of the condition with a cracked tooth.

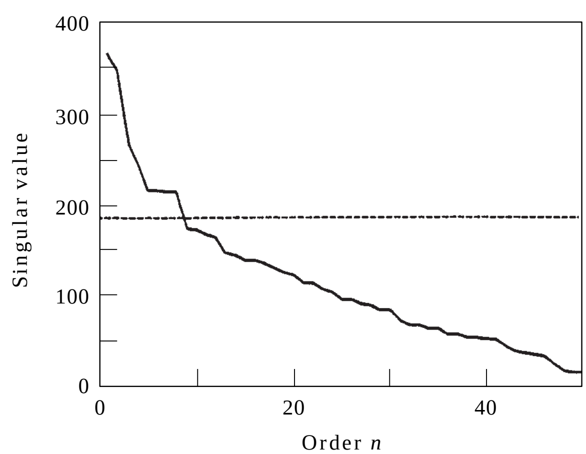

Figure 5. The singular spectrum of raw time series measured from the normal operating condition. The window length is 50 and the lag is 1.

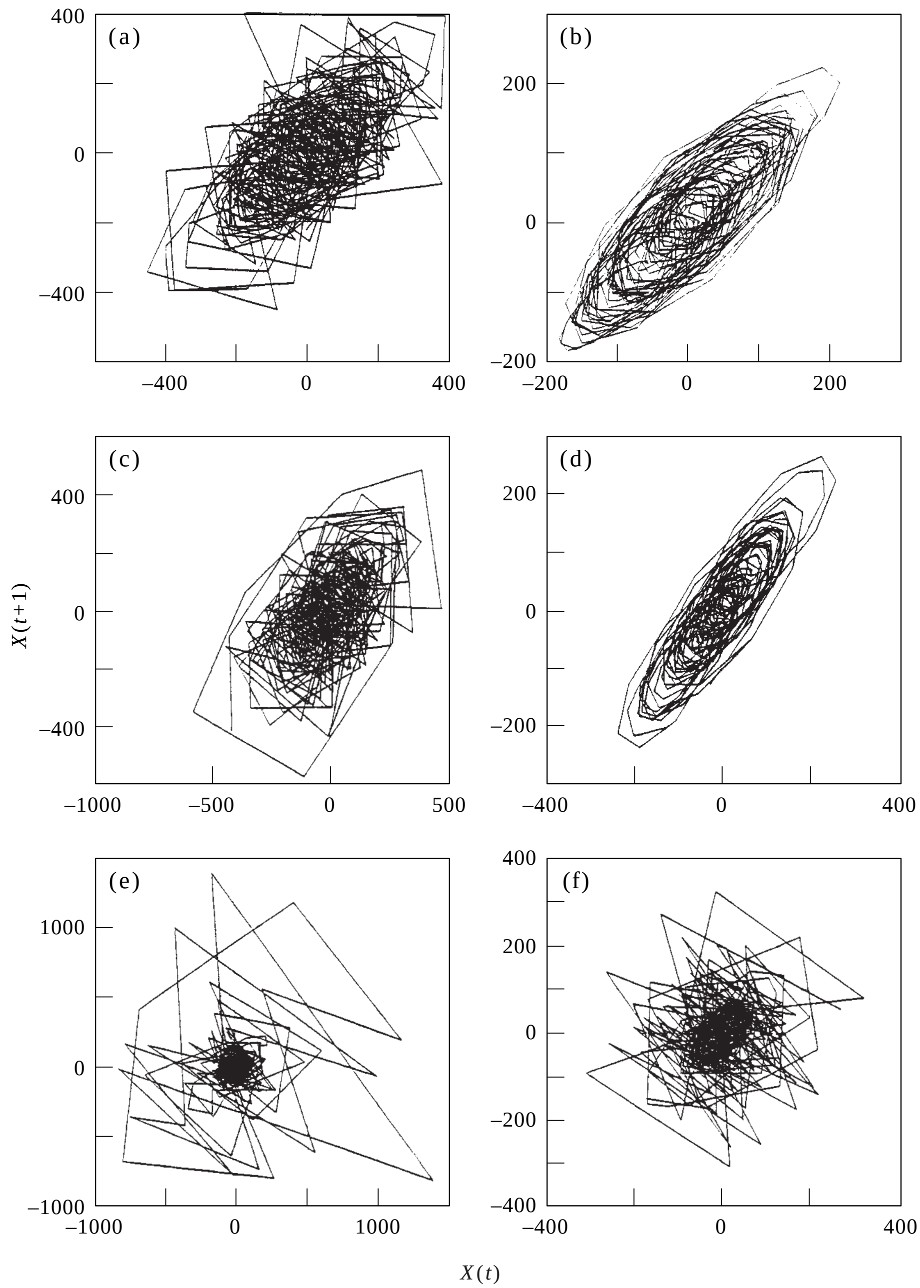

Figure 6. The pseudophase diagrams of raw vibration acceleration signals (a, c and e) against cleaned signals (b, d and f) correspond to different running conditions: (a), (b) normal condition; (c), (d) operating condition with crack tooth; (e), (f) operating condition with broken tooth.

4.2. ANALYSIS

As is known, it is difficult to identify the operating condition with fatigue cracks against the normal running state not only from the time domain but also from the frequency domain.

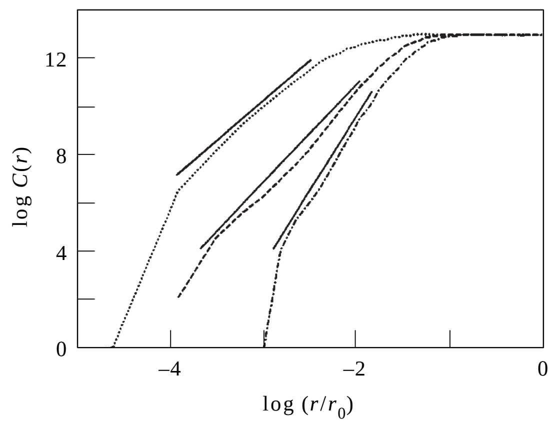

Figure 7. Log–log graph of correlation integral $C(r)$ versus normalized distance r. The embedding dimension is 15 and signal length is 1024. The numbers in the textbox demonstrate the correlation dimensions of gearbox vibration signals in different operating conditions. Key: -·-·-, normal $D = 6{\cdot}18$; ----, cracked $D = 4{\cdot}24$; ······, $D = 3{\cdot}39$.

The first stage of a non-linear analysis procedure is to reduce noise level from the measured vibration signals based on the singular value decomposition technique [20]. The singular spectrum of the normal condition vibration signal is illustrated in Figure 5. From the plot one can see clearly that the first several singular values are significantly large and possess the main component of the total signal energy. Therefore, the first six significant singular values are reasonably selected to reconstruct the "clean" signal.

Some non-linear diagnostic methods are available for practical application, such as time–frequency analysis based on the Wigner distribution, the Kullback index of complexity based on information theory, pseudophase diagrams, etc. The pseudophase diagrams are the most convenient for on-line diagnostics [21]. Some pseudophase diagrams of the raw vibration signals and constructed signals are illustrated in Figure 6.

