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PLUCKED STRINGS AND THE HARPSICHORD

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The excitation of a harpsichord string when it is set into motion, i.e., plucked, by a plectrum is studied. We find that the amplitude of the resulting string vibration is approximately independent of the velocity with which the key is depressed. This result is in accord with conventional wisdom, but at odds with a recent theoretical model. A more realistic theoretical treatment of the plucking process is then described, and shown to be consistent with our measurements. The experiments reveal several other interesting aspects of the plectrum–string interaction.

1. INTRODUCTION AND BACKGROUND

The harpsichord is a stringed instrument from which the piano evolved nearly 300 years ago. The two instruments differ in a number of ways, the most important being the manner in which the strings are excited. In a piano, the strings are set into motion by a blow from a felt covered hammer, while in a harpsichord the excitation is accomplished by plucking the string with a thin flexible "beam", called a plectrum. Originally, the plectrum was typically a piece of crow quill, while in modern instruments it is often a thin strip of plastic (delrin is a popular choice). While the piano has been the subject of a number of scientific studies [1–4], the harpsichord has attracted much less attention. A very nice, wide ranging, discussion of the physics of the harpsichord was given some years ago by Fletcher [5, 6], who gave a very insightful analysis of many aspects of the instrument. (More recent studies have given detailed discussions of several specific aspects of the harpsichord 7–9.) However, one topic which was not considered in any detail was the interaction of the plectrum and the string. This is the problem that is addressed in the present paper.

An important feature of the harpsichord is that the performer has relatively little control over the volume of a note. That is, the volume of a note is observed to be the "same" regardless of how fast a key is pressed. This is often viewed as a deficiency. Indeed, the desire to have an instrument in which the volume of a note can be made either soft or loud in accordance with the key velocity was a prime motivation for the invention of the piano.

The volume of a note produced by a harpsichord depends on the amplitude with which the string is plucked, i.e., the deflection of the string when it slips off the

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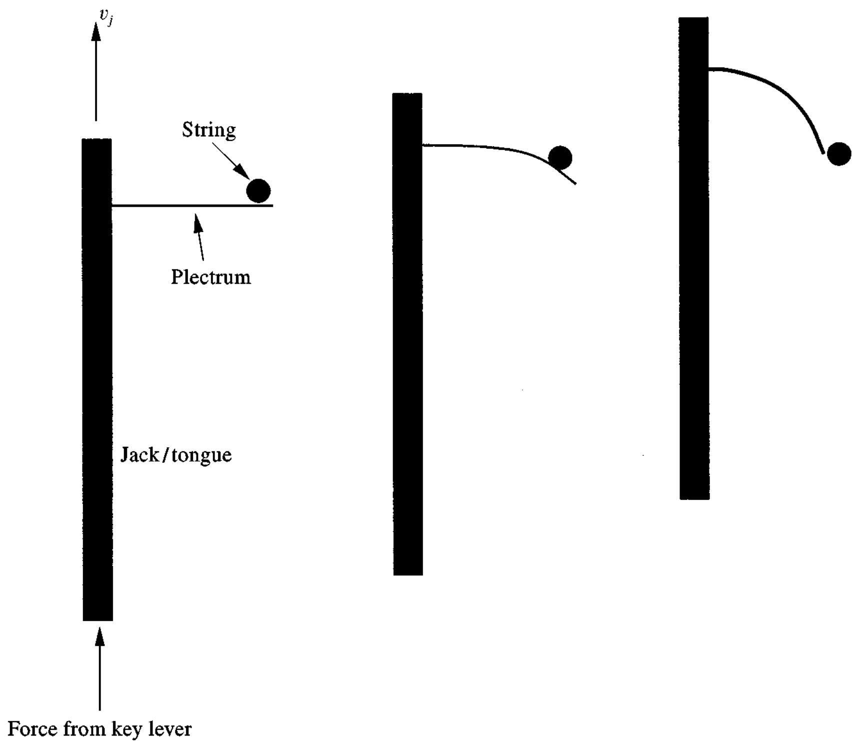

Figure 1. Qualitative picture of the excitation of a string by a flexible plectrum. The jack begins as shown at the left, with the plectrum unbent. As the jack moves upwards, the plectrum flexes, exerting a force on the string. Eventually, as shown at the right, the plectrum slips off the string, and thereafter the string vibrates freely.

plectrum. The interaction of the plectrum with the string is shown schematically in Figure 1. The plectrum is mounted in a holder called the tongue, which is mounted in the jack, which in turn sits on the end of the key lever. The tongue is able to rotate on an axle (attached to the jack) which runs parallel to the string. This enables the plectrum to move past (rotate out of the way of) the string, and hence be "reset", after a key is released. However, the tongue does not rotate when the plectrum is moving upwards, as in Figure 1. For our purposes, we can assume that the tongue and jack are rigidly connected, so that when a key is pressed the jack moves upwards, and the plectrum deflects as shown in Figure 1. This deflection increases as the jack moves, and eventually the plectrum slips past the string. The string then vibrates freely, producing its note. According to the conventional wisdom, the deflection of the plectrum and string when the two separate, and hence the amplitude of the pluck, are all approximately independent of the speed of the jack (i.e., the speed with which the key is pressed). This accounts for the observation that the volume of a note does not vary significantly with key velocity. However, a recent theoretical calculation has questioned this picture. Griffel [10] considered the physics of plucking using a model which treated the plectrum as a torsional spring, and included the inertia of the string. His calculation predicts that the string

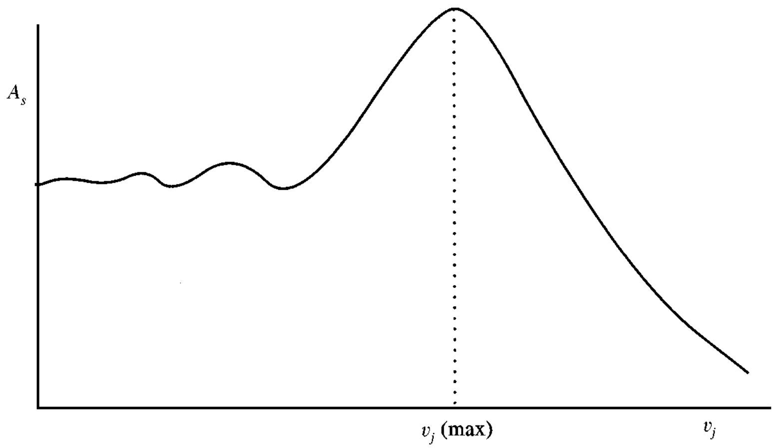

Figure 2. Prediction for the amplitude of the string motion as a function of the jack velocity, according to the model of Griffel [10].

vibration amplitude, A_s, can vary quite significantly with the jack velocity, v_j. While the detailed behavior depends on parameters such as the length of the plectrum and the frequency of the note, the qualitative prediction from Griffel's model is shown in Figure 2. As v_j is increased from small values, the model predicts oscillations of A_s, with a substantial peak followed by a rapid decrease, and with $A_s \to 0$ for large jack velocities. The value of A_s at v_j (max) is 50% larger than that found in the limit of small v_j. Griffel suggested that v_j in a real harpsichord is large enough to be outside the regime where A_s oscillates. However, if this is really the case, then the volume of a note should *decrease* substantially as the key velocity is increased, and we do not believe that this is observed in normal playing.

