Journal of Sound and Vibration (1999) **224**(4), 775–779
Article No. jsvi.1999.2220, available online at http://www.idealibrary.com on IDEAL®

LETTERS TO THE EDITOR

FUNDAMENTAL FREQUENCY OF AN ANNULAR CIRCULAR PLATE OF NON-UNIFORM THICKNESS AND AN INTERMEDIATE CONCENTRIC CIRCULAR SUPPORT

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(*Received* 30 *September* 1998)

1. INTRODUCTION

The present study deals with the problem defined in the title for the case of structural system shown in Figure 1 which, apparently, has not been treated previously in the literature [1].

The problem is solved by means of the optimized Rayleigh–Ritz method employing polynomial co-ordinate functions which identically satisfy the essential boundary condition and only two of the natural conditions at the outer and inner edge. It has been previously shown that this approach yields excellent accuracy in the case of a plate of uniform thickness [2].

2. APPROXIMATE ANALYTICAL SOLUTION

When the system executes transverse and normal modes of vibration its behaviour is described by the well-known functional

$$J[W]=\frac{1}{2}\iint_{A_p} D\left\{\left(\frac{\mathrm{d}^2W}{\mathrm{d}r^2}+\frac{1}{r}\frac{\mathrm{d}W}{\mathrm{d}r}\right)^2-2(1-\nu)\left[\frac{\mathrm{d}^2W}{\mathrm{d}r^2}\left(\frac{1}{r}\frac{\mathrm{d}W}{\mathrm{d}r}\right)\right]\right\} r\,\mathrm{d}r\,\mathrm{d}\theta$$
$$-\frac{1}{2}\rho\omega^2\iint_{A_p} hW^2r\,\mathrm{d}r\,\mathrm{d}\theta, \tag{1}$$

where A_p is the planform area of the plate and where the displacement amplitude W must satisfy appropriate boundary conditions.

Following reference [2] the following approximation will be used:

$$W\cong W_a=\sum_{j=1}^{N} G_j\varphi_j(r), \tag{2}$$

where

$$\varphi_j=\alpha_j r^{p+j-1}+\beta_j r^{j+2}+\gamma_j r^{J+1}+1, \tag{3}$$

and where p is the Rayleigh's optimization exponential parameter [3].

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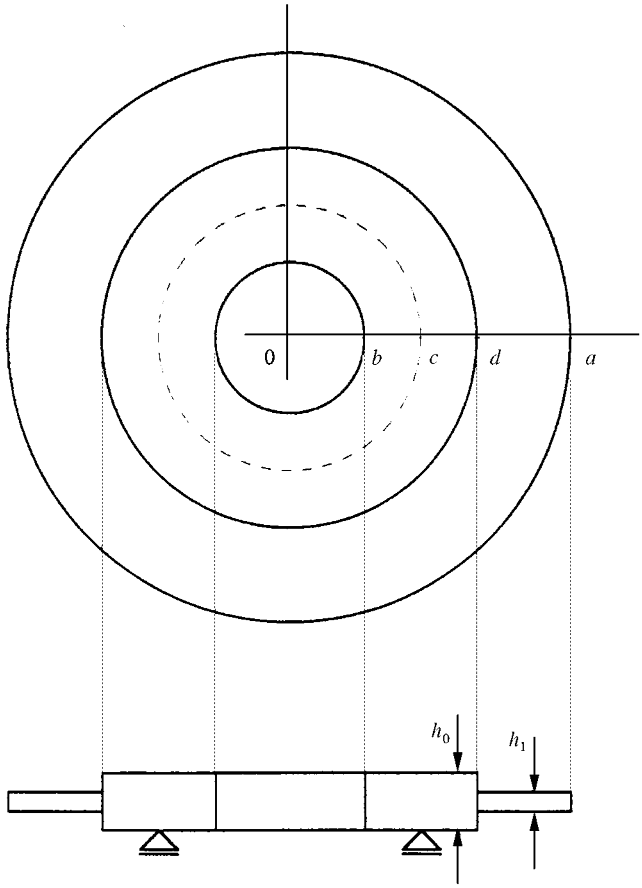

Figure 1. Annular plate of non-uniform thickness executing transverse vibrations considered in the present study.

The α_j's, β_j's and γ_j's are obtained substituting each co-ordinate function in the conditions

$$W(c) = 0, \qquad \left.\left(\frac{d^2W}{dr^2} + \frac{\nu}{r}\frac{dW}{dr}\right)\right|_{r=a,b} = 0. \tag{4a, b}$$

Equation (4a) describes the condition of zero displacement at the inner, concentric support while equation (4b) represents the fact that the bending moment is zero at the outer and inner contour. The condition of nulle Kirchhoff force at $r = a, b$ is not employed since this is a legitimate procedure on one hand since the Rayleigh–Ritz energy approach is used, the procedure being then simpler. On the other hand the analytical approach is now valid even if the support is placed at $r = a$ or b.

