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FUNDAMENTAL FREQUENCY OF VIBRATION OF RECTANGULAR MEMBRANES WITH AN INTERNAL OBLIQUE SUPPORT

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1. INTRODUCTION

A literature search performed by the authors reveals that no solution is available on the title problem [see Figure 1].

An approximate solution is obtained in the present note whereby the co-ordinate functions are generated using a truncated double Fourier series multiplied by a functional relation which takes into account the presence of the oblique support. The frequency determinant is generated by straightforward application of the classical Rayleigh–Ritz method.

A similar procedure has recently been applied in the case of vibrating rectangular plates with inner, oblique supports [1].

2. APPROXIMATE ANALYTICAL SOLUTION

The exact expression for the dynamic amplitude of the vibrating rectangular membrane without the intermediate, oblique support is

$$W(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} b_{nm} \sin(n\pi x/a) \sin(m\pi y/b). \tag{1}$$

Consequently, it seems reasonable to approximate the fundamental mode shape of the system shown in Figure 1 by means of an expression of the form

$$W(x, y) \approx W_a(x, y) = \sum_{n=1}^{N} \sum_{m=1}^{M} b_{nm}\, g(x, y) \sin(n\pi x/a) \sin(m\pi y/b), \tag{2}$$

where $g(x, y)$ is null for every (x, y) belonging to the straight line equation

$$y - \alpha x - \beta = 0. \tag{3}$$

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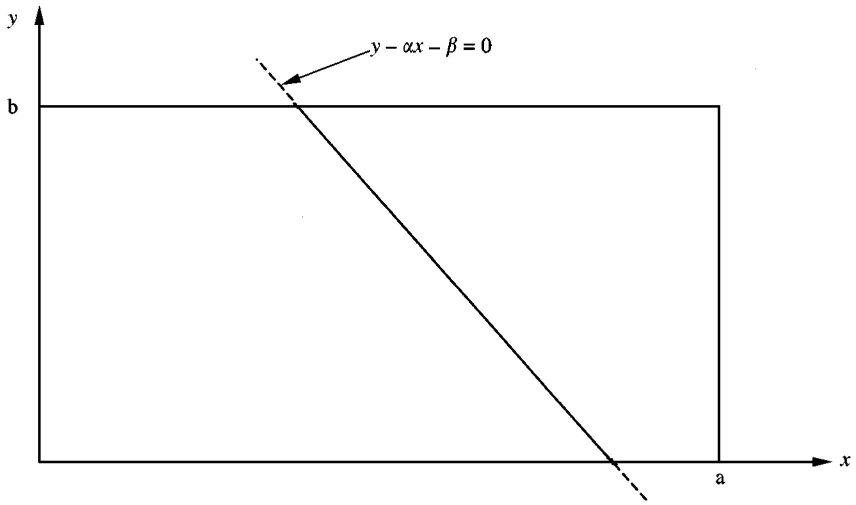

Figure 1. Vibrating rectangular membrane with an intermediate oblique support.

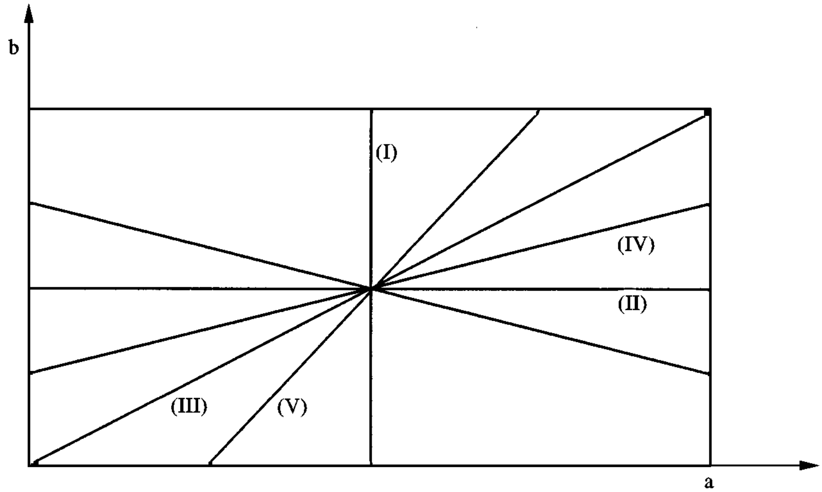

Figure 2. Cases of oblique supports considered in the present study.

