Journal of Sound and Vibration (1999) **224**(5), 937–940
Article No. jsvi.1997.1356, available online at http://www.idealibrary.com.on IDEAL®

LETTERS TO THE EDITOR

THE FINITE RESIDUAL MOTION OF A DAMPED THREE-DEGREE-OF-FREEDOM VIBRATING SYSTEM

E. V. Wilms and R. B. Pinkney
Department of Civil and Geological Engineering, University of Manitoba, Winnipeg, Canada

(*Received* 2 *October* 1997)

1. Introduction

In Figures 1 and 2 are shown two systems which oscillate in the horizontal direction. The three bodies each have unit mass, the springs have unit stiffness, and the dashpots have constant c. One of the systems oscillates indefinitely, while all oscillations are eventually damped out for the other one. Which one oscillates indefinitely?

2. Analysis

It is readily shown that the equations of motion of system A are

$$\ddot{x} + c(\dot{x} - \dot{z}) + 2x - y = 0, \qquad \ddot{y} + 2y - x - z = 0, \qquad \ddot{z} + c(\dot{z} - \dot{x}) + 2z - y = 0, \qquad (1\text{–}3)$$

where x, y and z are the displacements as indicated in Figure 1.

Insight into the problem may be obtained by calculating the natural frequencies and mode shapes corresponding to the underdamped case. If we set c, the damping constant, equal to zero and assume $x = X \sin \lambda t$, $y = Y \sin \lambda t$ and $z = Z \sin \lambda t$ we obtain: $Y = 0$ and $Z = X$ for the frequency $\lambda_1 = \sqrt{2}$, $Y = \sqrt{2}X$ and $Z = X$ for the frequency $\lambda_2 = (1 + \sqrt{2})^{1/2}$ and $Y = -\sqrt{2}X$ and $Z = X$ for the frequency $\lambda_3 = (2 - \sqrt{2})^{1/2}$. The mode shapes are plotted in Figure 3.

Guided by the mode shapes we make the variable changes:

$$x = A + B + D, \qquad y = \sqrt{2}A - \sqrt{2}B, \qquad z = A + B - D. \qquad (4\text{–}6)$$

Equations (1)–(3) then become

$$\ddot{A} + \ddot{B} + \ddot{D} + 2c\dot{D} + (2 - \sqrt{2})A + (2 + \sqrt{2})B + 2D = 0, \qquad (7)$$

$$\sqrt{2}\ddot{A} - \sqrt{2}\ddot{B} + (2\sqrt{2} - 2)A - (2\sqrt{2} + 2)B = 0, \qquad (8)$$

$$\ddot{A} + \ddot{B} - \ddot{D} - 2c\dot{D} + (2 - \sqrt{2})A + (2 + \sqrt{2})B - 2D = 0. \qquad (9)$$

Equation (7) minus equation (9) then yields

$$\ddot{D} + 2c\dot{D} + 2D = 0. \qquad (10)$$

0022–460X/99/300937 + 04 $30.00/0

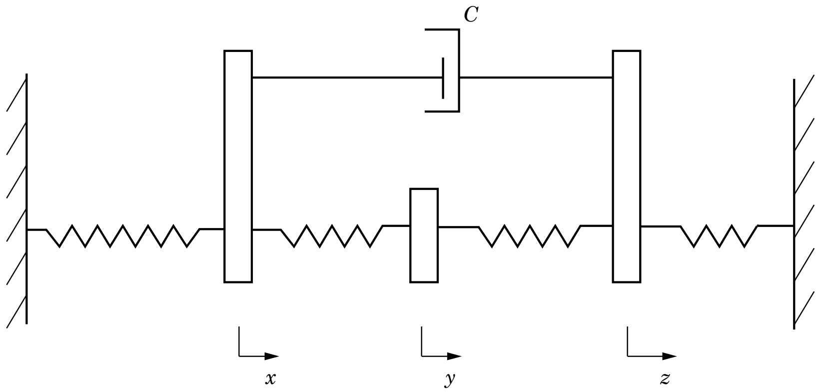

Figure 1. System A.

Equation (7) plus equation (9) plus $\sqrt{2}$ times equation (8) then yields

$$\ddot{A} + (2 - \sqrt{2})A = 0. \tag{11}$$

Similarly

$$\ddot{B} + (2 + \sqrt{2})B = 0. \tag{12}$$

Equation (10) indicates damped motion, and equations (11) and (12) indicate undamped motion. Hence for general initial values of $x, y, z, \dot{x}, \dot{y}$ and $\dot{z}$, finite motion remains for system A.

Similarly, for system B, we obtain

$$\ddot{A} + (c/2)\dot{A} + (2 - \sqrt{2})A = -c(3/2 + \sqrt{2})\dot{B}, \tag{13}$$

$$\ddot{B} + (c/2)\dot{B} + (2 + \sqrt{2})B = -c(3/2 - \sqrt{2})\dot{A}, \qquad \ddot{D} + c\dot{D} + 2D = 0. \tag{14, 15}$$

Equations (13)–(15) all indicate damped motion, so that all oscillations are eventually damped out. Both dashpots come into play for any motion of the system.

A similar system involving only two degrees of freedom was considered by Wilms and Pinkney [1], Stephen [2] and Gürgöze [3].

