Journal of Sound and Vibration (1999) **223**(5), 842–845
Article No. jsvi.1998.2150, available online at http://www.idealibrary.com on IDEAL®

TRANSVERSE VIBRATIONS OF A CIRCULAR PLATE WITH A FREE EDGE AND A CONCENTRIC CIRCULAR SUPPORT

P. A. A. Laura, R. H. Gutiérrez, S. A. Vera and D. A. Vega
Institute of Applied Mechanics (CONICET) and Departments of Engineering and Physics, *Universidad Nacional del Sur*, 8000 - *Bahía Blanca*, *Argentina*

(*Received* 4 *December* 1998)

1. introduction

An exact solution for the title problem in terms of Bessel functions was obtained by Bodine, see Figure 1 [1] as referenced by Leissa in his classical Monograph [2].

Very recently and during the course of a research program conducted at the Institute of Applied Mechanics dealing with the title problem but in the case of non-uniform thickness, serious discrepancies with the results depicted in references [1, 2] were found. Motivated by these uncertainties the present study was undertaken whereby two independent solutions were obtained: (1) the exact solution, as in reference [1]; (2) a variational formulation where the displacement amplitude of the fundamental mode was approximated and Rayleigh's optimization criterion was used to optimize the fundamental frequency coefficient.

Excellent agreement between both solutions was achieved but differences larger than 10% were observed, for some particular configurations when compared with the results obtained in reference [1].*

2. exact solution

For the case of normal modes the amplitudes of motion are (see Figure 1)

$$W_1 = A_1 \mathrm{J}_0(kr) + C_1 \mathrm{I}_0(kr), \quad 0 \leqslant r \leqslant b, \tag{1}$$

$$W_2 = A_2 \mathrm{J}_0(kr) + B_2 \mathrm{Y}_0(kr) + C_2 \mathrm{I}_0(kr) + D_2 \mathrm{K}_0(kr), \quad b \leqslant r \leqslant a, \tag{2}$$

where $k = \sqrt[4]{\rho h / D}\sqrt{\omega}$.

The determinantal equation is generated setting up the appropriate boundary and continuity conditions

*It is important to point out that the authors of the present study did not have access to reference [1] since it was not available to them in Argentina. Instead they had access to reference [2].

0022–460X/99/250842 + 04 $30.00/0

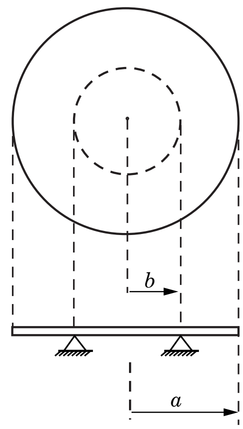

Figure 1. Structural system under study executing axisymmetric, transverse vibrations.

$$W_1(b) = 0, \quad W_2(b) = 0 \tag{3a, b}$$

$$(\mathrm{d}W_1/\mathrm{d}r)(b) = (\mathrm{d}W_2/\mathrm{d}r)(b), \quad (\mathrm{d}^2W_1/\mathrm{d}r^2)(b) = (\mathrm{d}^2W_2/\mathrm{d}r^2)(b), \tag{3c, d}$$

$$((\mathrm{d}^2W_2/\mathrm{d}r^2) + (\nu/r)\,\mathrm{d}W_2/\mathrm{d}r)|_{r=a} = 0, \quad \mathrm{d}/\mathrm{d}r\nabla^2 W_2|_{r=a} = 0. \tag{3e, f}$$

Setting up the frequency equation and then obtaining the fundamental eigenvalue has been greatly facilitated by the use of MAPLE [3].

3. APPROXIMATE ANALYTICAL SOLUTION

For the case where $b \neq 0$ the displacement amplitude has been approximated by means of

$$W \cong W_a = \sum_{j=1}^{3} C_j \varphi_j(r), \tag{4}$$

where

$$\varphi_1 = \alpha_1 r^p + \beta_1 r^2 + 1, \quad \varphi_2 = \alpha_2 r^{p+1} + \beta_2 r^3 + 1, \quad \varphi_3 = \alpha_3 r^{p+2} + \beta_3 r^4 + 1, \tag{5}$$

where the α's and β's are obtained substituting each co-ordinate function in equations (3a) and (3e) and p is Rayleigh's optimization parameter [4].