The results of Griffel have motivated us to carry out an experimental study of the plectrum–string interaction. Surprisingly, we know of no previous experiments of this kind. Our main goal was to measure the relation between A_s and v_j, and thereby test the prediction sketched in Figure 2. During the course of the measurements, we observed a number of associated phenomena, which we believe are also worthy of description. After presenting the experimental results, we consider a more realistic theoretical model of plucking, and compare it with our measurements.

2. EXPERIMENTAL SET-UP AND RESULTS

The apparatus is shown schematically in Figure 3. A string is looped over a hitchpin, runs between a bridge and an agraffe, and is then fastened to a tuning pin. The bridge sits on a thin piece of spruce, which acts as a soundboard. The string is plucked by a delrin plectrum, of thickness 0·5 mm, which is mounted in a plastic harpsichord jack obtained from Hubbard Harpsichords, Sudbury, MA. The jack is set into vertical motion by depressing, by hand, a key lever from a piano action. (We considered constructing a mechanical device to press the key, but the repeatability obtained by hand seemed satisfactory.) Piezoelectric accelerometers (both

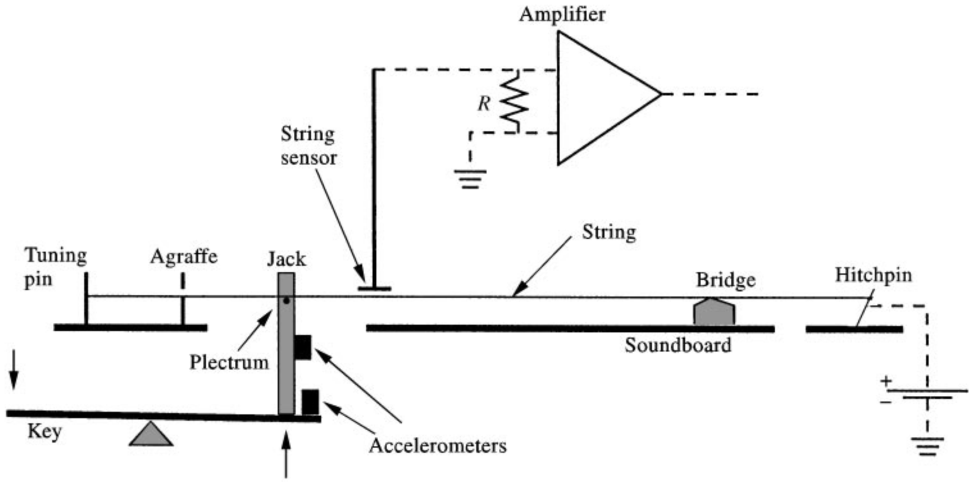

Figure 3. Schematic diagram of the apparatus, showing the location of the accelerometers, and the electronics associated with the string sensor.

from PCB Piezotronics) were attached to the jack, and to the key lever immediately adjacent to the jack. The accelerometer on the jack was a model 350A02, while the one on the key was a model 352B68.

A capacitive sensor [13, 14] was used to measure the motion of the string. This sensor was a small metal plate, $\sim 1 \times 1$ cm^2, which was positioned a few mm above the string. A DC potential (~ 20 V) was applied to the string, and the voltage across the sensor was fed to a preamplifier. The capacitance of the sensor (plus cables) was small, so the RC time constant for charging the sensor was very short (the resistor R was 10 kΩ). The sensor voltage was thus proportional to the charging current, which is proportional to the time derivative of the sensor capacitance. Since the sensor capacitance varies inversely with the spacing between the sensor plate and the string, the sensor output was proportional to the velocity of the string. In a separate arrangement (not shown in the figure), the plectrum force was measured by brushing the plectrum against the edge of a thin metal strip attached to a piezoelectric force sensor (model 208B03, obtained from PCB Piezotronics.) The thickness of this strip was 0·5 mm, which is the same as that of a typical harpsichord string. Any pair of signals (the two accelerometers, the string velocity, and the force sensor) could be recorded simultaneously using a personal computer, with a sampling rate of typically 22 kHz, and the results then processed and analyzed in detail.

Figure 4 shows the acceleration of the jack (top), and the velocity of the string (bottom), as functions of time for a pluck of moderate amplitude. A steel string with a diameter of 0·2 mm and length of 53 cm was used in this measurement. The plectrum contacted the string approximately 10% of the string length from the agraffe. The string velocity signal is simplest to understand. At early times, this signal was indistinguishable from the noise of the sensor plus preamplifier (note that while the origin of the time axis is arbitrary, it is the *same* in the two plots). At $t \approx 0{\cdot}24$ s, the string was set into motion, and it thereafter oscillated rapidly. We

