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LETTERS TO THE EDITOR

EFFECT OF INFINITE IMPEDANCE TESTING ON EQUIPMENT SUPPORTED BY A STRUCTURE OF A COMPARABLE MASS

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1. INTRODUCTION

The infinite impedance method is frequently used for vibration testing of sensitive equipment installed in airborne structures [1–4]. This method is based on records of flight vibrations data collected at the points of attachment of equipment to the supporting structure. Those records, representing typically acceleration versus frequency, usually show one or more characteristic peaks along with characteristic valleys. The characteristic peaks correspond to natural frequencies of the airborne structure as a whole, including equipment. That data is usually synthesized into a single spectrum for design and test requirements. Since a margin of safety is desirable, the synthesized spectrum is usually a smooth simple curve whose level is determined principally by the peaks of a spectrum that is a composite of the original field spectra. Then, the tested equipment is attached to a shaker programmed to deliver the synthesized spectrum. Input acceleration to the test specimen is maintained at the prescribed level regardless of the force magnitude required to sustain this acceleration, i.e., no matter what the reaction of the unit under test is. This amounts to testing with an infinite impedance vibration source and thus implies that the actual equipment support structure must have an infinite effective mass at all frequencies.

Of course, the supporting structure does not possess infinite effective mass at any frequency. The validity of this assumption depends on the relative mass of the tested equipment and the supporting structure. This will be a reasonable assumption for, e.g., an aircraft with a mass of 10 000 kg and a piece of equipment which has a mass of 100 kg. But, the applicability of the infinite mechanical impedance assumption is not so obvious for equipment which has a mass of the same order as the supporting structure. This is frequently the case for missiles or satellites.

The vibratory motion of the supporting structure can be significantly affected by the interface reactions of the coupled equipment item, i.e., by its dynamic loading [5–7]. At certain frequencies an equipment item may exert an unusually large reaction, or load, against support excitation. If the support excitation is a

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sinusoidal or random vibration, then the frequency response function of the support will exhibit a notch or characteristic valley at this frequency. That phenomenon is usually called antiresonance. Though quite common, the use of an envelope of spectral peaks to determine vibration test levels for use in standard test procedures does not account for the occurrence of antiresonance, i.e., the dynamic loading of the equipment against its support is neglected. Neglecting the dynamic interaction between equipment and support leads to a poor correlation between actual ("field") and test conditions. The resulting error may be large enough to cause a permanent damage to the tested equipment, which would not occur in real conditions.

The magnitude of overtest error is dependent on the amount of damping in the system and on the dynamic properties of both the equipment and supporting structure. The assumption that the dynamic properties of the equipment item are negligibly small compared to the dynamic characteristics of the supporting structure is often used as a justification for infinite impedance vibration testing. The objective of this paper is to show that this assumption may lead to unacceptable errors when masses of the tested equipment and the supporting structure are of the same order.

2. FORMULATION

To estimate the error involved in the applying the infinite impedance method to test equipment with mass comparable to the mass of supporting structure, a five-degree-of-freedom (5-DOF), "free–free" dynamic chain system pictured in Figure 1(a) is analyzed. This system is used to model the dynamical interaction between support, e.g., a missile structure, and equipment, e.g., an InfraRed seeker head.

A typical missile structure consists of several individual sections that are connected together to form the complete missile assembly. The individual missile sections perform unique functions and will be referred to as missile subsystems. Each subsystem usually consists of a number of levels or orders of structure. A level, or order, of a mechanical subsystem is that portion of a subsystem which can be identified as a single region in an overall model of the subsystem. For example, aerodynamic turbulence and rocket motor vibration excite the outermost airframe structure (first level), which drives the internal structure of the subsystem (second level), which carries an equipment mounting bracket (third level), to which is attached the case of an instrument (fourth level), which supports a module chassis (fifth level), and the module chassis is the mounting for a small component part (sixth level). The sixth level was neglected in the model used in this paper. It is assumed that the support is represented by masses m_1 and m_2, equipment base is modelled by m_3, and the equipment is represented by masses m_4 and m_5. To quantify the magnitude of error that can occur, a test specification based on the results of the 5-DOF system is developed and applied to the equipment only subsystem [m_4 and m_5 of Figure 1(a)].