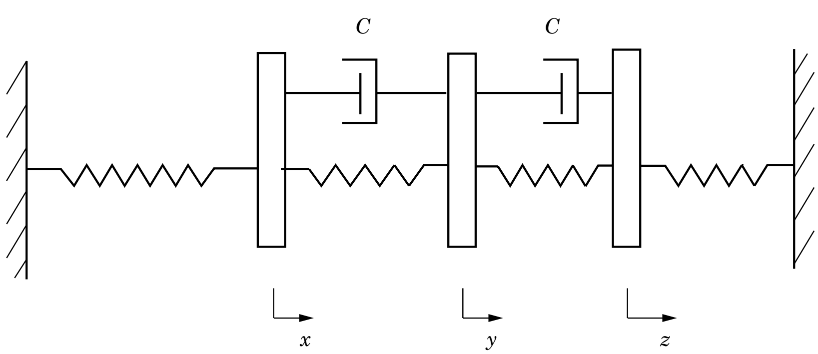

Figure 2. System B.

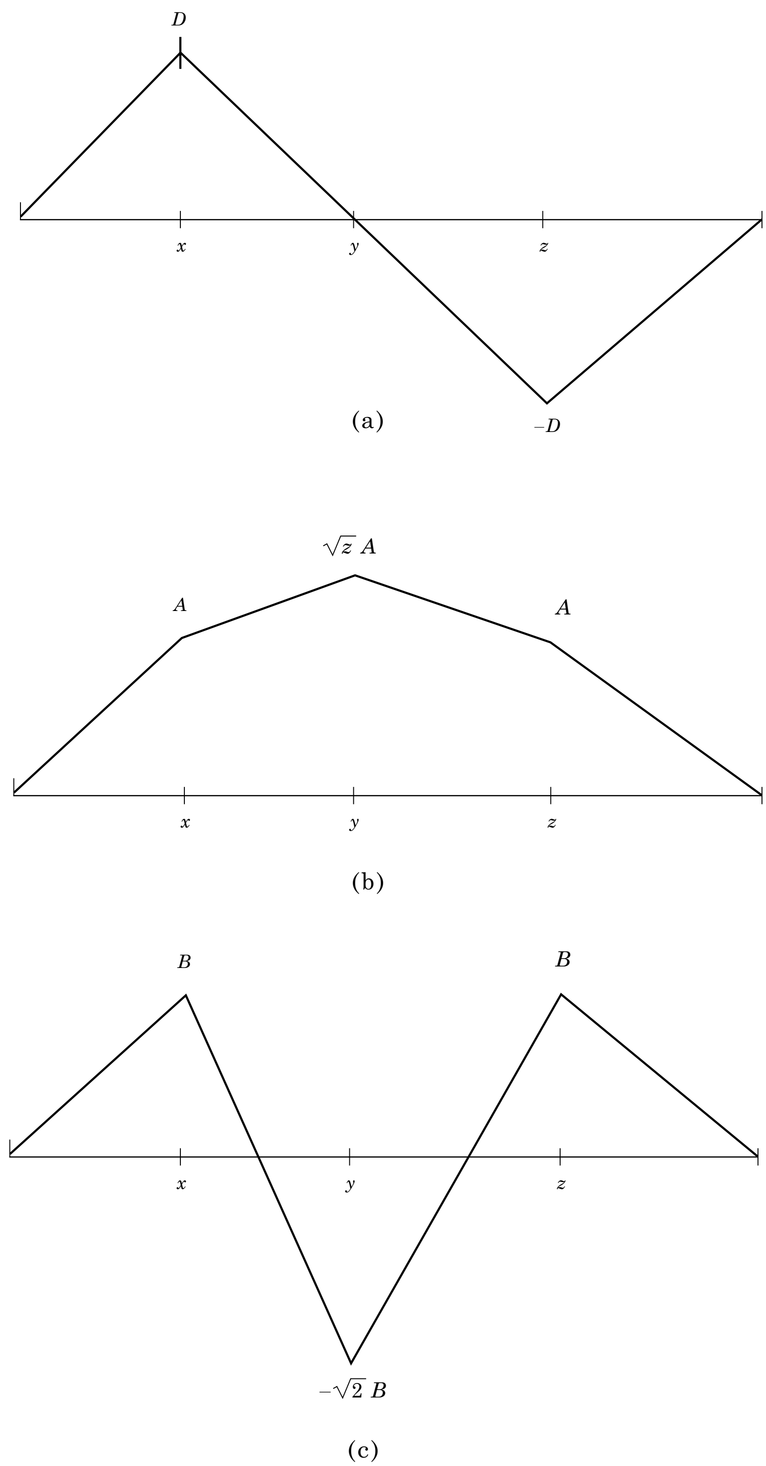

Figure 3. The undamped mode shapes: (a) $\lambda_1 = \sqrt{2}$, (b) $\lambda_2 = (2 + \sqrt{2})^{1/2}$, (c) $\lambda_3 = (2 - \sqrt{2})^{1/2}$.

REFERENCES

1. E. V. Wilms and R. B. Pinkney 1991 *Journal of Sound and Vibration* **148**, 533–534. The finite residual motion of a damped two-degree-of-freedom vibrating system.
2. N. G. Stephen 1992 *Journal of Sound and Vibration* **156**, 557. Intuitive approach to "the finite residual motion of a damped two-degree-of-freedom vibrating system".
3. M. Gürgöze 1994 *Journal of Sound and Vibration* **169**, 409–410. Another solution to "the finite residual motion of a damped two-degree-of-freedom vibrating system".