Journal of Sound and Vibration (1999) **223**(5), 834–841
Article No. jsvi1998.2115, available online at http://www.idealibrary.com on IDEAL®

VIBRATIONS OF CONTINUOUS, RECTANGULAR PLATES IN THE CASE OF OBLIQUE INTERMEDIATE SUPPORTS

P. A. A. LAURA, D. V. BAMBILL AND R. E. ROSSI
Institute of Applied Mechanics (CONICET) and Department of Engineering, Universidad Nacional del Sur, 8000 - *Bahía Blanca, Argentina*

(*Received* 30 *November* 1998)

1. INTRODUCTION

Well known treatises [1, 2] discuss the problem of vibrating, continuous, isotropic plates. On the other hand no studies are available on vibrating, continuous plates in the case where the internal support is oblique with respect to the plate sides. The present study deals with the determination of the fundamental frequency of transverse vibration of continuously clamped and simply supported rectangular plates with an oblique intermediate support whose position in the x–y plane is defined by the linear relation (Figure 1)

$$\beta y = \alpha x + \delta. \tag{1}$$

From the sake of generality it is assumed that the plate material possesses orthotropic characteristics, the isotropic case being then a particular constitutive case. It is important to point out that the case of oblique supports appears in practice due to operational requirements and also due to constructional imperfections. Two independent solutions are obtained in the present study: (1) the classical Rayleigh–Ritz method which is achieved by simple construction of the approximate fundamental mode shape, (2) the finite element method using a well known, standard finite element code [3].

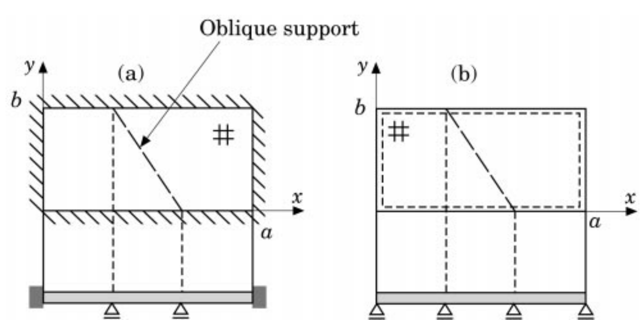

Figure 1. (a) Clamped and (b) simply supported rectangular plates, orthotropic plates with an oblique intermediate support considered in the present investigation.

0022–460X/99/250834 + 08 $30.00/0

TABLE 1

Fundamental frequency coefficients of plates shown in figure 2(a) ($(\Omega_1 = \sqrt{\rho h/D_1}\omega_1 a^2$)

Co-ordinate function	a/b				
	2/5	2/3	1	3/2	5/2
Isotropic Material					
1	61·685	65·200	74·296	101·447	208·331
2	60·830	64·542	73·899	101·270	208·084
3	60·815	64·631	73·880	101·266	208·084
4	60·649	64·376	73·870	101·222	206·860
5	60·706	64·471	74·000	101·295	205·833
F.E.	*57·883*	*63·221*	*73·519*	*100·223*	*189·523*
Orthotropic Materials					
$D_2/D_1 = 1/2$, $D_k/D_1 = 1/3$, $\mu_2 = 0{\cdot}30$					
1	61·551	64·436	71·133	89·787	163·051
2	60·690	63·765	70·726	89·653	163·019
3	60·672	63·754	70·720	80·648	163·012
4	60·501	63·530	70·582	89·640	162·436
5	60·555	63·612	70·673	89·736	161·971
F.E.	*55·141*	*61·943*	*69·845*	*89·411*	*154·914*
$D_2/D_1 = 1/2$, $D_k/D_1 = 1$, $\mu_2 = 0{\cdot}30$					
1	63·355	69·121	80·423	105·977	188·177
2	62·646	68·745	80·350	105·931	187·836
3	62·633	68·739	80·345	105·922	187·774
4	62·426	68·471	80·103	105·889	187·451
5	62·420	68·401	80·022	105·889	187·290
F.E.	*59·648*	*66·519*	*78·853*	*105·404*	*181·956*
$D_2/D_1 = 2$, $D_k/D_1 = 1$, $\mu_2 = 0{\cdot}30$					
1	63·612	70·914	87·990	132·971	291·83
2	62·900	70·536	87·887	132·965	291·71
3	62·893	70·530	87·882	132·957	291·70
4	62·698	70·362	87·861	132·775	289·69
5	62·721	70·408	87·888	132·619	287·768
F.E.	*60·659*	*69·428*	*87·622*	*129·944*	*259·800*

Notes: 1, equation (A.1); 2, equation (A.2); 3, equation (A.3); 4, equation (A.4); 5, equation (A.5); F.E., finite element results.