The equations of motion for the 5-DOF, free–free dynamic chain system pictured in Figure 1(a) are

$$[M]\{\ddot{x}\} = [C]\{\dot{x}\} + [K]\{x\} = \{F\}, \tag{1}$$

where

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 \\ 0 & 0 & 0 & 0 & m_5 \end{bmatrix},$$

$$[C] = \begin{bmatrix} c_1 & -c_1 & 0 & 0 & 0 \\ -c_1 & c_1 + c_2 & -c_2 & 0 & 0 \\ 0 & -c_2 & c_2 + c_3 & -c_3 & 0 \\ 0 & 0 & -c_3 & c_3 + c_4 & -c_4 \\ 0 & 0 & 0 & -c_4 & c_4 \end{bmatrix}, \tag{2, 3}$$

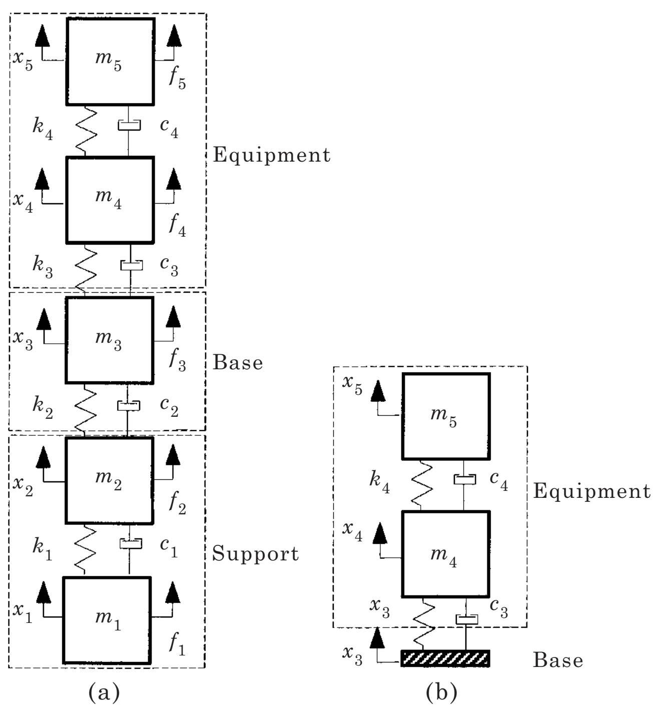

Figure 1. (a) Five-degree-of-freedom dynamic system, (b) two-degree-of-freedom system with base input.

$$[K] = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & 0 & -k_4 & k_4 \end{bmatrix},$$

$$\{F\} = \lfloor f_0, f_0, 0, 0, 0 \rfloor^{\mathrm{T}}. \qquad (4, 5)$$

System parameters have been chosen as $m_1 = 4m$, $m_2 = 3m$, $m_3 = 3m$, $m_4 = 2m$, $m_5 = m$, $c_1 = c_2 = c_3 = c_4 = c$, $k_1 = 5k$, $k_2 = 3k$, $k_3 = 4k$, $k_4 = k$, and $f_0 = F_0 \sin(2\pi f t)$ where $m = 0{\cdot}9068$ kg, $k = 1{\cdot}751 \times 10^6$ N/m, and $F_0 = 88{\cdot}96$ N. Amplitudes of steady state responses for $x_3(t)$, $x_4(t)$, and $x_5(t)$ have been obtained in terms of frequency f and gravitational acceleration $\boldsymbol{g}$ using Cramer's rule. All computations were performed using the *Mathematica*™ symbolic manipulation package. Final expressions for steady state acceleration amplitude have the form

$$\ddot{x}_i = \frac{(2\pi f)^2}{\boldsymbol{g}} \frac{|A_{Fi}|}{|A|}, \qquad i = 3, 4, 5, \qquad (6)$$

where

$$[A] = -\omega^2[M] + i\omega[C] + [K] \qquad (7)$$

and $[A_{Fi}]$ is matrix $[A]$ with the ith column replaced by the $\{F\}$ vector.

Accelerations of masses m_3, m_4 and m_5 model actual "field" responses of the base, and both components of the equipment subsystem, respectively. Test specifications are generated as envelopes of the m_3 base acceleration versus frequency characteristic peaks for the 5-DOF "field" configuration and five different damping levels: $c = 1$, 5, 10, 15 and 25 Ns/m. These envelope spectra are then applied to the equipment-only subsystem as inputs for a hypothetical infinite impedance vibration test. It was assumed that the 2-DOF equipment-only subsystem was excited by the motion of the base (i.e., by the motion of mass m_3) defined by the envelope of the m_3 base acceleration characteristic peaks in the 5-DOF field result. The equipment-only subsystem excited by the base is shown in Figure 1(b). Acceleration response of equipment subsystem components in terms of base acceleration can be represented using the concept of frequency response function:

$$\ddot{x}_4 = H_{\ddot{x}_4}\ddot{x}_3, \qquad \ddot{x}_5 = H_{\ddot{x}_5}\ddot{x}_3, \qquad (8, 9)$$

where $H_{\ddot{x}_4}$ and $H_{\ddot{x}_5}$ are frequency response functions for $\ddot{x}_4$ and $\ddot{x}_5$, respectively, and $\ddot{x}_3$ is defined by the envelope of base acceleration. Each subsystem component acceleration can be expressed in terms of relative displacements $y_4 = x_4 - x_3$, $y_5 = x_5 - x_4$, and non-dimensional system parameters ω_4, ω_5, ζ_4, ζ_5, M as