Substituting equation (2) in equation (1) and following the usual Rayleigh–Ritz procedure leads to a determinantal equation whose lowest root is the fundamental frequency coefficient

$$\Omega_1 = \sqrt{\rho h_0/D_0}\,\omega_1 a^2 \quad \text{where } D_0 = Eh_0^3/(12(1-\nu^2)),$$

see Figure 1.

Since

$$\Omega_1 = \Omega_1(p), \tag{5}$$

by minimizing equation (5) with respect to p one is able to optimize Ω_1.

3. NUMERICAL RESULTS

All calculations have been performed for $\nu = 0{\cdot}30$. The convergence of the results was studied as N was varied from 1–10.

In the case of a plate of uniform thickness the fundamental eigenvalues agreed with the exact ones for $N = 9$. Table 1 illustrates the convergence of the technique, in the case where $b/a = 0{\cdot}10$ and $h_1/h_0 = 0{\cdot}80$, for $N = 3$, 9 and 10. From the analysis of the table one concludes that for $N = 3$ the eigenvalues are rather high upper bounds. On the other hand, the differences of values of Ω_1 obtained for $N = 9$ and 10 are extremely small indicating the possibility of very good accuracy for these large values of N.

Tables 2 and 3 depict values of Ω_1 for several combinations of values of b/a, c/a and d/a for $h_1/h_0 = 0{\cdot}8$ and 0·6, respectively. Ten polynomial co-ordinate functions have been used for each determination of the fundamental frequency coefficient.

TABLE 1

Analysis of the convergence of the proposed approach ($N = 3$, 9 *and* 10) *when determining* Ω_1 *for* $b/a = 0{\cdot}10$ *and* $h_1/h_0 = 0{\cdot}80$

N	Values of Ω_1						
	c/a	$d/a = 0{\cdot}3$	0·4	0·5	0·6	0·7	0·8
3	0·2	4.66	5.03	5.41	5.66	5.73	5.71
	0·3		5·20	5·61	5·89	5·97	5·95
	0·4			6·18	6·56	6·70	6·70
	0·5				7·69	7·97	8·03
	0·6					9·68	9·82
	0·7						10·55
9	0·2	3·67	3·6	4·03	4·15	4·19	4·18
	0·3		4·39	4·62	4·79	4·87	4·87
	0·4			5·40	5·66	5·79	5·81
	0·5				6·79	7·03	7·10
	0·6					8·40	8·53
	0·7						8·81
10	0·2	3·67	3·86	4·03	4·15	4·19	4·18
	0·3		4·39	4·62	4·79	4·87	4·86
	0·4			5·40	5·66	5·79	5·81
	0·5				6·79	7·03	7·10
	0·6					8·40	8·52
	0·7						8·80

TABLE 2

Values of Ω_1 determined with $N = 10$ for several values of b/a, c/a and d/a ($h_1/h_0 = 0{\cdot}80$)

b/a	c/a	$d/a = 0{\cdot}4$	0·5	0·6	0·7	0·8
	0·3	3·84	4·03	4·19	4·25	4·26
	0·4		4·86	5·11	5·21	5·24
0·2	0·5			6·22	6·48	6·55
	0·6				7·88	8·03
	0·7					8·48
	0·4		4·14	4·34	4·49	4·52
	0·5			5·52	5·77	5·84
0·3	0·6				7·33	7·48
	0·7					8·40
	0·5			4·68	4·91	5·04
0·4	0·6				6·62	6·82
	0·7					8·46
0·5	0·6				5·63	5·92
	0·7					8·32
0·6	0·7					7·40

TABLE 3

Values of Ω_1 determined with $N = 10$ for several values of b/a, c/a and d/a ($h_1/h_0 = 0{\cdot}6$)

b/a	c/a	$d/a = 0{\cdot}3$	0·4	0·5	0·6	0·7	0·08
	0·2	3·29	3·71	4·14	4·46	4·55	4·48
	0·3		4·08	4·65	5·12	5·30	5·24
0·1	0·4			5·27	5·97	6·30	6·29
	0·5				6·95	7·58	7·71
	0·6					8·83	9·09
	0·7						8·98
	0·3		3·65	4·08	4·48	4·61	4·57
	0·4			4·79	5·41	5·66	5·66
0·2	0·5				6·34	6·99	7·09
	0·6					8·29	8·57
	0·7						8·66
	0·4			4·07	4·51	4·83	4·86
	0·5				5·63	6·20	6·31
0·3	0·6					7·73	8·03
	0·7						8·62
	0·5				4·72	5·21	5·41
0·4	0·6					6·94	7·34
	0·7						8·79
	0·6					5·78	6·34
0·5	0·7						8·78
0·6	0·7						7·75

A rather interesting consideration is the fact that when determining the exact eigenvalues in the case of uniform thickness the calculation of a single frequency coefficient took close to 12 min in a modern PC. On the other hand, when using the polynomial approach for $b/a = 0{\cdot}1$ and $h_1/h_0 = 0{\cdot}6$, the complete run took 3 min 50 s, for all the eigenvalues.

ACKNOWLEDGMENTS

The present study has been sponsored by CONICET Research and Development Program and by Secretaria General de Ciencia y Tecnologia of Universidad Nacional del Sur.

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