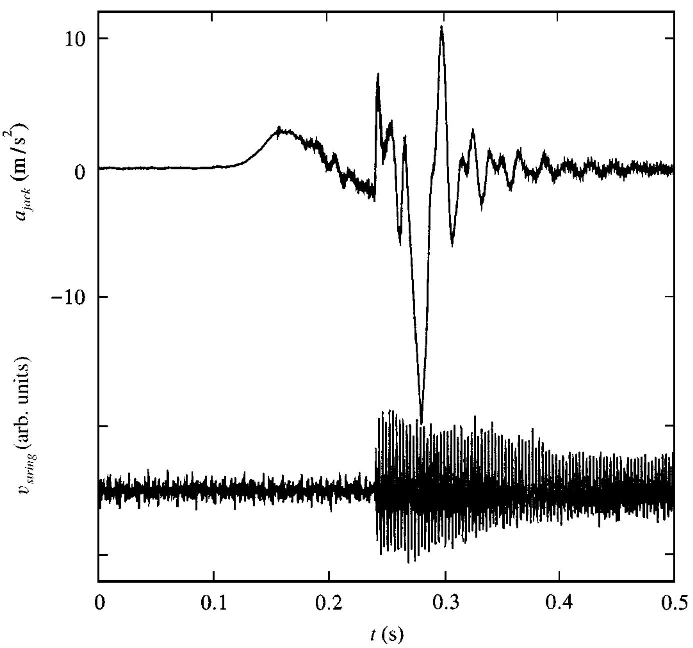

Figure 4. Results for the acceleration of the jack (top), and the string velocity (bottom), as a function of time, for a typical keypress. The origin of the time axis is arbitrary. The string was composed of steel, was 0·2 mm in diameter, and had a length of 53 cm. Here, and in Figures 5–7, the origin of the time axis is arbitrary; $t = 0$ simply corresponds to when the data recording was begun, and not to anything associated with the plectrum or string. However, the time scales for the jack acceleration and the string velocity are the *same*, so that one can use these data to determine the order of events for features in the two signals.

identify $t \approx 0{\cdot}24$ s as the time at which the plectrum left contact with the string. The key acceleration shows a more complicated behavior. The key, and hence also the jack and the plectrum, begin to accelerate significantly at about $t = 0{\cdot}1\,s$. The acceleration peaks at $t \sim 0{\cdot}16$ s, at about the same time the signal appears to become noisy. Figure 5 shows the acceleration signal on an expanded scale, and it is seen that this noise grows in strength up to $t \sim 0{\cdot}24$ s, at which time the acceleration changes very rapidly, and the string begins its free oscillation. As noted above, this was the time at which the plectrum lost contact with the string. It appears that the high-frequency "noise" in the jack acceleration signal begins when the plectrum makes its initial contact with the string. We believe that this "noise" is due to rubbing of the plectrum against the string, as it attempts to slide past. When we listen to this signal when played through a speaker, it does indeed sound like two objects rubbing together. Since the plectrum is in contact with the string during the time of these rubbing oscillations, one would expect them to be also present in the string signal. Evidently, the oscillations are too small to be evident above the noise in the string signal in Figures 4 and 5.

From the measured jack acceleration, one can integrate to obtain the velocity of the jack, and the results are shown in Figure 6. The jack velocity when the plectrum

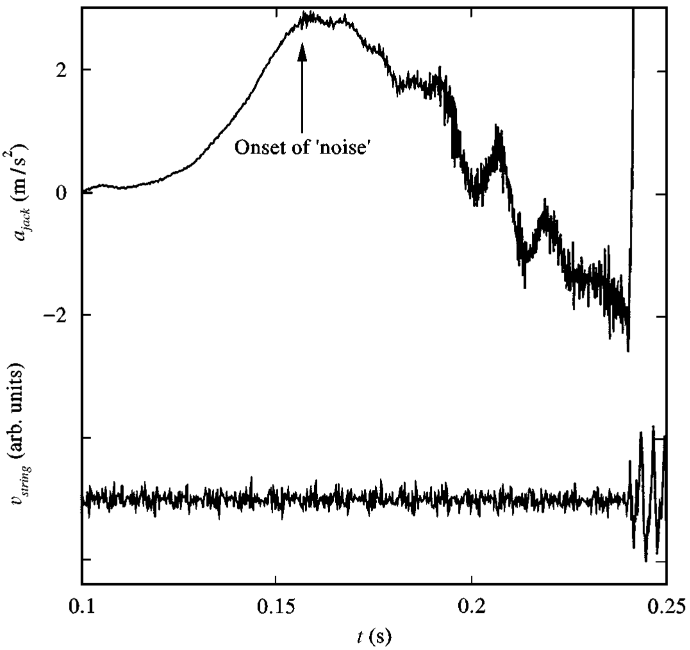

Figure 5. Expanded view of the results in Figure 4.

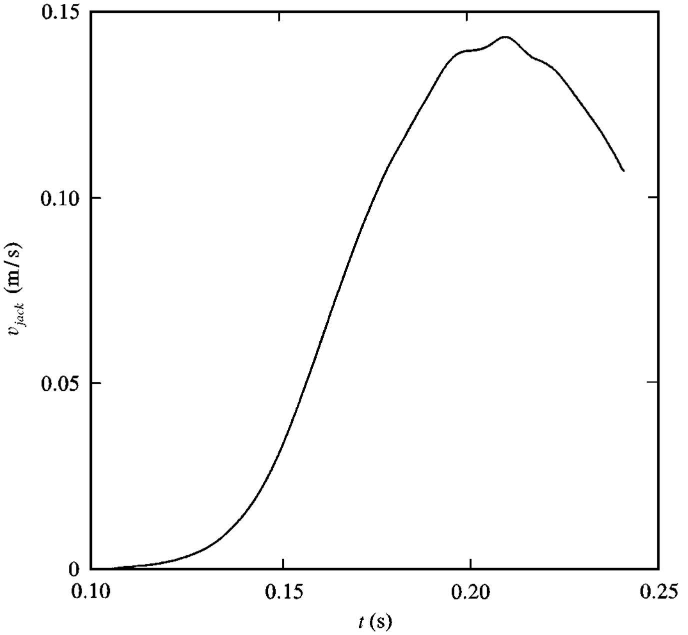

Figure 6. Velocity of the jack derived by integration of the acceleration data in Figure 4.

lost contact with the string was in this case approximately 0·11 m/s. Based on the experience of one of the authors (as an amateur harpsichord player), this keypress was typical for normal playing. It is also typical of values reported in the piano literature.