$$\ddot{x}_4 = -2\zeta_4\omega_4\dot{y}_4 - \omega_4^2 y_4 + 2M\zeta_5\omega_5\dot{y}_5 + M\omega_5^2 y_5, \qquad (10)$$

$$\ddot{x}_5 = -2\zeta_5\omega_5\dot{y}_5 - \omega_5^2 y_5, \qquad (11)$$

where

$$\omega_4 = \sqrt{\frac{k_3}{m_4}}, \qquad \omega_5 = \sqrt{\frac{k_4}{m_5}}, \qquad \zeta_3 = \frac{c_3}{2m_4\omega_4}, \qquad \zeta_4 = \frac{c_4}{2m_5\omega_5}, \qquad \text{and } M = \frac{m_5}{m_4}.$$

Relative displacements y_4 and y_5 can be written in terms of frequency response functions as

$$y_4 = H_{y_4}\ddot{x}_3, \qquad y_5 = H_{y_5}\ddot{x}_3. \tag{12, 13}$$

Equations (12) and (13) are substituted into equations (10) and (11) from which frequency response functions H_{y_4} and H_{y_5} are determined. These functions can be expressed as

$$H_{y_4} = \frac{n_{y_4}}{d_{y_4}}, \qquad H_{y_5} = \frac{n_{y_5}}{d_{y_5}}, \tag{14, 15}$$

where

$$n_{y_4} = \omega^2 - (1+M)\omega_5^2 - 2i(1+M)\omega\omega_5\zeta_4,$$

$$d_{y_4} = \omega^4 + \omega_4^2\omega_5^2 + 2i\omega\omega_4\omega_5(\omega_5\zeta_3 + \omega_4\zeta_4) - 2i\omega^3(\omega_4\zeta_3 + \omega_5\zeta_4 + M\omega_5\zeta_4) - \omega^2(\omega_4^2 + \omega_5^2 + M\omega_5^2 + 4\omega_4\omega_5\zeta_3\zeta_4),$$

$$n_{y_5} = \omega_4^2 + 2i\omega\omega_4\zeta_3,$$

$$d_{y_5} = -\omega^4 - \omega_4^2\omega_5^2 - 2i\omega\omega_4\omega_5(\omega_5\zeta_3 + \omega_4\zeta_4) + 2i\omega^3(\omega_4\zeta_3 + \omega_5\zeta_4 + M\omega_5\zeta_4) + \omega^2(\omega_4^2 + \omega_5^2 + M\omega_5^2 + 4\omega_4\omega_5\zeta_3\zeta_4).$$

Equations (14) and (15) are substituted into equations (8) and (9) from which frequency response functions $H_{\ddot{x}_4}$ and $H_{\ddot{x}_5}$ are determined. These functions can be expressed as

$$H_{\ddot{x}_4} = n_{\ddot{x}_4}/d_{\ddot{x}_4}, \qquad H_{\ddot{x}_5} = n_{\ddot{x}_5}/d_{\ddot{x}_5}, \tag{16, 17}$$

where

$$n_{\ddot{x}_4} = \omega_4^2\omega_5^2 - 2i\omega^3\omega_4\zeta_3 + 2i\omega\omega_4\omega_5(\omega_5\zeta_3 + \omega_4\zeta_4) - \omega^2\omega_4(\omega_4 + 4\omega_5\zeta_3\zeta_4),$$

$$d_{\ddot{x}_4} = \omega^4 + \omega_4^2\omega_5^2 + 2i\omega\omega_4\omega_5(\omega_5\zeta_3 + \omega_4\zeta_4) - 2i\omega^3(\omega_4\zeta_3 + \omega_5\zeta_4 + M\omega_5\zeta_4) - \omega^2(\omega_4^2 + \omega_5^2 + M\omega_5^2 + 4\omega_4\omega_5\zeta_3\zeta_4),$$

$$n_{\ddot{x}_5} = \omega_4^2\omega_5^2 - 4\omega^2\omega_4\omega_5\zeta_3\zeta_4 + 2i\omega\omega_4\omega_5(\omega_5\zeta_3 + \omega_4\zeta_4),$$

$$d_{\ddot{x}_5} = \omega^4 + \omega_4^2\omega_5^2 + 2i\omega\omega_4\omega_5(\omega_5\zeta_3 + \omega_4\zeta_4) - 2i\omega^3(\omega_4\zeta_3 + \omega_5\zeta_4 + M\omega_5\zeta_4) - \omega^2(\omega_4^2 + \omega_5^2 + M\omega_5^2 + 4\omega_4\omega_5\zeta_3\zeta_4).$$

Now, equations (8) and (9) can be used to compute the response of the 2-DOF subsystem subjected to an acceleration input enveloping the peaks of m_3 acceleration obtained for the 5-DOF system. Thus, the behavior of an equipment subsystem mounted on a shaker is modelled. Next, maximum accelerations of equipment subsystem components found from the 5-DOF system analysis were compared with maximum accelerations computed for the equipment as a separate 2-DOF system. Their ratio gave a measure of overtest in the infinite impedance method.

