Journal of Sound and Vibration (1999) **224**(5), 897–915
Article No. jsvi.1999.2226, available online at http://www.idealibrary.com on IDEAL®

THE RESONANT VIBRATION FOR A SIMPLY SUPPORTED GIRDER BRIDGE UNDER HIGH-SPEED TRAINS

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(*Received* 11 *September* 1998, *and in final form* 23 *February* 1999)

The dynamic response of a girder bridge under high-speed trains is studied, with an emphasis on the resonant vibration. Two dynamic models for the vehicle are presented to predict the dynamic response of the girder bridge. In the first model, the vehicles are simplified as a series of moving loads. In the second model, the vehicle–bridge interaction system is considered. The results show that the resonance for small and medium-span girder bridges may occur under a high-speed train consisting of a number of vehicles with similar types, and the resonance is caused by the free vibration. The main factors that influence the resonant vibration are investigated; the relations of resonance speeds to the parameters of the girder bridge as well as the vehicle are given.

1. INTRODUCTION

The investigation of dynamic behavior of a girder bridge under moving vehicle loads dates back to many years ago. In early studies, moving vehicles travelling along a bridge were modelled as moving loads or rolling masses [1, 2]. During the past three decades, considerable experimental and theoretical research on the dynamic response of bridges has been carried out. More sophisticated models that consider the various dynamic characteristics of vehicles have been implemented in the study of vehicle–bridge interactions [3, 4]. Meanwhile, the effects of track irregularities and wheel flatness on the dynamic response of a bridge have been investigated [5, 6]. Recently, the vehicle–bridge interaction element [7] has been developed for modelling the vehicle–bridge interaction in an analysis of railway bridges under high-speed trains, which may consist of a number of cars connected together.

It is generally known that if the forced-vibration term alone is considered, the resonance cannot occur for a short-span girder bridge under moving loads since the loading frequency is generally quite low compared with the natural frequency at the present speed of the train [8]. Even if the loading frequency and natural frequency coincide, the resonance is not a true one since there is only one-half cycle of loading as the force crosses the span. However, the results of field tests of railway bridges in Japan and China [9–11] have shown a heavy dynamic response exceeding the anticipation on certain short-span concrete and steel girder bridges. In this case, the

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resonance caused by a train consisting of a number of vehicles with similar types is the most important factor, especially at the high-speed range. Such phenomena of resonance have been studied by Yang *et al.* [12]. In their study, the resonant and cancellation effects of waves generated by the motion of wheel loads on the simple beam have been related to the ratio of the railroad car to bridge lengths, and the optimal design criteria that are effective for suppressing the resonant responses are proposed.

The main purpose of this paper is to further investigate the fundamental characteristics and the influential factors for the resonant vibration of a girder bridge under high-speed trains. The study includes two dynamic models for vehicles moving at a constant speed over a bridge. In the first model, a number of vehicles with similar types are considered as a series of moving loads moving over a girder bridge; in the second model, the dynamic interaction between the vehicle and bridge is adopted. Based on the analysis of the free- and forced-vibration responses of the girder bridge under a series of moving loads, the results show that the resonance is caused by the free vibration.

2. DYNAMIC RESPONSE OF A GIRDER BRIDGE UNDER MOVING LOADS

2.1. VIBRATION MODE EQUATION OF A GIRDER BRIDGE

Since this paper deals mainly with small- and medium-span simply supported girder bridges, it suggests that modes higher than the fundamental one may be neglected without serious loss of accuracy when calculating the deflections and bending moments [8]. The analysis of the vertical vibration can include the fundamental mode alone. It would be reasonable to expect the vertical displacement to vary with x and t as distinctly separate functions. We shall therefore assume that

$$y(x,t) = q(t)\phi(x), \tag{1}$$

where $y(x,t)$ is the vertical displacement, $\phi(x) = \sin(\pi x/L_b)$ is the fundamental mode shape, L_b is the span length, and $q(t)$ are the generalized co-ordinates that define the amplitude of vibration with time.

Let the first and last moving loads on the span be P_K and P_M at time t (Figure 1). If the fundamental mode of the girder bridge alone is taken into account, the equation of vertical motion in generalized co-ordinates under a series of moving loads at a constant speed can be expressed as

$$\ddot{q}(t) + 2\xi\omega\dot{q}(t) + \omega^2 q(t) = \frac{2}{mL_b}\sum_{i=K}^{M}\sin\frac{\pi(vt - a_i)}{L_b}P_i, \tag{2}$$

where m, ω and ξ are the mass per unit length, the first circular frequency and the damping ratio, v is the velocity of moving loads, P_i is the ith moving load and a_i is the distance between moving loads P_i and P_1 (Figure 1).

Let r_i be the distance between two adjacent loads P_i and P_{i-1} (Figure 1). When r_i is assumed to be constant, $r_i = r$, and the generalized load in equation (2) is

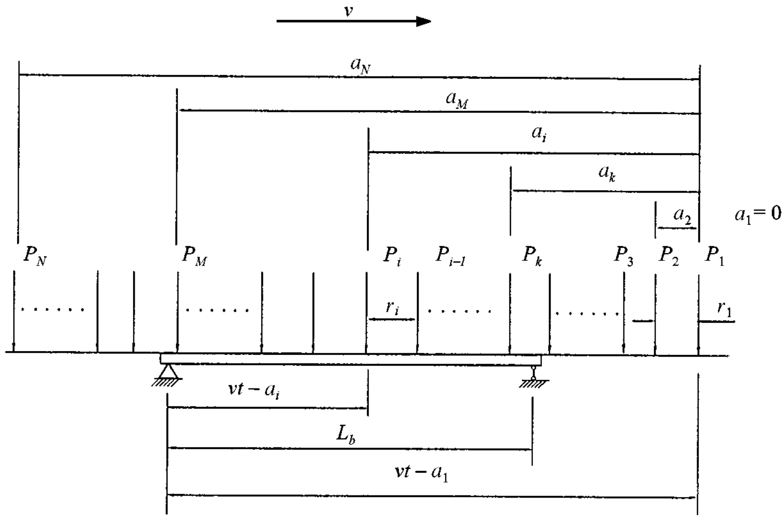

Figure 1. A girder bridge subjected to a series of moving loads.