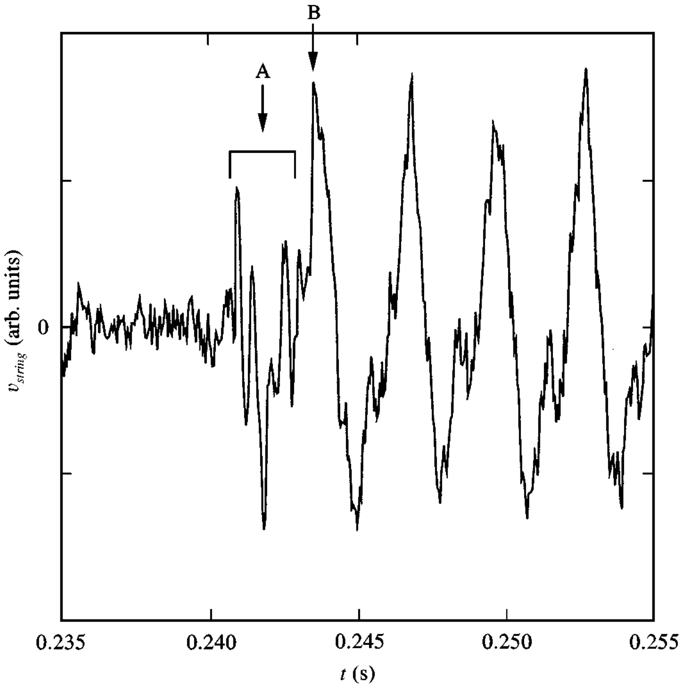

Figure 7. Expanded view of the string velocity data from Figure 4, just after the plectrum has lost contact with the string.

Figure 7 shows the string velocity just after the release of the plectrum. The repeating pattern of pulses after $t = 0{\cdot}243$ s is due to the "pluck" moving back and forth along the string. The repeat period of these pulses is approximately 3 ms, corresponding to the fundamental frequency of the string. Careful examination also reveals that an oscillation (indicated by arrow labeled A in Figure 7) at a much higher frequency is present just after the plectrum releases from the string. The frequency of this oscillation is approximately 2 kHz. We note that the frequency of this oscillation did not vary when the location of the plectrum was changed (we compared the behavior here with that found with the jack at locations 5 and 15% of the string length from the agraffe). This suggests that the high frequency oscillation is not due to a resonant mode of the string, but that it is connected with a mode of the plectrum. With this in mind, we measured the "free" vibrations of the jack/plectrum in the following way. With the jack resting on the key lever and the plectrum far from the string, the plectrum was "plucked" by hand (actually, with a Q-tip). The resulting signal from the accelerometer mounted on the jack is shown in Figure 8. The top of this figure shows the jack vibration in the time domain, while the bottom shows the power spectrum of this signal. The frequency composition is complex, but the strongest components are at approximately 1.8, 2.5, and 7 kHz. The lowest frequency component is close to that of the high-frequency oscillation of the string, as just observed in Figure 7. This supports our suspicion that this oscillation is due to vibrations of the plectrum.

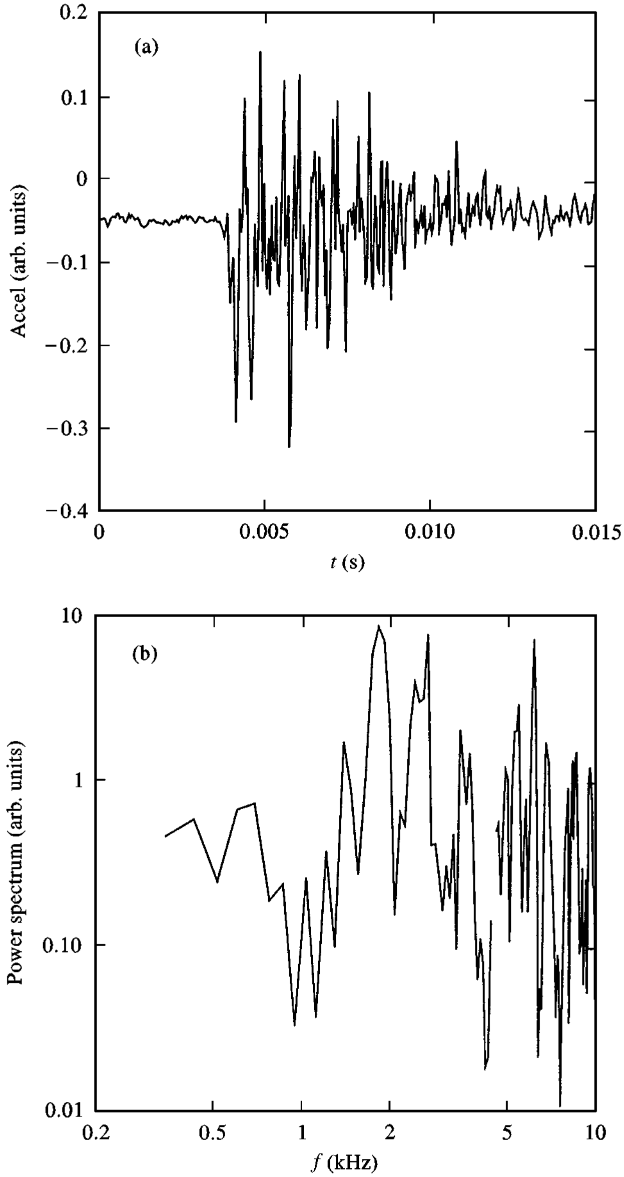

Figure 8. Acceleration of the jack body when the plectrum is set into free vibration. (a) Jack acceleration (along the vertical direction) as a function of time, (b) power spectrum of the acceleration signal.

Figure 9 shows the force exerted by the plectrum as it brushes by a metal blade attached to a force sensor (as described above). The set-up was arranged to mimic the interaction of the plectrum with a string, with a similar overlap between the plectrum and the blade. The maximum force was found to be of order 1 N, which is also a typical key force [15].

The results presented above show that the excitation of a harpsichord string by a plectrum is a complex process, and that the vibrational degrees of freedom of the

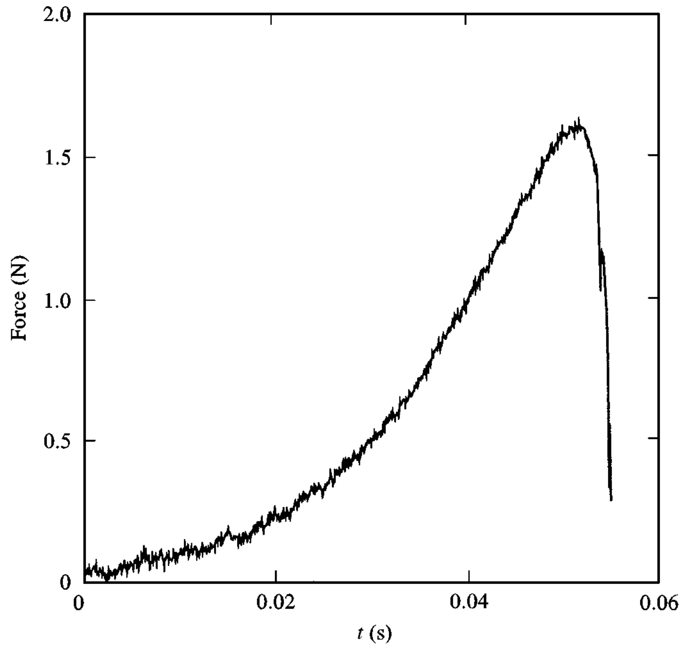

Figure 9. Plectrum force as a function of time, as the plectrum brushes past the edge of a metal blade.