The pseudophase diagrams of raw vibration signals are rather irregular, from which one can hardly extract characteristic information for diagnostics. However, the pseudophase diagrams become more regular after noise reduction processing, even though it is still difficult to identify the operating condition with the cracked tooth against the normal condition from the pseudophase diagram only. The reason lies in the fact that the pseudophase diagram only qualitatively displays geometric features of the phase space projection and cannot indicate any quantitative character. In contrast, the correlation dimension can indicate quantitative information about the phase space. Subsequently, the non-linear analysis method based on the correlation dimension is tried.

Signals preprocessed by the singular value decomposition technique are used to reconstruct the state space of underlying gearbox dynamic system. The choices of reconstruction parameters are based on the above-mentioned methods. Following the reconstruction operation, the correlation dimension of signals is calculated and the computational results are shown in Figure 7.

From the log–log graph, one can see that there are various slopes of different plots during the same scaling. The correlation dimension is estimated from the slope. Results shown in the figure indicate that the state spaces of different operating conditions construct into various low dimensional attractors due to different kinematics mechanisms. In the case of normal running condition, the state space dimension of the attractor reconstructed from the normal operating condition is near to 6, which is higher than that of the other two cases. During the fatigue testing process, the reconstructed attractor dimension decreases as all the gear fatigue crack appears. When one of the meshing teeth is broken, the attractor dimension drops to the bottom limit.