Then, the level of interaction between equipment and support was studied and the effect the two systems had on each other was demonstrated. The mean square acceleration of m_4 was computed for different combinations of the following dimensionless parameters: the mass ratio M, the frequency ratio ω_5/ω_4 between the uncoupled natural frequencies of each spring–mass system and the damping ratios ζ_3, ζ_4 associated with each spring-mass-damper system. The mean square accelerations, $\Phi(\ddot{x}_4^2)$ and $\Phi(\ddot{x}_5^2)$, are normalized by dividing by the mean square acceleration that the first mass would have had under the same excitation if m_5 had been removed completely. That corresponds to a 1-DOF system response.

The mean square $\Phi(Y^2)$ of a stationary response process Y can be obtained from

$$\Phi(Y^2) = S_0 \int_{-\infty}^{\infty} |H(\omega)|^2 \, \mathrm{d}\omega, \tag{18}$$

assuming that the input excitation is ideal white noise, so that the power spectral density of the input excitation is $S_x(\omega) = S_0$. All of the frequency response functions used previously can be represented in a following general form, given by Crandall and Mark [8], as

$$H(\omega) = \frac{-i\omega^3 B_3 - \omega^2 B_2 + i\omega B_1 + B_0}{\omega^4 A_4 - i\omega^3 A_3 - \omega^2 A_2 + i\omega A_1 + A_0}\,. \tag{19}$$

The integral in the expression for $\Phi(Y^2)$ is given by Gatscher and Kawiecki [9] as

$$\int_{-\infty}^{\infty} |H(\omega)|^2 \, \mathrm{d}\omega =$$

$$\pi \frac{\left(\dfrac{B_0^2}{A_0}\right)(A_2A_3 - A_1A_4) + A_3(B_1^2 - 2B_0B_2) + A_1(B_2^2 - 2B_1B_3) + \left(\dfrac{B_3^2}{A_4}\right)(A_1A_2 - A_0A_3)}{A_1(A_2A_3 - A_1A_4) - A_0A_3^2}\,. \tag{20}$$

The mean response $\Phi(\ddot{x}_4^2)$ can be obtained by substitution of the appropriate A and B constants, from each frequency response function [equation (16)], into the general solution formula. A complete expression for $\Phi(\ddot{x}_4^2)$ is given by Gatscher [10].

The absolute acceleration complex frequency response functions $H_{\ddot{x}}(\omega)$ for the 1-DOF system are obtained from the 2-DOF results by setting $M = 0$ in equation (16):

$$H_{\ddot{x}}(\omega) = \frac{2i\omega\zeta_3\omega_4 + \omega_4^2}{\omega_4^2 - \omega^2 + 2i\zeta_3\omega_4\omega}\,. \tag{21}$$

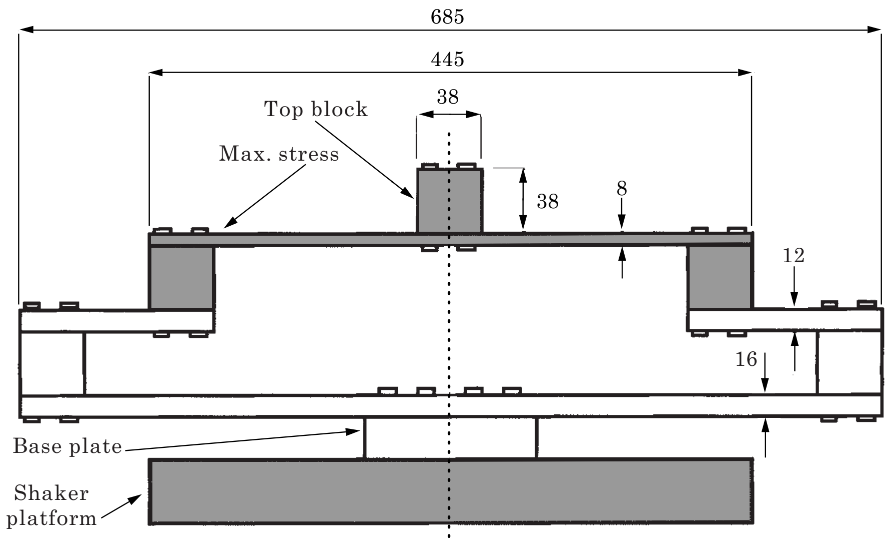

Figure 2. Experimental set-up.