plectrum are quite evident in the motion of the jack and string. Nevertheless, we would for the moment like to ignore such complications, and explore general trends as a function of the jack velocity. In particular, we would like to consider how the amplitude of the string vibration depends on the speed of the jack, v_j, and compare with the model calculations of Griffel as sketched in Figure 2. v_j can be derived by integration of the key acceleration, as shown above in Figure 6. The string amplitude can be measured in several ways. One is to simply measure the pulse height of the string velocity signal as in Figure 7. Another way would be to measure the r.m.s. of the string signal over a fixed interval after the plectrum leaves the string. We have used both of these approaches, and find that they give very similar results. In the figures below we have measured the string amplitude from the height of the first large pulse in the string signal; an example is the pulse labeled "B" in Figure 7.

Figure 10 shows results for the amplitude of the string velocity, v_s, as a function of the jack velocity. This result was obtained for a relatively thick brass string, 0·5 mm diameter, as would typically be found in the bass region. Over this range of v_j, which corresponds to the range of normal playing, the peak value of v_s varies weakly with the jack velocity. There appears to be an increase of approximately 10% in v_s, but this increase is comparable to the experimental uncertainties (caused, we believe, by variations in the precise time variation of the string velocity, from measurement to measurement), so our results are also consistent with v_s being a constant, independent of v_j. According to the model of Griffel, the peak in the string amplitude in this case should occur near $v_j \sim 0{\cdot}9$ m/s, at which point,

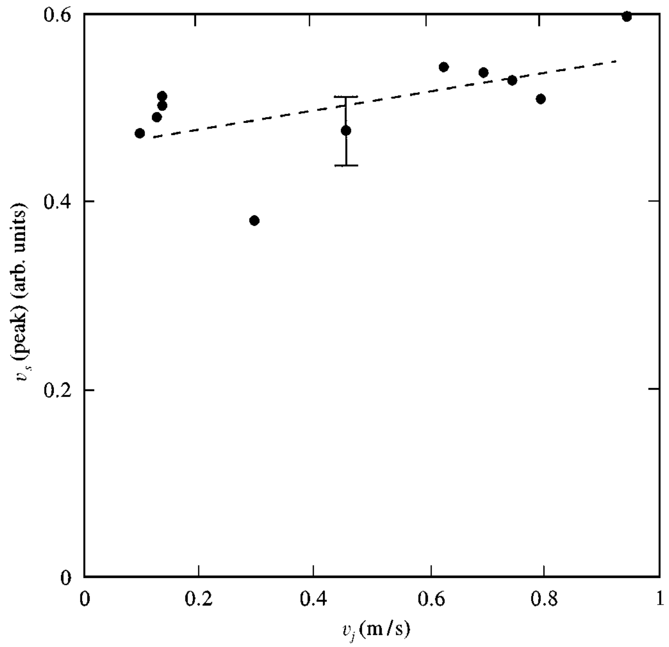

Figure 10. Amplitude of the string velocity, v_s, as a function of jack velocity, v_j, for a brass harpsichord string with diameter 0·50 mm and length 59 cm; $f = 150$ Hz. A typical error bar for v_s is shown. The dashed line is a guide to the eye.

according to his model, the string amplitude should be about 50% larger than at very small v_j. Hence, the model of Griffel greatly overestimates the dependence of v_s on jack velocity in this case.

It is interesting to compare the results in Figure 10 to what would be found if the string were excited by a blow from a hammer, i.e., for a piano. For the case of a piano, with the same range of key velocities (the key velocity would correspond to our jack velocity), it has been shown [16] that the string amplitude would vary by about a factor of 10. The very weak variation of the string amplitude found for our plucked string is thus quite different from what is observed for a piano.

Results for the string amplitude for a thinner string are shown in Figure 11. This string was composed of steel, was 0·2 mm in diameter, and is typical of a treble string. Here we find that v_s is, to within the experimental uncertainties, independent of v_j. For this case the Griffel model predicts that the main peak in the string amplitude should occur at $v_j \sim 2$ m/s, which is at a much higher key velocity than would be used in practice (and is much larger than we were able to attain here).

Figure 12 shows more results for the thick brass string, but with a much longer length, and correspondingly lower fundamental frequency, than in Figure 10. Here, there does appear to be some variation of v_s with jack velocity, with an increase of perhaps 30% over the range of v_j studied. Griffel's model predicts that for this case the main peak in v_s should occur at a jack velocity of 0·2 m/s, and that v_s should decrease by about a factor of 2 at the largest values of v_j considered here. Hence, the model of Griffel again does not give a very good account of the measurements.

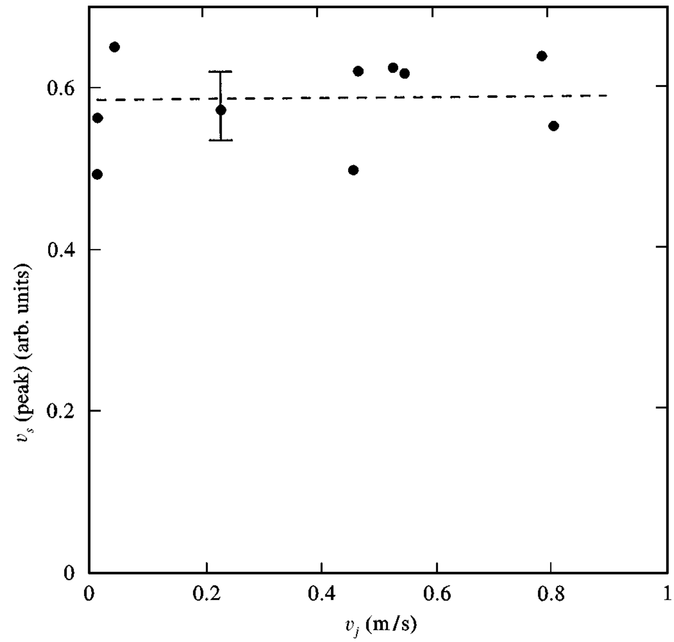

Figure 11. Amplitude of the string velocity, v_s, as a function of jack velocity, v_j, for a steel harpsichord string with diameter 0·20 mm and length 53 cm; $f = 300$ Hz. A typical error bar for v_s is shown. The dashed line is a guide to the eye.