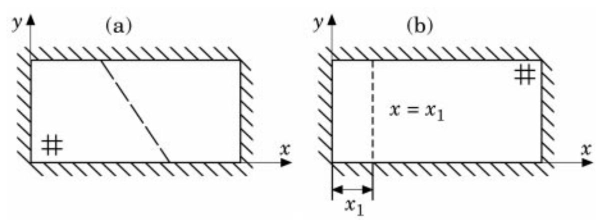

Figure 2. Clamped orthotropic (and isotropic) plates considered in the present investigation. (a) oblique support, $y = 3bx/a + 2b$; (b) support parallel to the y-axis.

2. APPROXIMATE ANALYTICAL SOLUTION

Using Lekhnitskii's well known notation [4] one expresses the governing energy functional in the form

$$J(W) = \tfrac{1}{2}\int\int\left[D_1\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + 2D_1\mu_2\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + D_2\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 4D_k\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)^2\right]\mathrm{d}x\,\mathrm{d}y$$

$$-\tfrac{1}{2}\rho h\omega^2\int\int W^2\,\mathrm{d}x\,\mathrm{d}y, \qquad (2)$$

TABLE 2

Fundamental frequency coefficients of the isotropic plates shown in Figure 2(b) $(\Omega_1 = \sqrt{\rho h/D_1}\,\omega_1 a^2)$

x_1/a	Co-ordinate function	a/b = 2/5	2/3	1	3/2	5/2
0·10	1	31·750	34·696	42·677	65·941	151·718
	4	28·605	31·900	40·277	64·263	150·687
	F.E.	*27·552*	–	*39·272*	–	*149·622*
0·20	1	35·840	38·693	46·342	68·910	153·805
	4	33·405	36·452	44·483	67·631	153·074
	F.E.	*33·036*	–	*44·132*	–	*152·519*
0·30	1	43·124	45·906	53·164	74·711	158·059
	4	41·760	44·687	52·203	74·135	157·836
	F.E.	*40·946*	–	*51·458*	–	*157·125*
0·40	1	55·211	58·039	65·033	85·460	166·508
	4	54·436	57·385	64·577	85·276	166·501
	F.E.	*52·401*	–	*62·504*	–	*164·668*
0·50	1	64·357	67·297	74·296	94·273	173·918
	4	63·191	66·281	73·534	93·894	173·885
	F.E.	*63·085*	–	*73·402*	–	*173·818*

Notes: Co-ordinate function as in Table 1.

TABLE 3

Fundamental frequency coefficients of the orthotropic plates shown in Figure 2(b) ($D_2/D_1 = 1/2; D_k/D_1 = 1/3; \mu_2 = 0 \cdot 30; \Omega_1 = \sqrt{\rho h/D_1}\omega_1 a^2$)

x_1/a	Co-ordinate function	a/b 2/5	2/3	1	3/2	5/2
0·10	1	31·616	33·888	39·453	55·170	114·430
	4	28·457	30·9184	36·849	53·165	113·088
	F.E.	*27·380*	–	*35·729*	–	*111·958*
0·20	1	35·717	37·958	43·368	58·648	117·132
	4	33·272	35·671	41·377	57·145	116·186
	F.E.	*32·880*	–	*40·955*	–	*115·708*
0·30	1	43·014	45·268	50·552	65·295	122·561
	4	41·645	44·028	49·535	64·630	122·274
	F.E.	*40·806*	–	*48·685*	–	*121·574*
0·40	1	55·113	57·503	62·848	77·240	133·079
	4	54·335	56·839	62·369	77·028	133·070
	5	–	–	–	–	133·056
	F.E.	*52·274*	–	*60·135*	–	*130·962*
0·50	1	64·263	66·809	72·338	86·790	142·070
	4	63·093	65·780	71·546	86·366	142·026
	F.E.	*62·962*	–	*71·273*	–	*142·002*

Notes: see Table 1.

where $W(x, y)$ is the amplitude of the fundmantal modeof vbration which will be approximated by the functional relation $W_a(x, y)$ in view of the fact that finding an exact solution will be, at best, an exceedingly difficult task.