After some simple numerical experiments it was found convenient to express $g(x, y)$ in the form

$$g(x, y) = (y - \alpha x - \beta)^{n+m-1}. \tag{4}$$

Substituting equation (2) in the governing functional

$$J[W] = \frac{1}{2} S \int_{A_0}\!\!\int \left[\left(\frac{\partial W}{\partial x} \right)^2 + \left(\frac{\partial W}{\partial y} \right)^2 \right] \mathrm{d}x \, \mathrm{d}y - \frac{\rho \omega^2}{2} \int_{A_0}\!\!\int W^2 \, \mathrm{d}x \, \mathrm{d}y \tag{5}$$

TABLE 1

Values of Ω_1 for a rectangular membrane with intermediate supports; see Figure 2

$a/b = 1$

	(I) $x = a/2$*	(II) $y = b/2$†	(III) $y = (b/a)x$	(IV) $y = (b/2a)x + (b/4)$	(V) $y = (2b/a)x - (b/2)$
$N = M = 1$	7·09547	7·09547	7·09547	7·09547	7·09547
$N = M = 2$	7·02623	7·02623	7·08394	7·05332	7·05332

* Exact: 7·02480.
† Exact: 7·02480.

$a/b = 2$

	(I) $x = a/2$*	(II) $y = b/2$†	(III) $y = (b/a)x$	(IV) $y = (b/2a)x + (b/4)$	(V) $y = (2b/a)x - (b/2)$
$N = M = 1$	4·47086	6·55313	5·60946	6·19294	4·95778
$N = M = 2$	4·44344	6·47809	5·41618	5·94971	4·87298

* Exact: 4·44287.
† Exact: 6·47655.

$a/b = 3$

	(I) $x = a/2$*	(II) $y = b/2$†	(III) $y = (b/a)x$	(IV) $y = (b/2a)x + (b/4)$	(V) $y = (2b/a)x - (b/2)$
$N = M = 1$	3·79037	6·447695	5·28865	6·01095	4·45063
$N = M = 2$	3·77601	6·37141	4·930176	5·65107	4·26040

* Exact: 3·77572.
† Exact: 6·3698.

$a/b = 4$

	(I) $x = a/2$*	(II) $y = b/2$†	(III) $y = (b/a)x$	(IV) $y = (b/2a)x + (b/4)$	(V) $y = (2b/a)x - (b/2)$
$N = M = 1$	3·52127	6·41038	5·17167	5·94594	4·25888
$N = M = 2$	3·51258	6·33365	4·72190	5·53155	3·98641

* Exact: 3·51240.
† Exact: 6·33207.

and applying the minimization condition

$$\frac{\partial J}{\partial b_{nm}} = 0, \tag{6}$$

one obtains a linear system of equations in b_{nm}'s. The non-triviality condition yields a determinantal equation whose lowest root constitutes the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho/S}\omega_1 a$.

3. NUMERICAL RESULTS

Calculations were performed for rectangular membranes ($a/b = 1, 2, 3$ and 4) for the following cases of intermediate supports passing through the center of the membrane (see Figure 2):

(I) $x - a/2 = 0$,

(II) $y - b/2 = 0$,

(III) $y - (b/a)x = 0$,

(IV) $y - (b/2a)x = 0$,

(V) $y - (2b/a)x - b/2 = 0$.

Cases I and II do possess an exact solution since they are modes of higher order of the simple rectangular membrane without intermediate support.

Table 1 depicts values of Ω_1 determined using one and four-term approximations. In the case of the square membrane ($a/b = 1$) the values of Ω_1 are approximately equal in view of the fact that the resulting modes are degenerate cases of the membrane without internal support [2]. The value corresponding to case III (diagonal support) should be exactly the same as for cases I and II while present calculations yield a value of Ω_1 which is less than 1% higher since the Rayleigh–Ritz method yields upper bounds.

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