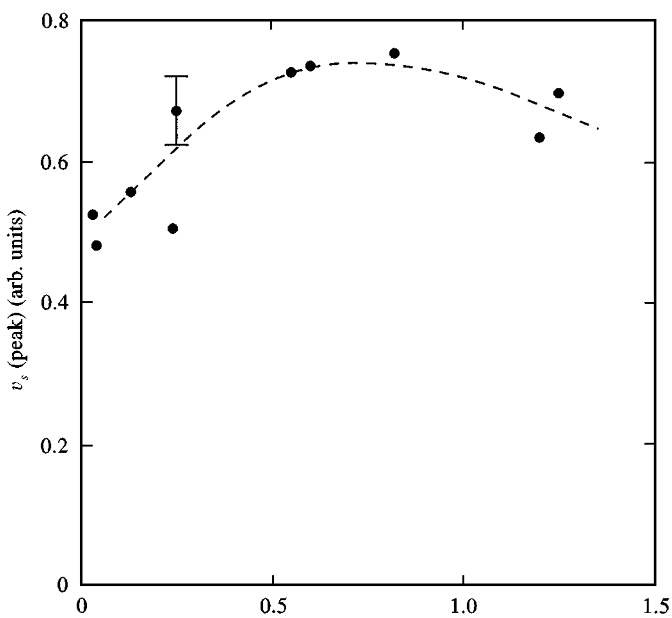

Figure 12. Amplitude of the string velocity, v_s, as a function of jack velocity, v_j, for a brass harpsichord string with diameter 0·50 mm and length 2·00 m; $f = 35$ Hz. A typical error bar for v_s is shown. The dashed line is a guide to the eye.

3. THEORETICAL MODEL AND RESULTS

While the Griffel theory [10] is very interesting, our measurements suggest that it has omitted some important physics of the problem. Let us therefore first review the ingredients of Griffel's model, and thereby consider what additional physics should be included. He treated the problem as essentially two coupled springs, one associated with the plectrum and one with the string. The plectrum was treated as a rigid, massless rod, connected to the jack by a hinge. This hinge was assumed to contain a torsion spring, with the force from this spring being proportional to the angular deflection of the plectrum with respect to the jack. As the jack moved, the plectrum hinge rotated, and the plectrum exerted a force on the string, with the force derived from the torsion spring. The string was modeled by a simple harmonic oscillator, with a single effective mass and spring constant. The frequency of this oscillator was equal to the fundamental frequency of the string. We should note that Griffel actually considered two models of the plectrum. In one case, just described, the plectrum was assumed to be a rigid rod, while in the other it was allowed to deform, via a spring-hinge, at its center. The two models gave similar results, so we will only consider the first one here.

The solution of this model yields results like those shown qualitatively in Figure 2. For small jack velocities, v_j, the resulting string amplitude, A_s, exhibits small oscillations around an approximately constant value. There is a substantial peak at a velocity v_j (max), with the peak value of A_s being 1·5 times the average value at small v_j. At larger jack velocities the string amplitude falls rapidly. The physical origin of these oscillations at small v_j is easy to understand. The string is treated as a harmonic oscillator, so if the excitation period (the time over which the plectrum accelerates the string) matches the period of the string oscillator, or one of its multiples, the energy transferred to this oscillator will be maximal. The largest peak occurs when the excitation period matches the fundamental of the string oscillator. For larger v_j, the excitation time is shorter, and the mass representing the string does not have time to move appreciably, so the string amplitude decreases.

While one would not expect Griffel's model to be quantitatively accurate, it is worthwhile to estimate the value of v_j (max) for a realistic harpsichord string. For typical parameters (essentially dependent on just the ratio of the plectrum length to the distance of the plucking point from the end of the plectrum; see reference [10]), the model yields v_j (max) $\approx 0{\cdot}2\, d\omega$, where d is the initial distance from the jack to the string, and ω is the fundamental (angular) frequency of the string. For a note an octave below middle C, with a plectrum length of $d \sim 5$ mm, one finds $v_{\max} \sim 0{\cdot}8$ m/s. As we have seen in the previous section, this is a readily accessible jack velocity. Hence, according to Griffel's calculation, it should be possible to significantly affect the volume of such a note by an easily realizable change in the key velocity. This prediction is at odds with both the "conventional" harpsichord wisdom, and with our measurements.

While Griffel's model seems reasonable at first glance, we believe that its treatment of the string degrees of freedom is too simple. On the other hand, one might argue (as does Griffel) that treating the string as an oscillator with a single resonant frequency amounts to ignoring all but the fundamental mode of vibration of the string. However, we do not like this argument for the following reason. The fundamental frequency of the string corresponds to motion when it is not in contact with the plectrum; i.e., when the vibrating length is the full length of the string. When the string is in contact with the plectrum, it is effectively divided into two pieces; each will have its own resonant frequency (determined by its length), and these will be different from the fundamental frequency after the plectrum loses contact with the string. Hence, it is probably not a good approximation to model the string with a single frequency which is the same before and after the plectrum loses contact. We believe that it is worth examining this problem with the more realistic model that we now describe.

We assume a flexible, lossless string, which can be described by the usual wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}, \tag{1}$$

where the undisturbed string runs along x, the jack moves along the transverse direction y, and the parameter c is the wave speed. The force from the plectrum will have a component along y, and also along the other transverse direction, z, so we must deal with a similar wave equation for the z displacement of the string; this equation will have the same form as equation (1).

The force from the plectrum acts only on a localized portion of the string, and is zero elsewhere, as determined by the location of the plectrum and its width. In the spirit of the finite-difference numerical approach which will be taken below (and which is explained in detail elsewhere [17]), it is convenient to think of the string as composed of short segments of mass Δm. Adding the force of the plectrum explicitly to equation (1) then leads to the equation of motion [17]

$$\Delta m \frac{\partial^2 y}{\partial t^2} = c^2 \Delta m \frac{\partial^2 y}{\partial x^2} + F_{plectrum}(x), \tag{2}$$

where Δm is the mass of a short segment of the string, and $F_{plectrum}(x)$ is the force from the plectrum, which will be zero except for the value(s) of x at which the plectrum contacts the string. A similar equation of motion is again obtained for the z degrees of freedom of the string. While the notation in equation (2) is perhaps less than elegant, it does make clear the connection to Newton's second law.