The mean square absolute acceleration $\Phi(\ddot{x}^2)$ is obtained similarly and has the form

$$\phi(\ddot{x}^2) = \frac{\pi\omega_4 S_0(1 + 4\zeta_3^2)}{2\zeta_3}. \tag{22}$$

3. EXPERIMENTAL VALIDATION

Experimental validation was done using a steel beam/block structure designed to represent a support/equipment unit system. This structure is shown in Figure 2, where dimensions are given in millimeters. The depth of all structural elements is 50 mm. The shaded top beam/block assembly could be detached from the combined structure and bolted directly to the shaker platform. Therefore, the top beam/block assembly represents an equipment unit for which a vibration test is to be developed. The combined structure was instrumented with six accelerometers attached to each of the five blocks and to the base plate. A constant 3 *g* amplitude logarithmic sine sweep (20–2000 Hz, $t = 1500$ s) was performed on the structure configured as shown in Figure 2 and for the top beam/block subsystem only. Then, a finite element model of the structure was developed using the ANSYS commercial finite element package. The purpose of finite element modelling was to evaluate stress at various locations on the structure. Bolted connections were modelled using spring elements. The advantage of using local springs, at the bolted connection locations, was the ability to adjust the finite element model to match the natural frequencies of the real test structures. Spring stiffnesses and Rayleigh damping coefficients were fine-tuned until a satisfactory agreement between acceleration frequency response functions was achieved for all instrumented locations. Once the finite element model performance was found to be adequate, bending stress frequency response functions were obtained for the maximum bending stress location indicated in Figure 2. These frequency response functions were obtained for the configuration shown in Figure 2 and for the top beam/block assembly mounted directly to the shaker platform.

4. THEORETICAL RESULTS

Figure 3 shows test specifications as an envelope of the m_3 base characteristic peaks from the 5 DOF field result for a system with very low damping ($c = 1$ Ns/m, $\zeta_3 = 0{\cdot}00014$, $\zeta_4 = 0{\cdot}00040$). This envelope is then applied to the equipment only 2-DOF subsystem as an input for a hypothetical infinite impedance vibration test.

Figures 4 and 5 display the results of the equipment only subsystem test superimposed on the field results from the original 5-DOF system. As shown in these figures, for the system with very low damping the amount of overtest is in error by a factor of 35 for m_4 response (4940 gs field results versus 175 000 gs test) and by a factor of 58 for m_5 response (10 700 gs field versus 619 000gs test).

Similar computations were made for a system with higher damping ($c = 25$ Ns/m, $\zeta_3 = 0{\cdot}0035$, $\zeta_4 = 0{\cdot}0099$). For this system the amount of overtest, although significantly reduced, was still unnecessarily high. It was in error by a factor of 1·4 for m_4 response (198 gs field results versus 279 gs test) and by a factor of 2·3 for m_5 response (429 gs field versus 990 gs test). Similar procedure was repeated for systems with $c = 5$, 10 and $c = 15$ Ns/m. Obtained results were used to plot the amount of overtest for m_4 and m_5 as a function of damping ratio ζ; see Figure 6. We can note that the amount of overtest for systems with damping ratio magnitudes typical for many metallic structures ($\zeta \approx 0{\cdot}002$–$0{\cdot}005$) can be in

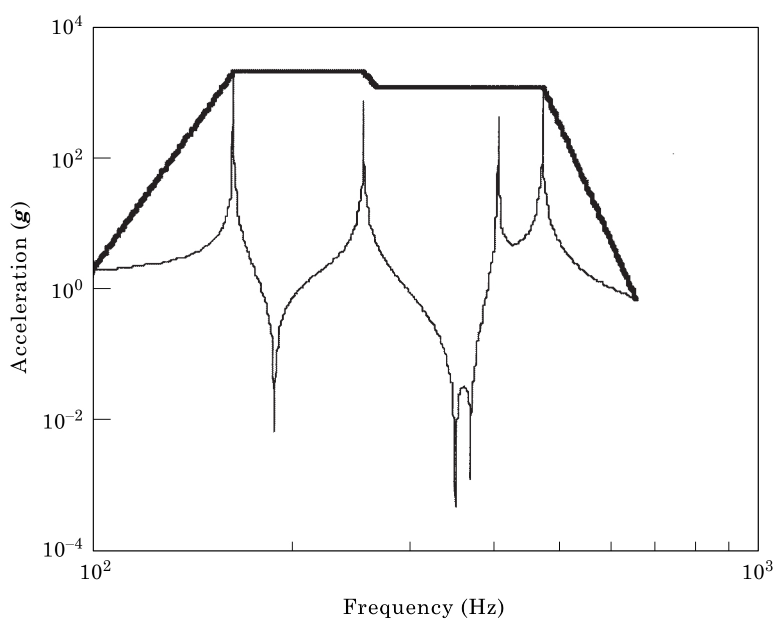

Figure 3. Acceleration test specification, $c = 1$ Ns/m; —, base field acceleration; ——, acceleration envelope.