It was considered extremely convenient to construct $W_a(x, y)$, in the case of the clamped plate, using the simple polynomial approximations employed in [5] for the case of "plain"* rectangular plate multiplied by functional relations which take into account the presence of an intermediate, oblique support. After several numerical experiments the following approximation was selected when using a two-term solution in view of the fact that it provided minimum upper bounds, the procedure being greatly facilitated by the use of Mathematica [6]:

$$W_a(x, y) = f_1(x)g_1(x)(A_1 r_1(x, y) + A_2 r_1(x, y)^3), \tag{3}$$

where

$$f_1(x) = x^4 + \alpha_3 x^3 + \alpha_2 x^2, \quad g_1(x) = y^4 + \beta_3 y^3 + \beta_2 y^2, \quad r_1(x, y) = \beta y - \alpha x + \delta. \tag{4a–c}$$

*The plain plate is defined as the structural element without the intermediate, oblique support.

TABLE 4

Fundamental frequency coefficients of the plates shown in Figure 3(a) ($\Omega_1 = \sqrt{\rho h/D_1}\omega_1 a^2$)

	a/b			
2/5	2/3	1	3/2	5/2
Isotropic Material				
40·336 (1)	43·661	50·965	69·495	131·505
42·837 (2)	46·224	52·933	69·805	130·227
38·190 (F.E.)	*42·051*	*49·462*	*66·661*	*118·045*
Orthotropic Materials				
$D_2/D_1 = 1/2$, $D_k/D_1 = 1/3$, $\mu_2 = 0{\cdot}30$				
40·233 (1)	43·166	49·134	63·237	109·531
42·664 (2)	45·758	51·381	64·409	108·170
37·980 (F.E.)	*41·485*	*47·697*	*61·442*	*102·343*
$D_2/D_1 = 1/2$, $D_k/D_1 = 1$, $\mu_2 = 0{\cdot}30$				
42·277 (1)	48·285	58·875	79·758	136·496
44·854 (2)	50·700	60·871	80·936	135·651
40·433 (F.E.)	*46·817*	*57·603*	*78·288*	*131·608*
$D_2/D_1 = 2$, $D_k/D_1 = 1$, $\mu_2 = 0{\cdot}30$				
42·421 (1)	49·244	62·716	92·928	183·760
45·021 (2)	51·530	64·095	92·453	183·541
40·647 (F.E.)	*47·820*	*61·120*	*88·927*	*164·577*

Notes: (1) $n = 3$, $m = 1$, equation (5): (2) $n = 1$, $m = 3$, equation (5): (F.E.) finite element results.

and where α_3, α_2 and β_3, β_2 are obtained substituting equations (4a) and (4b) in the essential boundary conditions of the clamped plate. Three term approximations have also been experimented (see Appendix A). When considering the simply supported case the following approximation was employed:

$$W_a(x, y) = \left(A_1 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b} + A_2 \sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}\right)(\beta y - \alpha x - \delta), \tag{5}$$

where n and m are integer optimization parameters which allow for minimization of the fundamental eigenvalue [7].

3. NUMERICAL RESULTS

Table 1 shows values of the fundamental frequency coefficient for an isotropic and orthotropic clamped rectangular plates considering a particular location of the oblique support; see Figure 2(a). The frequency coefficients have been obtained using different co-ordinate functions and are compared with results

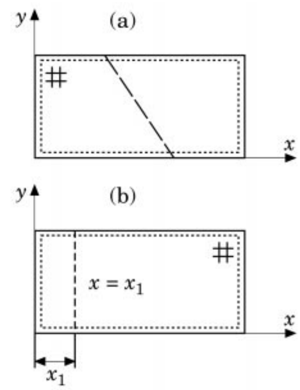

Figure 3. Simply supported orthotropic (and isotropic) plates considered in the present investigation. (a) oblique support, $y = 3bx/a + 2b$; (b) support parallel to the y-axis.

obtained using the finite element method*. The minimum analytical result has been underlined. In general the agreement with the finite element approach is very good from a practical view point.

Tables 2 and 3 deal with clamped isotropic and orthotropic plates, respectively when the intermediate support is parallel to the y-axis. The agreement with the values obtained by means of the finite element method is very good for all the situations considered.

TABLE 5

Fundamental frequency coefficients of the isotropic plates shown in Figure 3(b) ($\Omega_1 = \sqrt{\rho h/D_1}\omega_1 a^2$)

	a/b				
x_1/a	2/5	2/3	1	3/2	5/2
0·10	20·267[(1)] *19·080*[(F.E.)]	22·570 –	27·340 *26·073*	38·769 –	77·240 *75·675*
0·20	23·571[(1)] *22·581*[(F.E.)]	25·854 –	30·566 *29·644*	41·859 –	80·087 *79·183*
0·30	28·863[(1)] *27·651*[(F.E.)]	31·202 –	35·964 *34·899*	47·249 –	85·328 *84·481*
0·40	36·880[(1)] *34·864*[(F.E.)]	39·423 –	44·481 *42·495*	56·152 –	94·152 *92·502*
0·50	42·472[(1)] *41·059*[(F.E.)]	45·200 –	50·554 *49·351*	62·682 –	101·828 *101·172*

Superscripts as for Table 4.