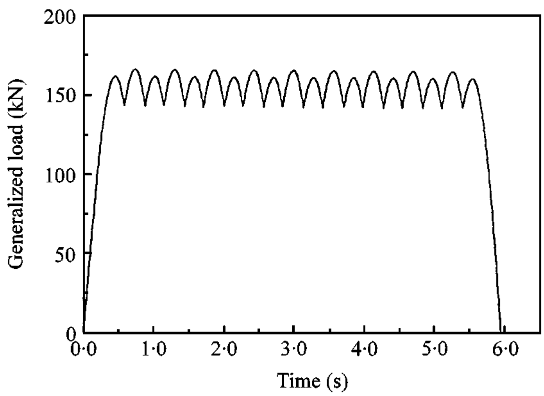

Figure 2. Generalized load against time.

a periodic function with two frequency components (Figure 2). One is the frequency, f_{v1} $(= v/2L_b)$, of each moving load travelling across the bridge. Another is the frequency, f_{v2} $(= v/r)$, of a series of moving loads acting periodically on the bridge. Figure 2 represents the generalized load against time when 10 moving loads move over a 32 m girder bridge at a constant speed of $v = 37$ m/s, in which case the distance r between two adjacent loads is 21 m and the magnitude of each moving load is 160 kN.

2.2. SOLUTION OF THE MODE EQUATION

Before considering the general case of a girder bridge under a series of moving loads, it is instructive to examine the response to a single moving load. If there is

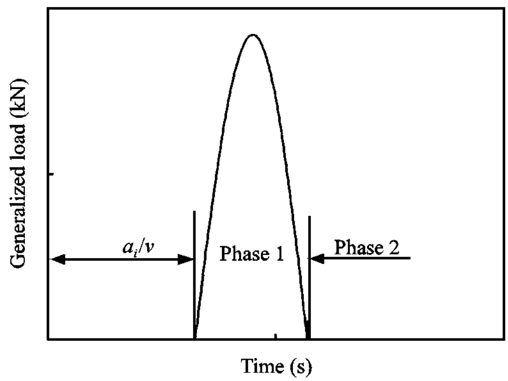

Figure 3. Half-sine-wave impulse.

only the ith moving load P_i moving over the bridge at a constant speed, the generalized load on the right hand side of equation (2) consists of a half-sine-wave impulse with the frequency $f_{v1} = v/2L_b$, as shown in Figure 3. The response will be divided into two phases, corresponding to the interval during which the moving load acts [13].

Phase 1. When the ith moving load is on the bridge, ignoring the difference between the damped and undamped frequencies, the response, including the free-vibration term (the transient term) as well as the forced-vibration term (the steady state term), can be written in the form

for $\dfrac{a_i}{v} \leqslant t \leqslant \dfrac{a_i}{v} + \dfrac{L_b}{v}$,

$$q^i(t) = y_{st} \frac{(1-\beta^2)\sin\dfrac{\pi(vt-a_i)}{L_b} - 2\xi\beta\cos\dfrac{\pi(vt-a_i)}{L_b}}{(1-\beta^2)^2 + (2\xi\beta)^2}$$

$$+ y_{st}\frac{e^{-\xi\omega(t-a_i/v)}}{(1-\beta^2)^2+(2\xi\beta)^2}\left\{[2\xi^2\beta - (1-\beta^2)\beta]\sin\omega\left(t-\frac{a_i}{v}\right)\right.$$

$$\left. + 2\beta\xi\cos\omega\left(t-\frac{a_i}{v}\right)\right\}, \tag{3}$$

where $y_{st} = 2P_iL_b^3/\pi^4EI \approx P_iL_b^3/48EI$ is the static deflection at midspan when the moving load P_i acts on midspan; $\beta = \pi v/L_b\omega$ is the speed parameter which is the ratio of the circular frequency of a single load travelling across the bridge to the natural frequency ω.

It is clear that the damping ratio of the actual bridge is smaller ($\xi < 0{\cdot}05$) and the loading circular frequency $\pi v/L_b$ is generally quite low compared with the natural frequency ω. Thus, neglecting the term $\xi\beta$ in equation (3), the response is written as

$$q^i(t) = \frac{y_{st}}{1-\beta^2}\sin\frac{\pi(vt-a_i)}{L_b} - y_{st}\frac{\beta}{1-\beta^2}\,\mathrm{e}^{-\xi\omega(t-a_i/v)}\sin\omega\left(t-\frac{a_i}{v}\right). \tag{4}$$

Phase 2. After the load P_i leaves the span, the free-vibration motion that occurs during this phase will depend on the generalized displacement $q^i(t')$ and the velocity $\dot{q}^i(t')$ existing at the end of phase 1, and it can be expressed as follows:

for $\quad t \geqslant t'$,

$$q^i(t) = \mathrm{e}^{-\xi\omega(t-t')}\left[\frac{\dot{q}^i(t') + \xi\omega q^i(t')}{\omega}\sin\omega(t-t') + q(t')\cos\omega(t-t')\right], \tag{5}$$

where $t' = a_i/v + L_b/v$ is the time when load P_i leaves the span. Substituting t' and $q^i(t')$, $\dot{q}^i(t')$ into equation (5), the response of free vibration becomes:

for $\quad t \geqslant \dfrac{a_i}{v} + \dfrac{L_b}{v}$,

$$q^i(t) = -\,\mathrm{e}^{-\xi\omega(t-a_i/v-L_b/v)}\frac{y_{st}\beta}{1-\beta^2}\sin\omega\left(t-\frac{a_i}{v}-\frac{L_b}{v}\right)$$

$$-\,\mathrm{e}^{-\xi\omega(t-a_i/v)}\frac{y_{st}\beta}{1-\beta^2}\sin\omega\left(t-\frac{a_i}{v}\right). \tag{6}$$

The dynamic response of the bridge under a series of moving loads can be obtained by the principle of superposition. Thus, the total responses to a series of moving loads are the sum of the response to each moving load.