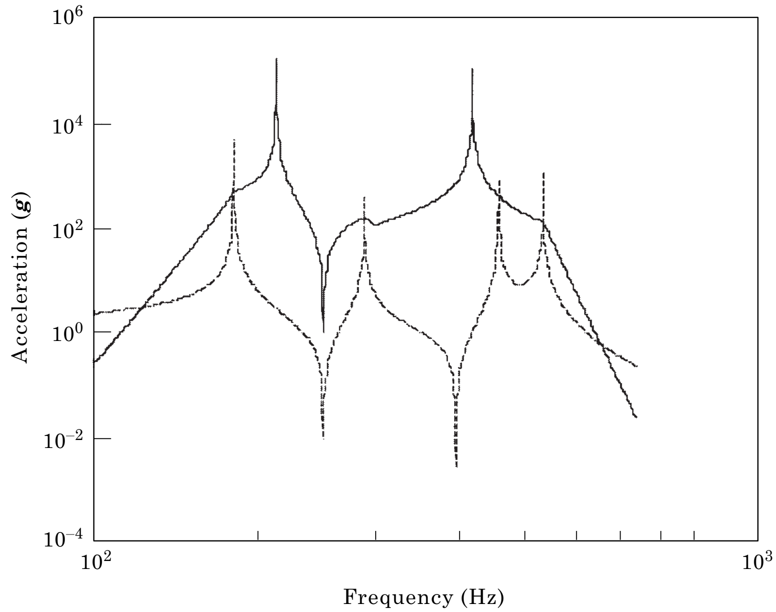

Figure 4. Frequency response function of m_4, $c = 1$ Ns/m; - - - , "field" results from 5-DOF system; —, infinite impedance equipment test.

error by as much as by a factor of 10. Even the damping ratios representative of composite structures ($\zeta \approx 0{\cdot}01$) can be associated with overtest errors by a factor of 2. This is still an unacceptable amount of overtesting.

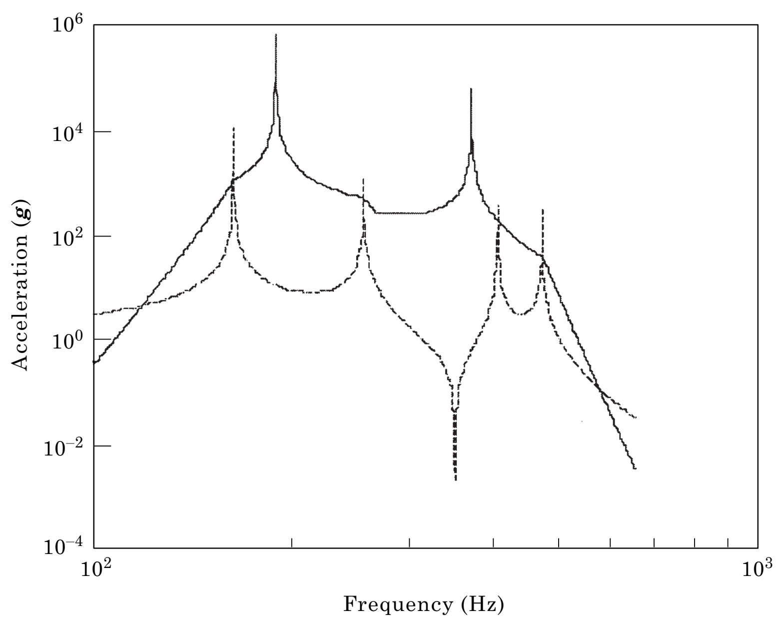

Figure 5. Frequency response function of m_5, $c = 1$ Ns/m; - - - , "field" results from 5-DOF system; —, infinite impedance equipment test.

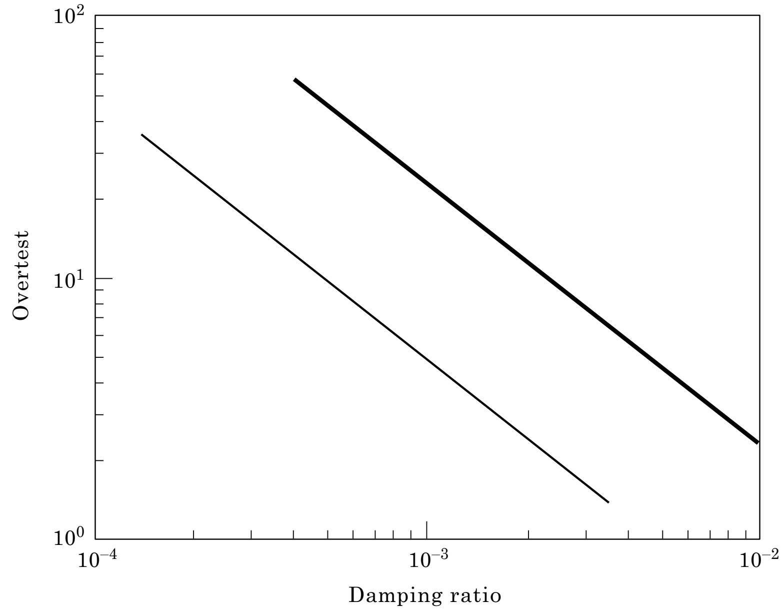

Figure 6. Amount of overtest versus damping ratio: —, m_4; ——, m_5.