In the case where $b = 0$ the following expressions have been used:

$$\varphi_1 = r^p + \alpha_1 r^2, \quad \varphi_2 = r^{p+1} + \alpha_2 r^3, \quad \varphi_3 = r^{p+2} + \alpha_3 r^4. \tag{6}$$

The generation of the frequency determinant follows the usual Rayleigh–Ritz energy procedure and the final step is the minimization of the fundamental frequency coefficient with respect to p,

$$\mathrm{d}\Omega_1/\mathrm{d}p = 0, \tag{7}$$

where $\Omega_1 = \sqrt{\rho h/D}\,\omega_1 a^2$, and which allows for optimization of Ω_1.

TABLE 1
Values of Ω_1 *of the system shown in Figure* 1 ($\nu = 1/3$)

b/a	Exact	Optimized Rayleigh–Ritz
0	–	3·774
0·1	3·9323	3·935
0·2	4·3026	4·313
0·3	4·8863	4·896
0·4	5·7516	5·760
0·5	6·9883	6·999
0·6	8·4701	8·482
0·7	9·0304	9·031
0·8	7·8680	7·872
0·9	6·2881	6·293
1·0	4·9838	4·984

TABLE 2
Values of Ω_1 *of the system shown in Figure* 1 ($\nu = 0{\cdot}3$)

b/a	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
Exact	3·9093	4·2750	4·8517	5·7069	6·9295	8·3968	8·9599	7·8094	6·2354	4·9351

4. NUMERICAL RESULTS

In order to compare with the results presented in reference [1], all the numerical determinations have been performed first for Poisson's ratio (ν) equal to 1/3. Table 1 depicts the fundamental frequency coefficients Ω_1 for different values of the parameter b/a obtained (1) by the exact approach and (2) making use of the optimized Rayleigh–Ritz method.

It is observed that the agreement between both sets of results is excellent for all values of b/a, the maximum difference being of the order of 0·2% for $b/a = 0{\cdot}4$. The results obtained in the present study* have been plotted in Figure 2 in order to compare with the results obtained in references [1, 2]. Apparently the results of reference [1] are extremely high for $0 < b/a < 0{\cdot}68$ (for $b/a = 0{\cdot}5$ the difference is of the order of 13%) and on the low side for $0{\cdot}7 < b/a < 1$. Both curves practically coincide for the maximum value which is the frequency coefficient corresponding to the first axisymmetric mode of the completely free plate. Table 2 shows values of Ω_1 for $\nu = 0{\cdot}3$. $b/a = 1$ is the case of a simply supported circular plate and Ω_1 coincides to five significant figures with the very accurate result from the literature.

*The differences between both sets of results cannot be distinguished when plotting the results in the scale used to prepare Figure 2.

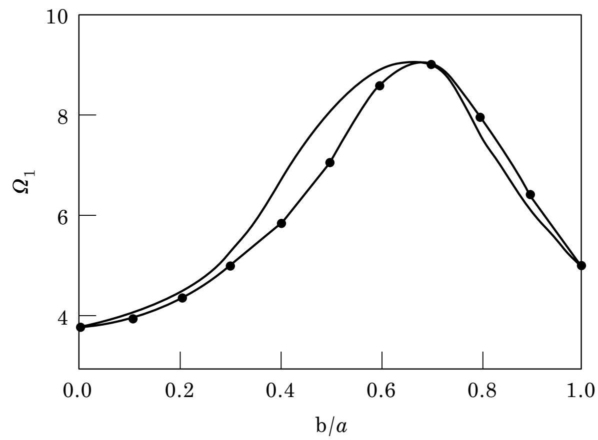

Figure 2. Fundamental frequency coefficient as a function of b/a: ——, references [1, 2]; ●—●, present study.

ACKNOWLEDGMENTS

The present study has been sponsored by CONICET Research and Development Program and by Secretaría General de Ciencia y Tecnología of Universidad Nacional del Sur (Departments of Engineering and Physics). S. A. Vera and D. A. Vega were supported by CONICET Fellowships.

REFERENCES

1. R. V. BODINE 1959 *Journal of Applied Mechanics* **26**, 666–668. The fundamental frequencies of a thin flat circular plate simply supported along a circle of arbitrary radius.
2. A. W. LEISSA 1969 *NASA SP*160. Vibration of plates.
3. B. W. CHAR, K. O. GEDDES, G. H. GONNET, B. L. LEONG, M. B. MONNAGAN and S. M. WATT 1991 *MAPLE V*.5, *Library Reference Manual*.
4. P. A. A. LAURA 1995 *Ocean Engineering* **22**, 235–250. Optimization of variational methods.