If the first and last moving loads on the span are P_1 and P_M at time t (Figure 4), the response may be expressed as follows:

for $\quad 0 \leqslant t \leqslant \dfrac{L_b}{v}$,

$$q(t) = \frac{y_{st}}{1-\beta^2}\sum_{i=1}^{M}\sin\frac{\pi(vt-a_i)}{L_b} - y_{st}\frac{\beta}{1-\beta^2}\sum_{i=1}^{M}\mathrm{e}^{-\xi\omega(t-a_i/v)}\sin\omega\left(t-\frac{a_i}{v}\right). \tag{7}$$

If the number of moving loads out of the span is $K-1$ and the number of moving loads on the span is $M-K+1$, at time t (Figure 1), the response becomes:

for $\quad \dfrac{L_b}{v} \leqslant t \leqslant \dfrac{a_N}{v} + \dfrac{L_b}{v}$,

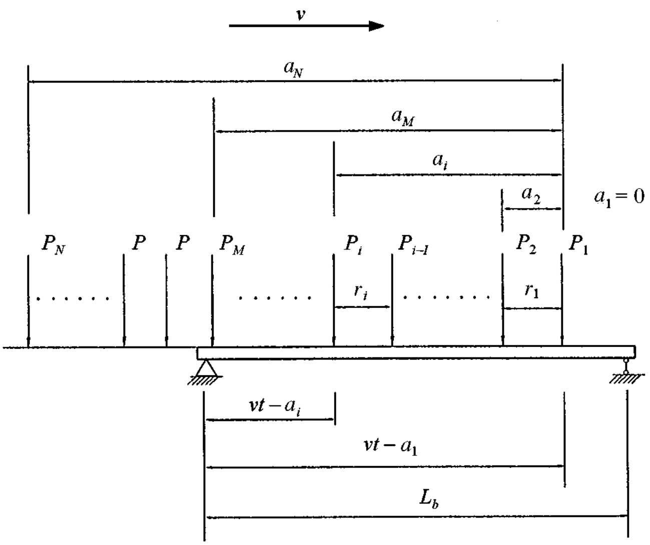

Figure 4. A girder bridge under moving loads from P_1 to P_M.

$$q(t) = \frac{y_{st}}{1-\beta^2} \sum_{i=K}^{M} \sin \frac{\pi(vt - a_i)}{L_b} - y_{st} \frac{\beta}{1-\beta^2} \sum_{i=k}^{M} e^{-\xi\omega(t - a_i/v)} \sin \omega\left(t - \frac{a_i}{v}\right)$$

$$- y_{st} \frac{\beta}{1-\beta^2} \sum_{i=1}^{K-1} e^{-\xi\omega(t - a_i/v)} \sin\omega\left(t - \frac{a_i}{v}\right)$$

$$- y_{st} \frac{\beta}{1-\beta^2} \sum_{i=1}^{K-1} e^{-\xi\omega(t - a_i/v - L_b/v)} \sin \omega\left(t - \frac{a_i}{v} - \frac{L_b}{v}\right). \tag{8}$$

It is noted in equation (8) that the response consists of four terms. The first term on the right-hand side of equation (8) represents the response component at the frequency of each moving load travelling across the bridge; it is the forced-vibration response. The second and third terms are the response component at natural frequency and are the free-vibration effects caused by the sudden application of the force. The fourth term is also the free vibration after moving loads leave the span.

2.3. FORCED-VIBRATION RESPONSE

If the fundamental mode of the girder bridge alone is taken into account, the deflection at midspan is given by

$$y(t) = \phi\left(\frac{L_b}{2}\right) q(t) = q(t) \sin \frac{\pi}{2} = q(t).$$

Clearly, dynamic response components due to forced vibration in equations (7) and (8) increase with speed. Therefore, it is natural to wonder what happens if the frequency of each moving load travelling across the bridge and the natural

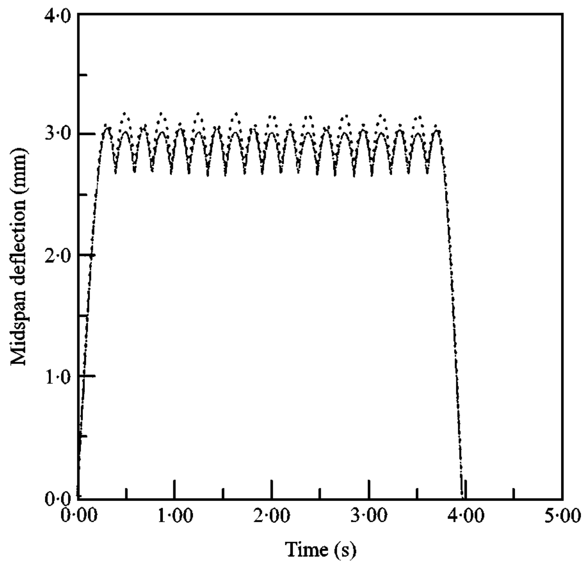

Figure 5. Midspan deflection under ten moving loads. —, The crawl deflection, ---- due to enforced vibration.

frequency coincide. In this case, the resonance due to forced vibration does not really occur since there is only one-half cycle of loading as each moving load crosses the span. However, for the small- and medium-span girder bridges, the frequency of each moving load travelling across the bridge in actual speed ranges is generally quite low compared with the natural frequency. If the frequency of each moving load travelling across the bridge agrees with the natural frequency, the speed will be too high for actual vehicles to reach. For example, Figure 5 represents the crawl deflection and the deflection due to forced-vibration components at midspan of a 32 m deck girder bridge subjected to 10 moving loads at a constant speed $v = 55{\cdot}56$ m/s. The natural frequency of the girder is 5·3 Hz and the distance between two adjacent moving loads is 21 m. The magnitude of each moving load is 160 kN. It can be seen from Figure 5 that the deflection at midspan caused by the forced-vibration components approaches the crawl deflection. In this case, the speed at which the frequency of each moving load travelling across the bridge agrees with the natural frequency would be 339·4 m/s.