Clearly, the amount of damping significantly affects the response of the tested mechanical system. Figures 6 and 7 reveal the influence the system damping has on dynamic loading.

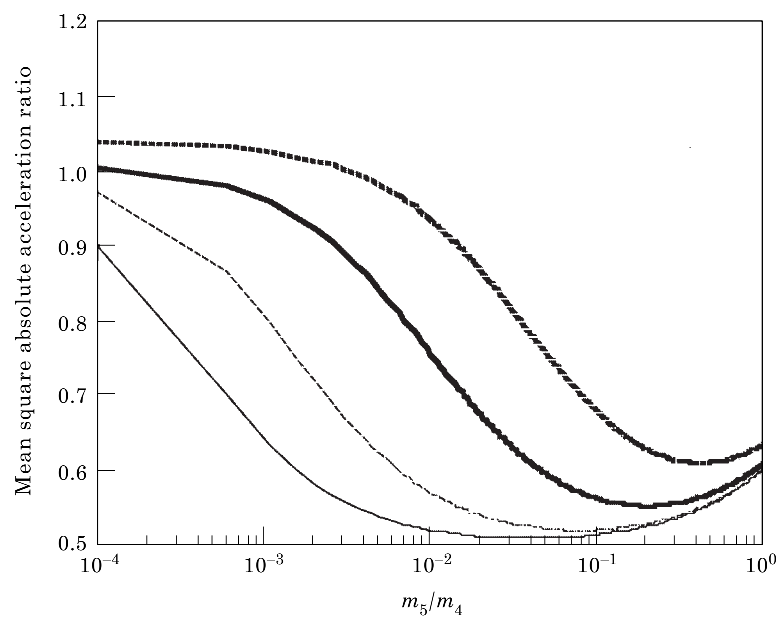

Figure 7. Mean square relative acceleration of m_4 as a function of M and damping ratios $\zeta_3 = \zeta_4$ for $\omega_5/\omega_4 = 1\%$ —, $\zeta_3 = \zeta_4 = 1\%$; - - - , $\zeta_3 = \zeta_4 = 2\%$; —, $\zeta_3 = \zeta_4 = 5\%$; — — — , $\zeta_3 = \zeta_4 = 10\%$.

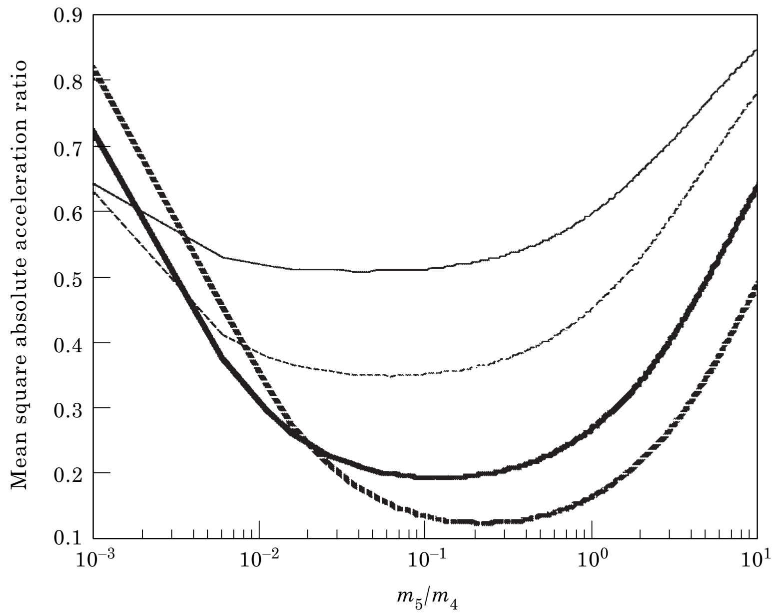

Figure 8. Mean square relative acceleration of m_4 as a function of M and ζ_4 for $\omega_5/\omega_4 = 1$ and $\zeta_3 = 1\%$; —, $\zeta_4 = 1\%$; - - - , $\zeta_4 = 2\%$; —, $\zeta_4 = 5\%$; – – – , $\zeta_4 = 10\%$.

Figure 7 shows that equal damping in each spring–mass–damper system ($\zeta_3 = \zeta_4$) dynamic loading is relatively independent of the damping ratio for mass ratios close to 1. Also, for mass ratios $< 0{\cdot}1$ an increase in damping ratio will decrease the amount of dynamic loading.