The plectrum is a flexible "beam", and in principle it could be modeled using the full equation of motion for a thin plate [6, 18]. However, we will take a simpler approach, which ignores the possibility of waves propagating along the plectrum. The force required to bend a one-dimensional beam is given by [6]

$$F_{plectrum} = F_0 \frac{\partial^3 y_p}{\partial z^3}, \tag{3}$$

where $y_p(z)$ describes the shape of the plectrum. The shape of a deformed beam is a complicated problem; here we wish to approximate this shape by a simple function consistent with the use of equation (3). A convenient choice for such a function is

$$y_p = \alpha z^3. \tag{4}$$

The shape of the plectrum will also be needed for two other aspects of the modelling. (1) It will give the direction of the force on the string from the plectrum; here we will *assume*, following Griffel, that this force is normal to the plectrum at the contact point (and hence ignore the frictional rubbing observed in the measurements). (2) The plectrum shape function will determine when the plectrum slips past the string. While the use of equation (4) may seem somewhat arbitrary, we believe that any reasonable choice (we have investigated several others) will lead to similar results; we will comment on this further below.

Equations (2)–(4) define our model. The actual calculation was carried out numerically as follows. The equation of motion (2) for the string displacement y, and the analogous equation for z, were written in finite-difference form, as described in detail elsewhere [17, 19]. The jack was assumed to move with a constant velocity, v_j, so its equation of motion was simply

$$y_{jack} = v_j t. \tag{5}$$

(Note that we also investigated the behavior with an accelerated jack, and found that only the velocity of the jack just prior to the release of the plectrum was important.) The plectrum was assumed to be massless, so the force on the string was $F_{plectrum}$ in equation (3). As noted above, the direction of $F_{plectrum}$ in the model was perpendicular to the plectrum; this direction was calculated from equation (4), so that F_x and F_y could be found (see Figure 13). Furthermore, the plectrum force was assumed to be distributed along the string according to the width of the plectrum, which in our experiments was $\sim 1{\cdot}5$ mm. In our specific calculation, the spatial step size along x was typically $\Delta x = 0{\cdot}5$ mm, so we distributed the force $F_{plectrum}$ over three of these spatial units. After each discrete time step of the simulation, the distance along the plectrum from the jack to the string was calculated. When this distance exceeded the length of the plectrum, contact with the string was lost, and the string then vibrated freely.

Calculated results for the amplitude of the string vibration, y_{string}, as a function of v_j are given in Figure 14. The model parameters used here, which are all given in the figure caption, were chosen to correspond to the experiment in Figure 10. It is seen that the string amplitude is essentially independent of the jack velocity. The explanation for this result seems straightforward; to a first approximation the location of the jack at which the plectrum releases from the string varies little with v_j, so the amplitude of the pluck is also independent of v_j. This simple picture is confirmed by examining other results of the model (such at the jack location at the release point), but it is only a first approximation. The small but noticeable variations of the string amplitude at small v_j are similar to those found in the Griffel model, and have the same origin. However, here they lead to a much smaller peak in y_{string}, which is less than 10% in this case, and occurs at $v_j \sim 0{\cdot}3$ m/s. At large

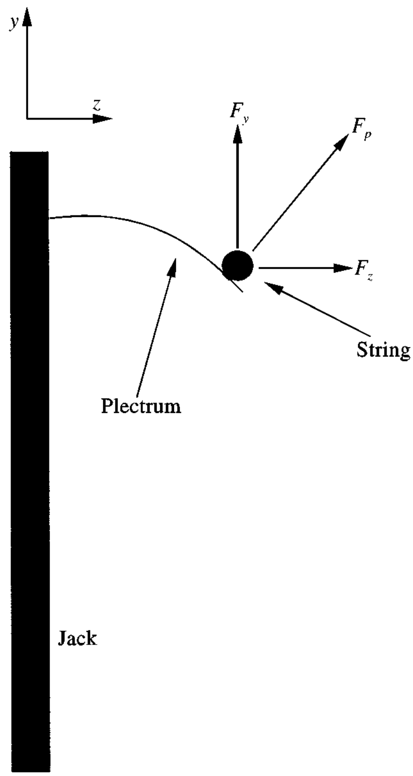

Figure 13. Schematic diagram of our model of the plucking process.

v_j there is a slight gradual decrease of the string amplitude with increasing v_j. This decrease appears to be due to "inertial effects". That is, the effective mass of the string is too large to move instantaneously with the plectrum, giving a smaller string displacement at the release point for large v_j. In other words, if one attempted to excite a heavy cable with the plectrum, the resulting amplitude would be very small. With regards to the modelling, we should add that the basic results do not depend strongly on the parameter values (such as the plectrum force constant, F_0), so long as they are within reason.

Similar results are found from the model for parameters appropriate for the experiment in Figure 11. The model prediction for this case is shown in Figure 15. The predicted variation of the string amplitude is very slight, and would not be discernible in our measurements given the experimental uncertainties.

Our model does not do quite as well as for the case considered experimentally in Figure 12. Here the measurements showed a significant $\approx$30%, increase in the string amplitude with increasing jack velocity. The model, Figure 16, shows a somewhat smaller increase of about 10%. Interestingly, in both the experiment and the model this increase occurs abruptly, and at similar values of v_j (at 0·2 m/s in the experiment, and 0·1 m/s in the model). Indeed, all of our calculations exhibit such abrupt increases in the string amplitude, although the data are sometimes not precise enough to confirm such a variation. This behavior seems to be associated