2.4. FREE-VIBRATION RESPONSE

Let us consider a series of moving loads with uniform intervals r. The distance a_i between moving loads P_i and P_1 is given by

$$a_i = (i - 1)r \quad (i = 1, 2, 3, \ldots, N). \tag{9}$$

When J times of the frequency component due to a series of moving loads acting periodically on the bridge equal to the natural frequency, it is evident that

$$J\frac{v}{r} = \frac{\omega}{2\pi} \quad (J = 1, 2, 3, \ldots\ ). \tag{10}$$

Substituting equation (10) into equations (7) and (8), the free-vibration response components can be written as follows:

$$\text{for} \quad 0 \leqslant t \leqslant \frac{L_b}{v},$$

$$q'(t) = -y_{st}\frac{\beta}{1-\beta^2}\sum_{i=1}^{M} \mathrm{e}^{-\xi\omega[t-(i-1)r/v)]}\sin\omega t; \tag{11}$$

$$\text{for} \quad \frac{L_b}{v} \leqslant t \leqslant \frac{a_N}{v} + \frac{L_b}{v},$$

$$q'(t) = -y_{st}\frac{\beta}{1-\beta^2}\sum_{i=1}^{M} \mathrm{e}^{-\xi\omega[t-(i-1)r/v]}\sin\omega t$$

$$-y_{st}\frac{\beta}{1-\beta^2}\sum_{i=1}^{K-1} \mathrm{e}^{-\xi\omega[t-(i-1)r/v-L_b/v]}\sin\omega\left(t-\frac{L_b}{v}\right). \tag{12}$$

Neglecting damping, equation (12) becomes

$$q'(t) = A\sin(\omega t - \theta), \tag{13}$$

where

$$A = \frac{y_{st}\beta}{1-\beta^2}\sqrt{\left[M + (K-1)\cos\left(\omega\frac{L_b}{v}\right)\right]^2 + \left[(K-1)\sin\left(\omega\frac{L_b}{v}\right)\right]^2},$$

$$\mathrm{tg}\,\theta = \frac{M + (K-1)\cos(\omega L_b/v)}{(K-1)\sin(\omega L_b/v)}.$$

When equation (10) holds, the difference of the phase angles for the free-vibration components in equations (7) and (8) caused by each moving load passing over the bridge is $2\pi m$ ($m = 1, 2, 3, \ldots$). In this case, the sum of free-vibration-response components generated by each moving load acting successively on the bridge results in resonance if the total number of moving loads is large enough. The velocities of moving loads for resonant response can be obtained from equation (10) and are given by

$$v = \frac{\omega}{2\pi J}r \quad (J = 1, 2, 3, \ldots). \tag{14}$$

For the example of a 32 m girder bridge subjected to 10 moving loads, the velocity corresponding to $J = 2$ for resonance is 55·56 m/s. In this case, Figure 6 shows the response-amplitude at midspan caused by free-vibration components when the resonant response occurs if the damping of the girder is neglected.

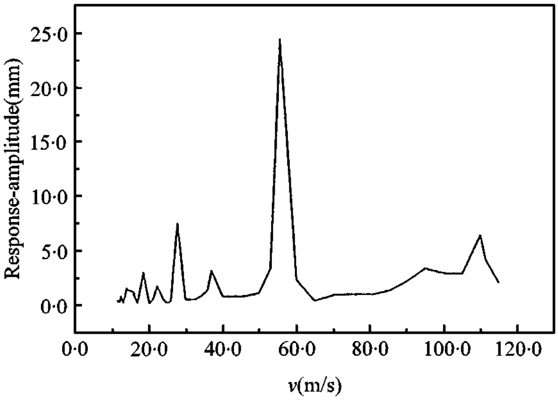

Figure 6. Response-amplitude due to free-vibration components corresponding to $J = 2$.

2.4.1. *Main factors influencing the resonant response*

(1) *The values of J*: In a practical case, the damping is always present in the bridge. The physical meaning of J may be understood from equation (10). After the free-vibration response caused by moving load P_{i-1} has decayed for J cycles due to the damping, this free-vibration response adds to that caused by moving load P_i (Figure 7). Thus, the resonant response-amplitude decreases as the value of J increases.

(2) *The ratio of the span to the velocity of moving loads*: If J times of the frequency component due to a series of moving loads acting periodically on the bridge agree with the natural frequency, it can be seen from equation (12) that the ratio of the span to the velocity is an important factor influencing the resonant response-amplitude.

In case the ratio of the span to the velocity is integer times the natural period of the girder, namely

$$\frac{L_b}{v} = \frac{2\pi k}{\omega} \quad (k = 1, 2, 3, \dots)$$

that is, $\cos(\omega L_b/v) = 1$, equation (12) becomes

$$q'(t) = -\left\{y_{st}\frac{\beta}{1-\beta^2}\sum_{i=1}^{M} \mathrm{e}^{-\xi\omega[t-(i-1)r/v]} + y_{st}\frac{\beta}{1-\beta^2}\sum_{i=1}^{K-1} \mathrm{e}^{-\xi\omega[t-(i-1)r/v-L_b/v]}\right\} \times \sin\omega t.$$

The difference of the phase angles between the first term and second term on the right-hand side of equation (12) is $2\pi m$ ($m = 1, 2, 3, \dots$). The two terms get in phase and the resonant response-amplitude reaches the maximum.

However, if the ratio of span to velocity satisfies the following condition:

$$\frac{L_b}{v} = \frac{k\pi}{\omega} \quad (k = 1, 3, 5, \dots),$$

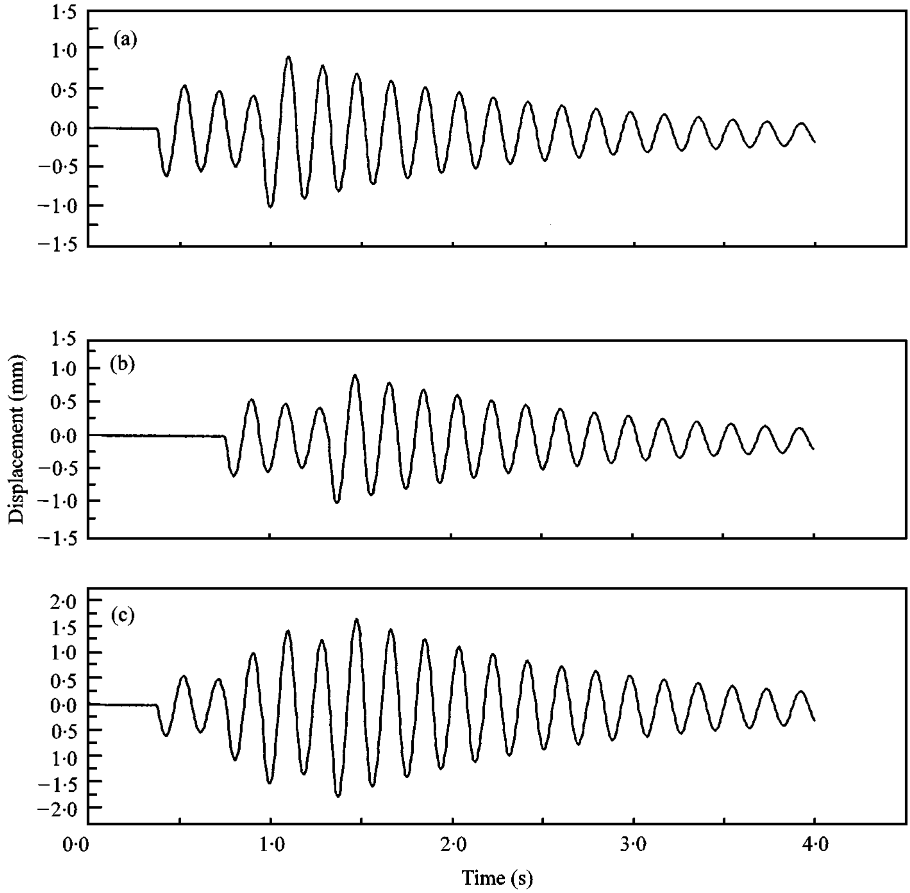

Figure 7. Response due to P_{i-1} and P_i. (a) due to P_{i-1}; (b) due to P_i; and (c) total response due to P_{i-1} and P_i.

namely, $\cos(\omega L_b/v) = -1$, equation (12) becomes

$$q'(t) = -\left\{ y_{st}\frac{\beta}{1-\beta^2}\sum_{i=1}^{M} e^{-\xi\omega[t-(i-1)r/v]} - y_{st}\frac{\beta}{1-\beta^2}\sum_{i=1}^{K-1} e^{-\xi\omega[t-(i-1)r/v-L_b/v]} \right\} \times \sin\omega t.$$

The difference of the phase angles between the first term and the second term in equation (12) is πm ($m = 1, 3, 5, \ldots$). The two terms mentioned above counteract each other and the resonant response amplitude is effectively suppressed.

For the 32 m girder bridge, the variation of the response-amplitudes of the midspan deflection caused by the free-vibration components with the velocities and damping ratios is shown in Figure 8. The total number of moving loads is 20 and the magnitude of each moving load is 160 kN. The distance between two adjacent loads is 21 m, and the natural frequency is 5·3 Hz. The velocities, the values of J and the span to velocity ratios are shown

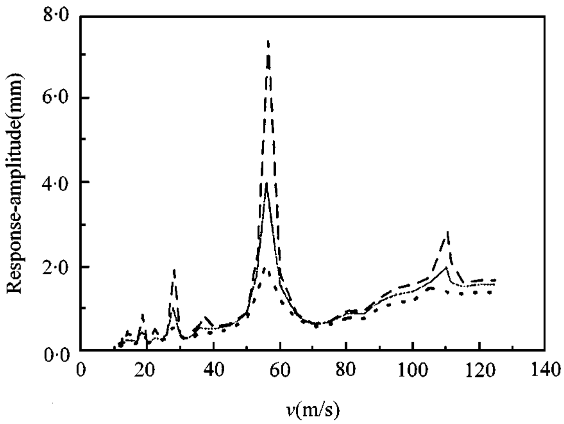

Figure 8. Variation of response-amplitude with velocity and damping. - - - $\xi = 0{\cdot}01$; —— $\xi = 0{\cdot}02$; · · · $\xi = 0{\cdot}04$

TABLE 1

The values of J, the velocities, and the span to velocity ratios

	$J=1$	$J=2$	$J=3$	$J=4$	$J=5$
Velocity (m/s)	111·3	55·65	37·10	27·82	22·59
$\cos(\omega L_b/v)$	− 0·99	0·96	− 0·90	0·826	− 0·733

in Table 1. It can be seen from Figure 8 and Table 1 that the response-amplitude reaches its maximum when $J = 2$ (corresponding velocity $v = 55{\cdot}65$ m/s), and the response-amplitude is smaller when $J = 1$ (corresponding velocity $v = 111{\cdot}3$ m/s). This is because the span to velocity ratios have influenced the response-amplitude. It can be seen from Table 1 that $\cos(\omega L/v) = 0{\cdot}96$ (approaching 1·0) when $J = 2$, and $\cos(\omega L/v) = 0{\cdot}99$ (approaching − 1·0) when $J = 1$.

(3) *The number of the moving loads*: Figure 9 shows how the response of a 32 m girder bridge builds up in the case where resonant response occurs when $J = 2$ (corresponding velocity $v = 55{\cdot}65$). In both cases with and without damping, the response builds up gradually as the number of moving loads increases. In the damped system as shown in Figure 9(b), the damping limits the resonant response-amplitude. The number of moving loads required by this damped resonant response to reach essentially its maximum amplitude depends on the amount of damping. It is clearly seen from Figure 9(b) that when damping ratio $\xi = 0{\cdot}02$, about 10 moving loads are required to reach nearly the full response-amplitude.

(4) *The damping ratio*: It can be seen from Figure 8 that the resonant response-amplitude decreases very quickly as the damping ratio grows. Thus, the increase of the damping for the bridge is an effective measure to reduce resonant response-amplitudes.