Figure 8 indicates that for constant ζ_3 an increase in ζ_4 will increase the dynamic absorber effect on the bottom mass, or as the damping in the upper mass system increases, it is more efficient as a dynamic vibration absorber for given M and ζ_3.

Similarly, it is possible to show that for constant ζ_4 an increase in ζ_3 will decrease the dynamic absorber effect on the bottom mass, or as the damping in the lower mass system increases, the upper mass, for given M and ζ_4, is less efficient as a dynamic vibration absorber.

5. EXPERIMENTAL AND NUMERICAL RESULTS

Figure 9 shows a comparison of the experimental versus calculated acceleration response for the top block. It is apparent that the finite element model was able to represent the system response very well. Similarly good results were obtained for the remaining accelerometer locations.

Figure 10 demonstrates bending stress frequency response functions for the configuration shown in Figure 2 and for the top beam/block assembly mounted directly to the shaker platform. The maximum bending stress computed for the "field" configuration was 240 MPa. The bending stress determined for the same location during an infinite impedance test reached 6794 MPa, indicating the failure of the tested "equipment unit," with an overtest factor of 28. This result confirmed the overtest results obtained theoretically.

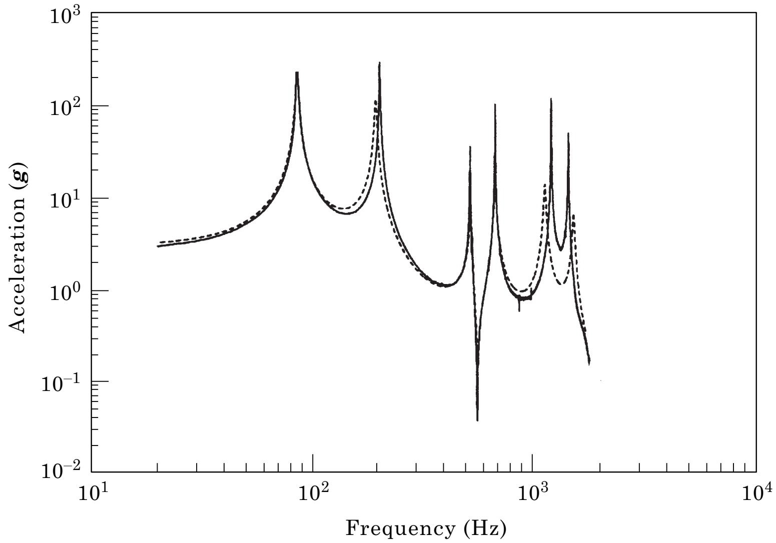

Figure 9. Comparison of experimental versus calculated response: —, experimental; - - - , finite element results.

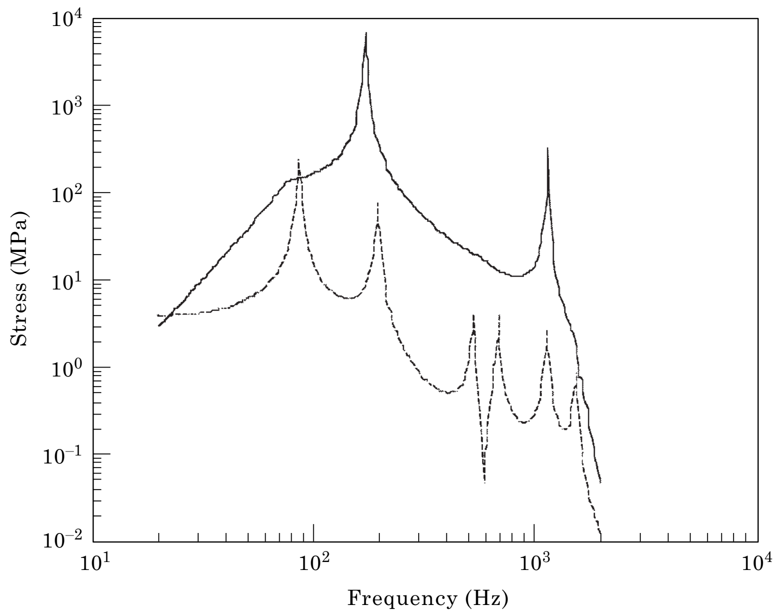

Figure 10. Bending stress frequency response functions comparison; - - - , "field" results; —, infinite impedance test results.

6. CONCLUSIONS

This analysis thoroughly emphasizes the effect, for a wide range of dimensionless parameters, that an equipment item has on its supporting structure. It is shown that for lightly damped systems where masses of the supporting structure and of the equipment are of the same order, the infinite impedance test can result in equipment damage.

The conclusion can be made that very few lightly damped systems with comparable masses of equipment and the supporting structure can neglect the dynamic loading interaction between equipment and support, without risking a severe overtest at the equipment laboratory vibration test. The dynamic loading an equipment item exerts against its support must be considered.

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