*The results have been determined using between 9000 and 10 000 elements, depending upon the geometric configurations.

TABLE 6

Fundamental frequency coefficients of the isotropic plates shown in Figure 3(b) $(D_2/D_1 = 1/2; D_k/D_1 = 1/3; \mu_2 = 0{\cdot}30; (\Omega_1 = \sqrt{\rho h/D_1}\omega_1 a^2)$

	a/b				
x_1/a	2/5	2/3	1	3/2	5/2
0·10	20·198$^{(1)}$ *19·010*$^{(F.E.)}$	22·260 –	26·252 *24·948*	35·140 –	63·273 *61·414*
0·20	23·506$^{(1)}$ *22·514*$^{(F.E.)}$	25·570 –	29·269 *28·622*	38·473 –	66·642 *65·583*
0·30	28·801$^{(1)}$ *27·587*$^{(F.E.)}$	30·944 –	35·074 *33·986*	44·196 –	72·719 *71·745*
0·40	36·818$^{(1)}$ *34·800*$^{(F.E.)}$	39·183 –	43·693 *41·683*	53·477 –	83·209 *80·835*
0·50	42·408$^{(1)}$ *40·993*$^{(F.E.)}$	44·965 –	49·809 *48·588*	60·198 –	91·123 *90·392*

Superscripts as for Table 4.

Table 4 depicts fundamental frequency coefficients for simply supported isotropic and orthotropic plates for a particular location of the oblique support; see Figure 3(a). It is again observed that the agreement with the values determined by means of the finite element method is good from an engineering viewpoint.

Tables 5 and 6 present fundamental eigenvalues for simply supported isotropic and orthotropic plates, respectively, when the support is parallel to the y-axis. The agreement with the finite element method is quite good for all the situations considered.

In summary: the proposed analytical approach yields reasonable accuracy for a rather complex elastodynamics problem. Certainly its scope and range of validity is modest when one compares with the finite element algorithmic procedure but allows for the determination of first order design values with a minimum amount of difficulty. Admittedly the analytical approach yields, in some instances, numerical results which are rather high upper bounds.

ACKNOWLEDGMENTS

The present study has been sponsored by CONICET Research and Development Program and by Secretaría General de Ciencia y Tecnología of Universidad Nacional del Sur.

REFERENCES

1. R. D. Blevins 1979 *Formulas for Natural Frequency and Mode Shape*. New York: Van Nostrand Reinhold.
2. A. W. Leissa 1969 *NASA SP* **160**. Vibration of plates.
3. Reference Manual 1997 *Linear Stress and Vibration Analysis Processor Part Number* 6000.501, *revision* 2.02. Pittsburgh. Professional Mech/E.
4. S. G. Lekhnitskii 1968 *Anisotropic Plates*. New York: Gordon & Breach.
5. P. A. A. Laura and B. F. Saffel 1967 *The Journal of the Acoustical Society of America* **41**, 836–838. Study of small amplitude vibrations of clamped rectangular plates using polynomial approximations.
6. S. Wolfram 1996 *The Mathematica Book*. Cambridge: Wolfram Media/Cambridge University Press; third edition.
7. P. A. A. Laura 1995 *Ocean Engineering* **22**, 235–250. Optimization of variational methods.

APPENDIX A

Different approximations used in the case of the clamped plate:

$$W_a(x, y) = A_1 f_1(x) g_1(x) r_1(x, y), \tag{A.1}$$

$$W_a(x, y) = f_1(x) g_1(x) r_1(x, y)(A_1 + A_2 r_1(x, y)^2), \tag{A.2}$$

$$W_a(x, y) = f_1(x) g_1(x) r_1(x, y)(A_1 + A_2 r_1(x, y)^2 + A_2 r_1(x, y)^4), \tag{A.3}$$

$$W_a(x, y) = f_1(x) g_1(x) r_1(x, y)(A_1 + A_2 r_1(x, y)^2 + A_2 x^2 y^2), \tag{A.4}$$

$$W_a(x, y) = f_1(x) g_1(x) r_1(x, y)(A_1 + A_2 r_1(x, y)^4 A_2 x^2 y^2). \tag{A.5}$$