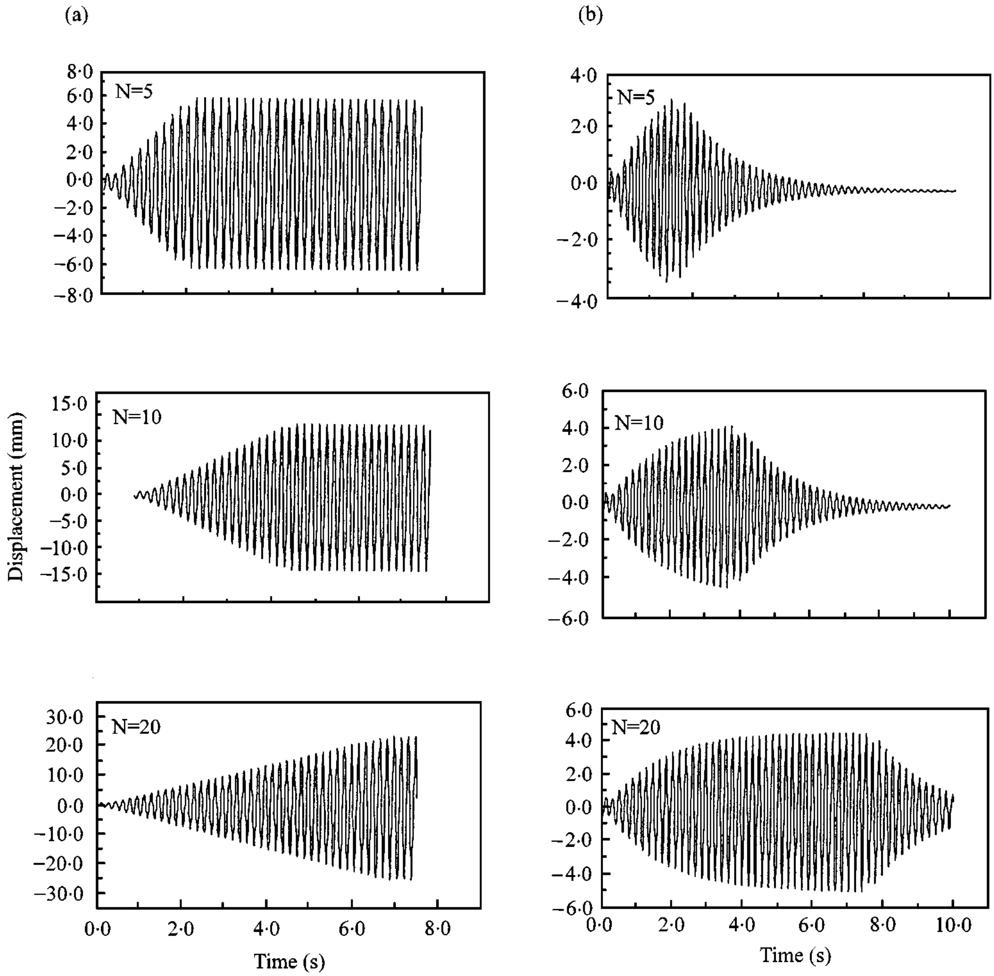

Figure 9. The influence of the number of moving loads on resonant response amplitude. (a) without damping; (b) the damping ratio $\xi = 0{\cdot}02$.

3. VEHICLE–BRIDGE SYSTEM

The above analyses of a bridge under a series of moving loads are a useful introduction to the vibration of a bridge under moving vehicles. However, in practice the masses of vehicles may be significant compared with the mass of a bridge. It is necessary to investigate the dynamic response of the vehicle–bridge system. A simplified model of the vehicle–bridge system is shown in Figure 10. This model consists of a series of successive vehicles and a bridge. Vehicle mass, wheel mass as well as vehicle suspension are all considered.

3.1. THE VEHICLE MODEL

Each vehicle may be idealized as a model of a rigid body and four wheel–axle sets, with two degrees of freedom if the vertical motion alone is considered [14]

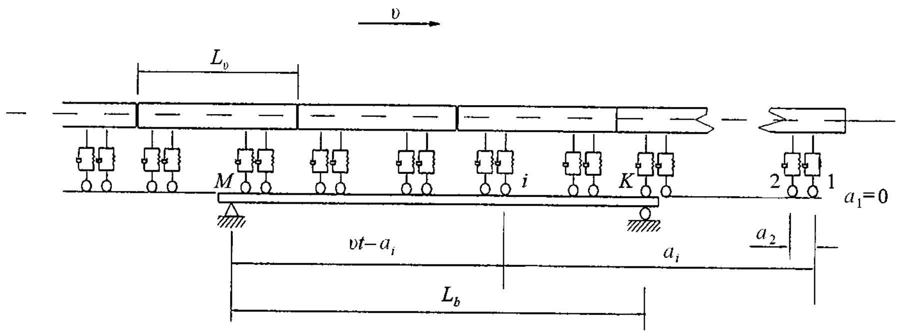

Figure 10. The vehicle–bridge system.

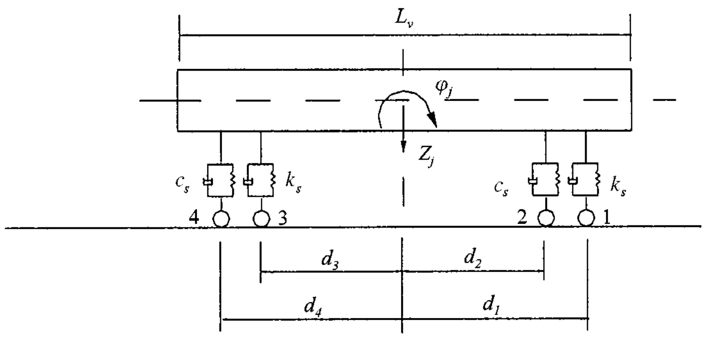

Figure 11. The vehicle model.

(Figure 11). The two trucks in the actual vehicle are assumed to form a part of the car body. The two degrees of freedom of the vehicle correspond to bounce and pitch motion. Let the subscripts j denote the vehicle number and k denote wheel-axle set number of the jth vehicle; M_j, J_j are the mass and pitch moment of inertia respectively, for each vehicle; Z_j and φ_j are vertical and pitch displacements of the jth vehicle from the equilibrium position. Referring to Figure 11, the vertical motion equations for each vehicle can be expressed as

$$M_j\ddot{Z}_j + \sum_{k=1}^{4} k_s u_{jk} + \sum_{k=1}^{4} c_s \dot{u}_{jk} = 0,$$

$$J_j\ddot{\varphi}_j + \sum_{k=1}^{4} k_s u_{jk}(\pm d_k) + \sum_{k=1}^{4} c_s \dot{u}_{jk}(\pm d_k) = 0, \tag{15}$$

where $u_{jk}(t) = Z_j \pm d_k\varphi_j(t) - [y_{jk}(x,t) - w_{jk}(x)]$; $y_{jk}(x,t)$ is the vertical displacement of the wheel/rail contact point at time t; d_k is the longitudinal distance between the centroid of the jth vehicle and its kth wheel–axle set; $w_{jk}(x)$ is the rail irregularity at the point beneath each wheel–axle set; and c_s and k_s represent the spring damping and stiffness of the vehicle's suspension system for each wheel-axle set.

