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A SIMPLE DERIVATION OF THE ACOUSTIC BOUNDARY CONDITION IN THE PRESENCE OF FLOW

F. FARASSAT
NASA Langley Research Center, Hampton, Virginia 23681, *U.S.A.*

AND

M. H. DUNN
Old Dominion University, Norfolk, Virginia 23508, *U.S.A.*

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In modelling the noise propagation in a duct of a ducted fan engine, the authors needed the boundary condition satisfied by the acoustic pressure on a lined duct wall in the presence of flow. The result in the form used by the authors was derived by M. K. Myers [1] who presented a formal derivation of a related result by K. Taylor [2]. A good review of the subject can be found in reference [3]. Here, a brief and simple derivation of the acoustic boundary condition of Myers is given. The main difference in the derivation from that in reference [1] is that the Gaussian co-ordinate system $(q^1, q^2, q^3(q^1, q^2, t))$ is used to specify the instantaneous position, i.e., Lagrangian variable, of the point on the mean position of the wall with curvilinear (Gaussian) co-ordinates (q^1, q^2). Myers uses a locally orthogonal coordinate system which is somewhat less specific than what is used here.

One assumes that one has a base or background flow with velocity $\vec{u}_0(\vec{x})$ which is time independent perturbed by a small velocity distribution $\varepsilon\vec{u}_1(\vec{x}, t)$ where $0 < \varepsilon \ll 1$. One also assumes that the wall boundary's mean position S_0 is independent of time and specified by the position vector $\vec{x}_0(q^1, q^2)$ where (q^1, q^2) are the curvilinear (Gaussian) co-ordinates on the boundary surface. The position of the time dependent boundary S is given by

$$\vec{x}(q^1, q^2, q^3, t) = \vec{x}_0(q^1, q^2) + \varepsilon q^3(q^1, q^2, t)\vec{n}_0(q^1, q^2), \tag{1}$$

where $\vec{n}_0$ is the local unit normal to S_0 and εq^3 is the distance along the normal $\vec{n}_0$ from S_0 to S at (q^1, q^2, t). The fundamental physical requirement at the boundary is

$$(\vec{u}_0 + \varepsilon\vec{u}_1)\cdot\vec{n}_0 = \varepsilon\partial q^3/\partial t. \tag{2}$$

This is the instantaneous equality of the normal fluid velocity and the surface velocity. Note that the symbol ε will be retained for order of magnitude comparison for now.

The left side of equation (2) will now be expanded as follows and equated to the right side:

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$$[\vec{u}_0(\vec{x}_0 + \varepsilon q^3 \vec{n}_0) + \varepsilon \vec{u}_1(\vec{x}_0 + \varepsilon q^3 \vec{n}_0)] \cdot \vec{n}_0$$
$$= \vec{u}_0(\vec{x}_0) \cdot \vec{n}_0 + \varepsilon[q^3 \vec{n}_0 \cdot \nabla \vec{u}_0(\vec{x}_0) + \vec{u}_1(\vec{x}_0)] \cdot \vec{n}_0 = \varepsilon \partial q^3/\partial t. \tag{3}$$

The equality of the zeroth and first order terms from both sides gives

$$\vec{u}_0(\vec{x}_0) \cdot \vec{n}_0 = 0 \tag{4}$$

and

$$q^3 \vec{n}_0 \cdot [\vec{n}_0 \cdot \nabla \vec{u}_0(\vec{x}_0)] + \vec{u}_1(\vec{x}_0) \cdot \vec{n}_0 = \partial q^3/\partial t. \tag{5}$$

Equation (4) tells us that the normal velocity based on the mean flow is zero on the mean surface. Equation (5) can be written as

$$\vec{u}_1(\vec{x}_0) \cdot \vec{n}_0 = \partial q^3/\partial t - q^3 \vec{n}_0 \cdot [\vec{n}_0 \cdot \nabla \vec{u}_0(\vec{x}_0)]. \tag{6}$$

One now assumes $\varepsilon = 1$ and thus $|\vec{u}_1| \ll |\vec{u}_0|$ and q^3 is the local normal distance between S_0 and S as a function of time. Note that $\partial q^3/\partial t$ is the local normal velocity of S in terms of the Lagrangian variables (q^1, q^2). Using Eulerian variables $\vec{x}_0$, one can define a new function $g(\vec{x}_0, t)$ such that $q^3 = g(\vec{x}_0, t)$. Note that since q^3 is the normal distance between S_0 and S, one has $|\nabla g| = 1$. One notes that

$$(\partial q^3/\partial t)(q^1, q^2) = \partial g/\partial t + \vec{u}_0(\vec{x}_0) \cdot \nabla g. \tag{7}$$

Using this result in equation (6) gives equation (11) of Myers [1]:

$$\vec{u}_1(\vec{x}_0) \cdot \vec{n}_0 = \partial g/\partial t + \vec{u}_0(\vec{x}_0) \cdot \nabla g - g\vec{n}_0 \cdot [\vec{n}_0 \cdot \nabla \vec{u}_0(\vec{x}_0)], \tag{8}$$

which is the condition that the perturbation velocity $\vec{u}_1$ must satisfy on the mean surface S_0.

One now derives the liner boundary condition based on equation (8). For a time harmonic disturbance proportional to $e^{i\omega t}$, the complex acoustic pressure p and g are related to each other by the following relation on S_0:

$$g = -p/i\omega Z, \tag{9}$$

where Z is the complex normal impedance. Using this result in equation (8) gives

$$\vec{u}_1 \cdot \vec{n}_0 = -p/Z - (1/i\omega)\vec{u}_0 \cdot \nabla(p/Z) + (p/i\omega Z)\vec{n}_0 \cdot (\vec{n}_0 \cdot \nabla \vec{u}_0), \tag{10}$$

which is the liner boundary condition, equation (15), in Myers [1]. Equation (10) is implemented in a ducted fan noise prediction code developed for NASA Langley Research Center by the authors [4].

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