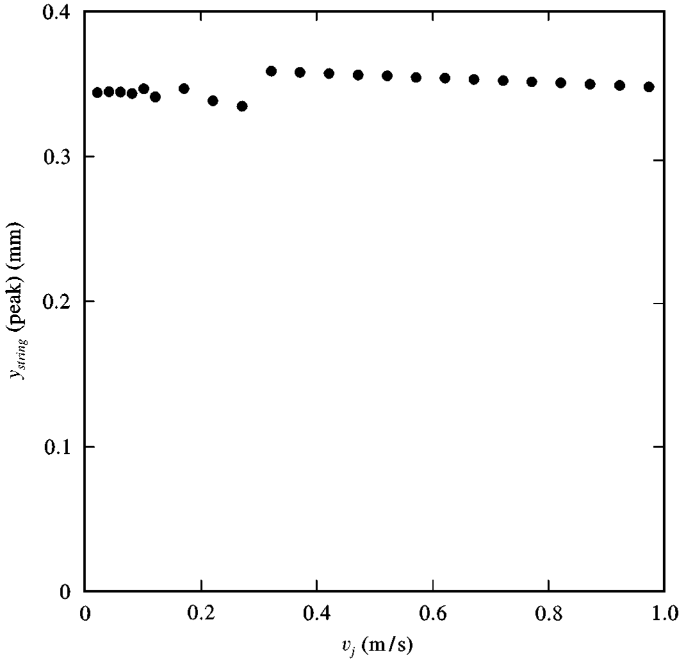

Figure 14. Results calculated for our model, with string parameters chosen to match those of the experiment in Figure 10; diameter of 0·5 mm, length of 59 cm; $f = 150$ Hz. The spatial step size for the string portion of the calculation was 0·5 mm, the time step was $2{\cdot}8 \times 10^{-6}$ s, and the string was plucked 10% from one end. The plectrum force constant was chosen to be $F_0 = 2 \times 10^{-5}$ N. This value was chosen to yield maximum plectrum forces in accord with the measurements shown in Figure 9. With this choice for F_0, the string deflection at the point of release was calculated to be $\sim$0·4 mm, which agreed with direct observation in the experiments.

with the constructive contributions of reflections from the ends of the string. When v_j is sufficiently large, these reflections can add constructively to the overall amplitude, so long as the plectrum has lost contact with the string when the reflections return. When v_j is small, the plectrum is still in contact with the string when the reflections return. The associated force on the plectrum then acts back on the plectrum, and is included in our model, and then leads to the small oscillations of the string amplitude seen at small v_j, as mentioned above.

The most serious disagreement between the measurement and the model is at high v_j, where the model shows a significant decrease in the string amplitude while the measurement shows only a small decrease. It is not currently clear to us where the problem lies. We have tried different forms for the plectrum shape function, but this has little effect on the behavior of the model. Other possible additions to the model which we have not yet explored are: (1) including the mass of the plectrum, and (2) including the effects of the frictional force between the plectrum and the string. We also suspect that the plectrum force law may differ from the simple assumptions entailed by equations (3) and (4). In any event, aside from the discrepancy at large v_j, our model does a reasonable qualitative job, as it predicts an abrupt increase in the string amplitude, at approximately the correct value of v_j.

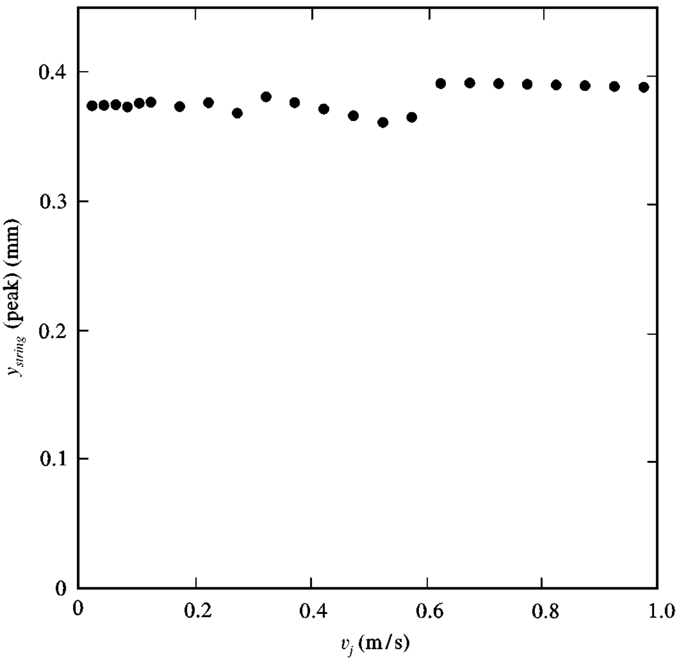

Figure 15. Results calculated for our model, with string parameters chosen to match those of the experiment in Figure 11; 0·2 mm diameter, 53 cm length; $f = 300$ Hz. The other model parameters (the step sizes, F_0, etc.) were the same as in Figure 14.

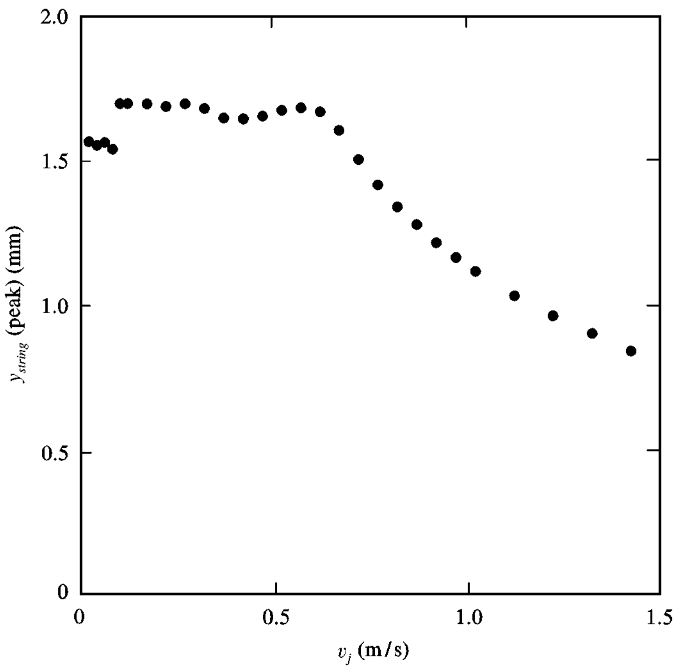

Figure 16. Results calculated for our model, with string parameters chosen to match those of the experiment in Figure 12; 0·5 mm diameter, 2·00 m length; $f = 35$ Hz. The other model parameters (the step sizes, F_0, etc.) were the same as in Figure 14.

4. SUMMARY

We have presented the results of an experimental study of the plucking of a harpsichord string. For normal playing conditions, we find that the amplitude of the string motion is approximately independent of the speed with which the key is pressed. This result is in agreement with the conventional wisdom, and with a simple model of the plucking process which we have investigated. The experiments also reveal effects due to the vibrations of the plectrum and jack, and to frictional rubbing of the plectrum against the string. The influence of these effects on tone production has not yet been determined. However, it seems likely that they are responsible for the difference in tone produced by different types of plectra. This is a problem that we will leave for the future.

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