5. CONCLUSIONS

Non-linear time series analysis theory based on correlation dimension for practical application, especially for gearbox fault diagnosis, is introduced in this paper. The influences of sample size and noise level on correlation dimension computational precision are discussed. Based on simulation experiments, some valuable principles are presented to improve the calculation precision of the correlation dimension:

(1) Sample size of $N \approx \Theta^D$ can meet the computation precision requirements, where N is the sample size, and Θ is of the order of 10.

(2) It is necessary to preprocess the data using noise reduction methods if non-linear time series analysis based on the correlation dimension is used to analyze practical signals.

The application of correlation dimension in industrial gearbox condition monitoring is presented. Analysis results show that the correlation dimension is capable of identifying industrial gearbox defects which occur naturally. The growth of gear fatigue crack leads to the decrease of state space dimension. It shows that the correlation dimension has great potential for the diagnosis of defects in industrial gearboxes.

Further work on this subject will be to investigate the relationship between gear crack extent and the correlation dimension.

REFERENCES

1. P. D. McFadden 1986 *Transactions of the ASME, Journal of Vibration, Acoustic, Stress and Reliability in Design* **108**, 165–170. Detecting fatigue cracks in gears by amplitude and phase demodulation of the meshing vibration.
2. W. J. Staszewski and G. R. Tomlinson 1994 *Mechanical System and Signal Processing* **8**, 289–307. Application of the wavelet transform to fault detection in a spur gear.
3. D. Brie, M. Tomczak, H. Oehlmann and A. Richard 1997 *Mechanical Systems and Signal Processing* **11**, 149–167. Gear crack detection by adaptive amplitude and phase demodulation.
4. F. Pfeiffer and A. Kunert 1990 *Nonlinear Dynamics* **1**, 63–74. Rattling models from deterministic to stochastic processes.

5. K. Kargiannis and F. Pfeiffer 1991 *Nonlinear Dynamics* **2**, 36–387. Theoretical and experimental investigations of gear-rattling.
6. R. J. Comparin and R. Singh 1990 *Journal of Mechanical Design* **112**, 237–245. An analytical study of automotive neutral gear rattle.
7. P. Grassberger and I. Procacia 1983 *Physica D* **9**, 189–208. Measuring the strangeness of strange attractors.
8. F. C. Moon 1992 *Chaotic and Fractal Dynamics*. New York: John Wiley.
9. H. D. I. Abarbanel, R. Brown, J. J. Sidrowich and I. S. Tsiming 1993 *Reviews of Modern Physics* **65**, 1331–1392. The analysis of observed chaotic data in physical systems.
10. K. J. Dooley 1995 *Proceedings of the Fourth Industrial Engineering Research Conference*. IIE, Norcross, GA, USA, 1025–1032. Estimating fractal dimension, with application to productive systems.
11. D. Logan and J. Mathew 1996 *Mechanical Systems and Signal Processing* **10**, 241–250. Using the correlation dimension for vibration fault diagnosis of rolling element bearing—1. Basic concepts.
12. G. L. Baker and J. P. Gollub 1990 *Chaotic Dynamics—An Introduction*. Cambridge: Cambridge University Press, p. 1.
13. F. Takens 1980 in *Dynamical Systems Turbulence* (edited by D. Rand and L.-S. Young). Lecture Note in Mathematics No. 898. Berlin: Springer.
14. H. Froehling, J. P. Crutchfield, J. D. Farmer, N. H. Packard and R. Shaw 1981 *Physica* **D3**, 605–617. On determining the dimension of chaotic flows.
15. G. Kember and A. C. Fowler 1993 *Physical Letters A* **179**, 72–80. A correlation function for choosing time delay in phase reconstruction.
16. J. Theiler 1990 *J. Opt. Soc. Am.* **A7**, 1055. Estimating fractal dimension.
17. L. A Smith 1988 *Physical Letters A* **133**, 283. Intrinsic limits on dimension calculations.
18. D. Ruelle 1990 *Proceedings Royal Society of London, Series A* **427**, 241.
19. G. Ch. Zhang, W. Y. Zhao, Y. D. Shen and F. R. Tu 1994 *Journal of Mechanical Transmission* **18**, 21–24. The feature of gear fatigue crack and its diagnostic method (in Chinese).
20. D. S. Broomhead and G. P. King 1986 *Physica D* **20**, 217–236. Extracting qualitative dynamics from experimental data.
21. L. S. Qu, A. L. Xie and X. Li 1993 *Mechanical Machine Theory* **28**, 699–713. Study and performance evaluation of some nonlinear diagnostic methods for large rotating machinery.