3.2. THE EQUATION OF MOTION FOR THE BRIDGE

When the interaction between the vehicle and bridge is taken into account, the equation of motion (2) for the bridge in generalized co-ordinates can be rewritten as

$$\ddot{q}(t) + 2\xi\omega\dot{q}(t) + \omega^2 q(t) = \frac{2}{mL_b} F(t), \tag{16}$$

where $F(t)$ is the interacting force between vehicles and a bridge expressed in the generalized co-ordinates.

For the sake of convenience, now, the serial number of the wheel–axle set starts from the first one of the first vehicle for a train. Let a_i be the longitudinal distance from the ith wheel–axle set to the first wheel–axle set of the train (Figure 10). If the first and last wheel–axle sets on the span are the Kth and Mth wheel–axle sets, respectively, at time t, $F(t)$ is given by

$$F(t) = \sum_{i=K}^{M} P_i \sin\frac{\pi(vt - a_i)}{L_b} - \sum_{i=K}^{M} m_s(t)\ddot{y}_{bi}(x,t)\sin\frac{\pi(vt - a_i)}{L_b}$$

$$+ \sum_{i=K}^{M} \sin\frac{\pi(vt - a_i)}{L_b}[c_s\dot{u}_i(t) + k_s u_i(t)], \tag{17}$$

in which P_i is the static weight of the ith wheel-axle, a quarter of the vehicle's weight, m_s is the mass of each wheel-axle set, $\ddot{y}_{bi}(x,t) = \ddot{q}(t)\sin\pi(vt - a_i)/L_b$ is the acceleration of the bridge on the ith wheel-axle set, and $\dot{u}_i$ and u_i are the velocity and the deformation of suspension springs.

The generalized loads on the right-hand side of equation (17) consists of three terms. The first term arises from the weight loads of a series of successive wheel–axle sets moving over the bridge at a constant speed. The second part is due to the negative inertia of the mass of the wheel–axle sets. The remaining term is caused by the deformation of the spring of the vehicle's suspensions.

The first term of the generalized loads in equation (17) is the same as the generalized loads caused by equivalent vehicles as a series of moving loads. If the train consists of a number of vehicles and each vehicle length L_v is constant, this term is a periodic function with two frequency components (Figure 12). The first frequency component f_{v1} $(= v/2L_b)$ is due to each wheel–axle set travelling across the bridge. The second frequency component f_{v2} $(= v/L_v)$ is caused by weight loads of a series of successive vehicles acting periodically on the bridge. Figure 12 shows periodic variation of the generalized load due to the weight loads of a series of successive vehicles with time t.

Equations (15) and (16) are coupled second-order equations with variable coefficients which can be solved by direct numerical integration. In this paper, the Wilson-θ method [13] is adopted.

Because the factors of the vertical vibration equations of the bridge–vehicle system are variable with time, the natural frequency of the bridge under loaded

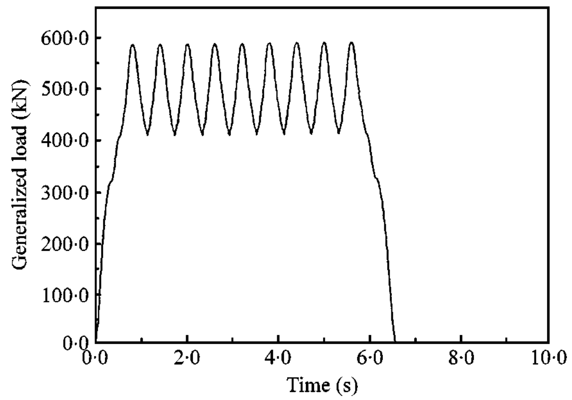

Figure 12. Generalized load due to the weight loads of a series of successive vehicles.

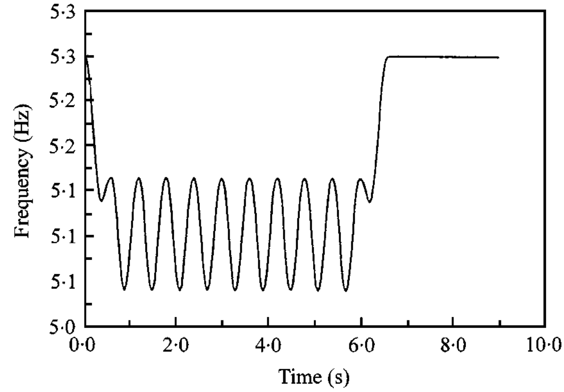

Figure 13. Variation of fundamental frequency with time.

conditions will be obtained by the solution of eigenvalue at each step of the numerical integration. Figure 13 shows the variation of the fundamental frequency with time when a series of successive vehicles move over a 32 m deck girder bridge. It can be seen from Figure 13 that the vertical fundamental frequency of the bridge varies periodically with time. The lower frequency may be regarded as the loaded frequency of the bridge.

From the analysis, it can be shown that if the integer times of the frequency component f_{v2} due to weight loads of a series of successive vehicles acting periodically on the bridge agree with the loaded frequency of the bridge, resonant vibration response will appear.

4. NUMBER EXAMPLES

In order to investigate the vibration behavior of the simply supported girder bridge under a high-speed train, the vertical vibration response for 16 and 32 m deck girder bridges under the quasi-high-speed train will be presented.

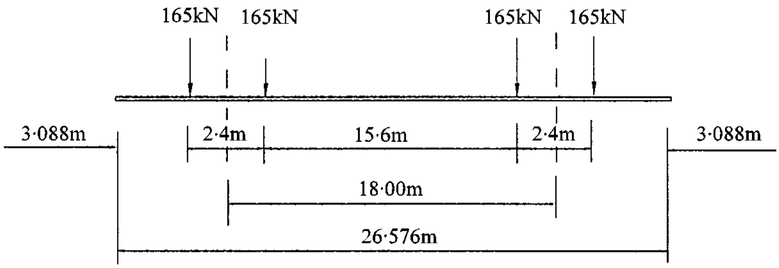

Figure 14. The static weight of the wheel–axle and the distance between the wheel–axle.

