Journal of Sound and Vibration (1999) **224**(2), 379–383
Article No. jsvi.1998.2152, available online at http://www.idealibrary.com on IDEAL®

TRANSVERSE VIBRATIONS OF A FREE–FREE CIRCULAR ANNULAR PLATE

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(*Received* 16 *December* 1998)

1. INTRODUCTION

Previous, recent studies performed by the authors have dealt with the determination of accurate eigenvalues of circular annular plates for the following situations: (1) clamped or simply supported at the outer boundary and free at the inner contour [1]; (2) the four combinations of clamped and simply supported edge conditions at both boundaries [2].

Very recently the authors were confronted with the necessity of knowing accurate values of the lower frequency coefficients of transverse vibration of the free–free circular annular plate; see Figure 1, Case I. In view of this fact it was decided to obtain the solution for this case, as well as for Case II (simply supported inner boundary and free on the outside) and Case III (clamped inner edge and free outer boundary), see Figure 1.

After considering the numerical information depicted in references [1, 2] and in the present study it is felt that accurate eigenvalues are now available for the circular, annular plate. The authors gratefully acknowledge the valuable information contained in Leissa's classical treatise [3] which was obtained several decades ago and was used as a guide when obtaining the eigenvalues presented in references [1, 2] and the present study.

2. ANALYTICAL SOLUTION

The amplitude of a normal mode of vibration is described by [3]

$$W(r, \theta) = [A\mathrm{J}_n(kr) + B\mathrm{Y}_n(kr) + C\mathrm{I}_n(kr) + D\mathrm{K}_n(kr)]\cos n\theta, \qquad (1)$$

where $k = \sqrt[4]{(\rho h/D)}\sqrt{\omega}$.

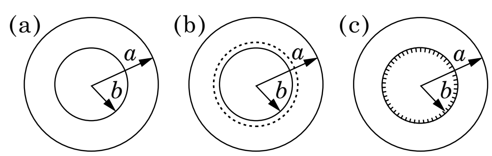

Figure 1. Vibrating circular annular plates executing transverse vibrations and considered in the present study: (a) Case I, (b) Case II, (c) Case III.

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TABLE 1

Frequency coefficients Ω_{01}, Ω_{10}, Ω_{20} *of free–free circular annular plate* (*Case* I)

	b/a								
n	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
	$\nu = 0·3$								
0	8·77454	8·44422	8·35347	8·61377	9·31348	10·65480	13·16304	18·47276	34·83311
1	20·40639	19·69460	18·29181	17·24282	17·19822	18·50431	21·91381	29·95546	55·71946
2	5·30338	5·14613	4·90598	4·60664	4·27111	3·92037	3·57250	3·24097	2·93379
	$\nu = 1/3$								
0	8·81865	8·44189	8·31611	8·55104	9·22876	10·54555	13·01859	18·26227	34·42881
1	20·44368	19·67466	18·16998	17·04300	16·94318	18·19279	21·51835	29·39408	54·65605
2	5·19907	5·04975	4·82044	4·53248	4·20712	3·86465	3·52332	3·19706	2·89431

TABLE 2

Frequency coefficients Ω_{00}, Ω_{10}, Ω_{20} of simply supported-free circular annular plate (Case II)

n	b/a								
	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
	$\nu = 0·3$								
0	3·44965	3·33950	3·42230	3·67273	4·12095	4·86995	6·18665	8·89209	17·11369
1	2·43765	2·88641	3·37379	3·99573	4·86159	6·16601	8·35063	12·73287	25·89422
2	5·42767	5·65193	6·08040	6·80406	7·98609	9·94705	13·42665	20·6352	42·61937
	$\nu = 1/3$								
0	3·44009	3·31314	3·38657	3·62965	4·07006	4·80873	6·10904	8·78224	16·90748
1	2·40782	2·84251	3·31598	3·92221	4·76829	6·04498	8·18534	12·48142	25·38841
2	5·31825	5·53557	5·95136	6·65470	7·80517	9·71629	13·11135	20·15150	41·63654

TABLE 3

Frequency coefficients Ω_{00}, Ω_{10}, Ω_{20} *of clamped–free circular annular plate* (*Case* III)

	b/a								
n	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
	$\nu = 0\cdot3$								
0	4·23737	5·18108	6·66036	9·02056	13·02425	20·52175	36·95263	84·49979	344·4022
1	3·47813	4·81302	6·55231	9·11549	13·28978	20·93595	37·49833	85·16191	345·1664
2	5·62270	6·44710	7·95648	10·46451	14·70381	22·50134	39·27686	87·19307	347·4657
	$\nu = 1/3$								
0	4·26294	5·21350	6·70117	9·07188	13·08941	20·60663	37·0690	84·67757	344·7610
1	3·47991	4·81710	6·56013	9·12933	13·31333	20·97559	37·56655	85·28927	345·4734
2	5·51642	6·34306	7·85498	10·36553	14·60840	22·41361	39·2079	87·1738	347·6189

As usual the frequency determinant is generated after imposing the boundary conditions, i.e.,

Case I:

$$M_r|_{r=a,\,b} = 0, \quad V_r|_{r=a,\,b} = 0. \tag{2a, b}$$

Case II:

$$M_r|_{r=a,\,b} = 0, \quad V_r|_{r=a} = 0, \quad W(b,\,\theta) = 0. \tag{3a–c}$$

Case III:

$$M_r|_{r=a} = 0, \quad V_r|_{r=3} = 0, \quad W(b,\,\theta) = 0, \quad \partial W/\partial r(b,\,\theta) = 0, \tag{4a–d}$$

where Leissa's standard notation has been employed [3].

3. NUMERICAL RESULTS

The first three eigenvalues $\Omega = \sqrt{\rho h/D}\omega a^2$ have been computed for ν (Poisson's ratio) equal to 0·3 and 1/3. Table 1 depicts values of Ω_{01}, Ω_{10} and Ω_{20} for the case of the free–free circular annular plate (Case I) while Tables 2 and 3 depict eigenvalues for Cases II and III respectively. The calculation procedure has been greatly facilitated by the use of MAPLE [4]. The tables contained in references [1, 2] and in the present study constitute improved data with respect to the one presently available in the open literature.

ACKNOWLEDGMENTS

The present study has been sponsored by CONICET Research and Development Program and by Secretaría General de Ciencia y Tecnología of Universidad Nacional del Sur (Departments of Physics and Engineering). S. A. Vera and D. A. Vega have been supported by CONICET Fellowships.

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