TABLE 2

Dynamic characteristic of the deck girder bridge

Type of bridge	Bending inertia moment (m^4)	Mass per unit length (t/m)	Fundamental frequency (Hz)	Loaded frequency (Hz)
16 m	0·0032	1·880	11·60	10·52
32 m	0·1592	2·880	5·300	5·040

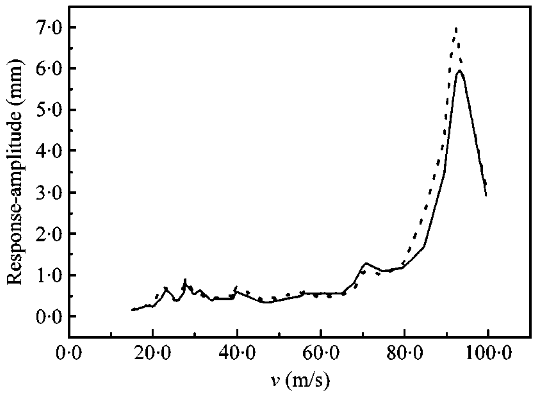

Figure 15. The deflection amplitude at midspan of the 16 m deck girder-bridge.

The static weight of the wheel–axle and the longitudinal distance between the wheel–axle for a vehicle of the quasi-high-speed train is shown in Figure 14. The dynamic characteristics of 16 and 32 m deck girder bridges are given in Table 2; the damping ratio of these bridges is supposed to be 0·02.

The relationships between the dynamic deflection amplitudes at the midspan for 16 and 32 m deck girder bridges and the speeds of the train are shown in Figures 15 and 16. Two curves are shown on each plot of Figures 15 and 16. The first curve (the dashed line) is the predicted bridge response from equivalent vehicles as a series of moving loads. The second curve is the result of the dynamic analysis using the vehicle–bridge system. The resonant velocities and the span to speed ratios are shown in Table 3. In this case, 10 vehicles travelling across the bridge at various speeds are considered. When the vehicles are considered as a series of moving loads, the natural frequency of the bridge is substituted by the loaded frequency.

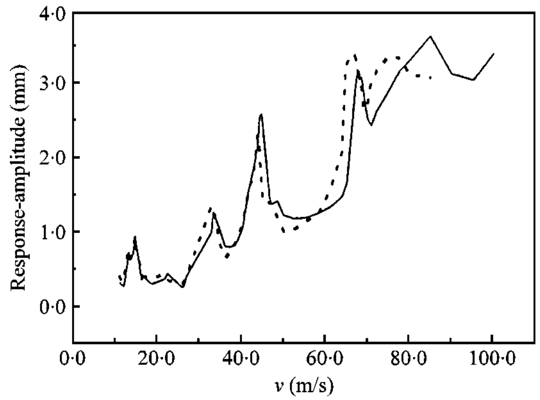

Figure 16. The deflection amplitude at midspan of the 32 m deck girder-bridge.

TABLE 3

The values of J, the peak-velocities and the ratios of span to velocities

		$J = 1$	$J = 2$	$J = 3$	$J = 4$	$J = 5$
16 m deck girder	Resonant velocity (m/s)	279·580	139·700	93·1940	69·896	55·916
	$\cos(\omega L_b/v)$	– 0·515	– 0·470	0·999	– 0·559	– 0·423
32 m deck girder	Resonant velocity (m/s)	133·950	66·980	44·650	33·490	26·790
	$\cos(\omega L_b/v)$	– 0·107	– 0·980	0·310	0·920	– 0·482

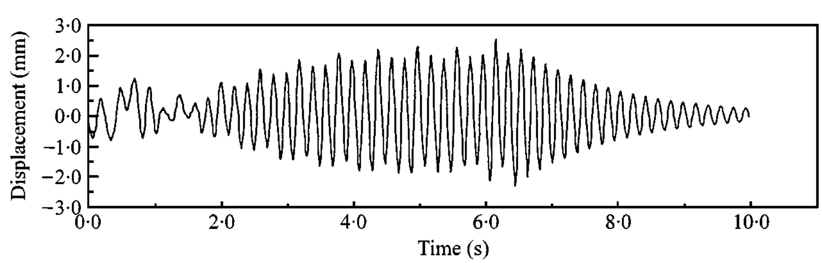

Figure 17. The dynamic deflection time curve at midspan of the girder calculated with the vehicle-bridge model.

From Figures 15 and 16, it can be seen that if the ratio of the vehicle body length L_v to the velocity v satisfies equation (10), both the curves reach their peak values. The ratios of the L_v/v and the values of J are the main factors to influence the peak values of the vibration response. For the 32 m deck girder bridge, when the train moves at 44·65 m/s (corresponding to $J = 3$) the greater peak values of the vibration response may appear.

Figure 17 shows the time-history curve of dynamic deflection at midspan calculated by using the vehicle–bridge system for a 32 m deck girder bridge when the train moves over the bridge at a speed of 44·65 m/s (corresponding to $J = 3$). The results show clearly that the vertical resonance phenomenon of the girder bridge is mainly caused by free vibrations arising from the weight loads of a series of successive wheel–axles.

5. CONCLUSIONS

Based on the analyses of the free and forced-vibration responses of the girder bridge using two dynamic models, the fundamental characteristics and the main influential factors for the resonant vibration caused by a train have been investigated in detail. Some useful results are obtained:

- When the periods due to weight loads of a series of successive vehicles acting periodically on the bridge agree with the integer times of the fumdamental loaded period, that is $L_v/v = 2\pi J/\omega$ ($J = 1, 2, 3 \ldots$), the sum of free-vibration response components generated by each moving load acting successively on the bridge will result in the resonance.
- If the ratio of span to velocity is constant, the resonant response-amplitudes increase as the values of J decrease. However, in case of J being a constant, the ratio of span to velocity, L_b/v, is the main factor that affects the resonant response-amplitude. If $\cos(\omega L_b/v)$ approaches $-1{\cdot}0$, the resonant response-amplitude may be suppressed.
- The number of vehicles travelling across the bridge is also an important factor. If the number of vehicles is very small, resonance may not occur.

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