Journal of Sound and Vibration (1999) **225**(1), 29–39
Article No. jsvi.1999.2196, available online at http://www.idealibrary.com on IDEAL®

FREE VIBRATION OF A SYSTEM OF TWO ELASTICALLY CONNECTED RECTANGULAR PLATES

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(*Received* 29 *July* 1998, *and in final form* 27 *January* 1999)

The free vibration problem of a system of two rectangular plates connected by a non-homogeneous elastic layer is considered. An integral formulation of the problem by using properties of Green's functions is achieved and by application of a quadrature method to the integral equation, the frequency equation of the combined system is obtained. The comparison of an exact solution with the numerical results obtained by using the presented method for a system of two identical plates is given. The numerical investigations have shown the effect of the area size as well as the stiffness of an elastic layer connecting the plates on the vibration frequencies of the combined systems.

1. INTRODUCTION

The analysis of the vibrations of compound systems is of a great importance for engineering applications. The determination of the influence of component substructures on the vibration of the whole compound system has a particular significance for designers. The prediction of the vibrational behaviour enables the proper choice of the system's parameters so that the resulting system has the desired vibrational characteristic.

The subject of the present paper is the free vibration problem of a system of two plates connected by means of an inhomogeneous elastic layer. The natural frequencies of the considered combined system depend on the vibration frequencies of the component plates as well as on properties of the connecting layer. Some frequencies of the plate system may equal the frequencies of a single plate on an elastic foundation. The free vibrations of rectangular plates on an inhomogeneous foundation have been investigated in references [1, 2].

The Green's function synthesis to a class of layered distributed parameter systems has been applied in reference [3] to an example of sandwiched Euler–Bernoulli beams. Reference [4] is devoted to the free vibrations of rectangular plates connected by a homogeneous elastic layer. The free vibration of a system of many plates connected by a discrete elastic elements was investigated in reference [5]. Reference [6] is devoted to the free vibration problem of a system of two line connected rectangular plates.

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The solution of the free vibration problem of a compound system may be often obtained only by the use of an approximate method. The analytical solution can be achieved for example for the vibration system, which consists of beams or rectangular Levy plates connected by discrete elastic elements. This solution (frequency equation, mode shapes) may be expressed by Green's functions, which correspond to substructures of the compound system (e.g. reference [5]). The application of the Green's functions for the solution of such vibration problems is particularly profitable. The Green's functions for beams and Levy plates are well known [7].

The natural frequencies of the combined system depend on the vibration frequencies of the component substructures as well as on a method of their connections. If the stiffness of the elastic connections is small then the vibration frequencies of the compound system are near the frequencies of the substructures. The increase of the connection stiffness results in an increase of the eigenfrequencies of the compound system. Therefore, among the eigenfrequencies of the compound system there can be distinguished subsequences, which correspond to separate substructures.

A particular situation appears when the substructures of the compound system are identical. For instance, in the case of two elastically connected identical beams are distinguished by two different sets of mode shapes which characterize the following property [3]: the corresponding points of the beams during the free vibration of the system, are moving together or in opposite directions. Moreover, the vibration frequencies of the compound system, which correspond to the modes when the beams move together, are independent of the connection stiffness of the beams. An analogous situation occurs in the case of a system of two identical elastically, line connected plates [6].

In the present paper, the free flexural vibrations of a system of two rectangular plates, which are connected by an elastic non-homogeneous layer, are analyzed. The plates of the system have the same geometrical dimensions and various physical properties and boundary conditions are considered. The non-homogeneous elastic layer is treated as a system of linear unconnected springs which is defined by a stiffness modulus $k(x, y)$. The application of the Green's functions to the differential eigenproblem corresponding to the considered system leads to integral equations of the Fredholm type. For numerical calculations of the eigenfrequencies of the combined system a quadrature method has been applied to the integral equation.

2. THEORY

Consider a system of two rectangular plates, which are connected by an elastic layer with the stiffness modulus $k = k(x, y), 0 \leqslant x \leqslant a, 0 \leqslant y \leqslant b$. The equations for small vibrations of the plates, according to the Love–Kirchhoff theory, are

$$D_1 \nabla^4 w_1 - \rho_1 \partial^2 w_1/\partial t^2 = k(x, y)[w_1(x, y, t) - w_2(x, y, t)], \tag{1}$$

$$D_2 \nabla^4 w_2 - \rho_2 \partial^2 w_2/\partial t^2 = k(x, y)[w_2(x, y, t) - w_1(x, y, t)], \tag{2}$$

where D_i, ρ_i $(i = 1, 2)$ are the flexural rigidity and plate mass per unit area, respectively, w_1, w_2 are the transverse displacements of the plates and ∇^4 is the biharmonic operator.

In order to find the natural frequencies of the system, ω, one assumes that

$$w_1(x, y, t) = \bar{W}_1(x, y)\,\mathrm{e}^{\mathrm{i}\omega t}, \qquad w_2(x, y, t) = \bar{W}_2(x, y)\,\mathrm{e}^{\mathrm{i}\omega t}. \tag{3}$$

Substituting equations (3) into equations (1) and (2) and introducing dimensionless quantities, one obtains

$$\mathscr{L}W_1(\xi, \eta) - \Phi^4\lambda_1^4 W_1(\xi, \eta) = K(\xi, \eta)[W_2(\xi, \eta) - W_1(\xi, \eta)], \tag{4}$$

$$\mathscr{L}W_2(\xi, \eta) - \Phi^4\lambda_2^4 W_2(\xi, \eta) = -\mu K(\xi, \eta)[W_2(\xi, \eta) - W_1(\xi, \eta)], \tag{5}$$

where $\xi = x/a$, $\eta = y/b$, $W_1 = \bar{W}_1/a$, $W_2 = \bar{W}_2/a$, $\Phi = b/a$, $K = b^4k/D_1$, $\mu = D_1/D_2$, $\lambda_1^2 = \omega a^2\sqrt{\rho_1/D_1}$, $\lambda_2^2 = \omega a^2\sqrt{\rho_2/D_2}$ and $\mathscr{L} \equiv \partial^4/\partial\eta^4 + 2\Phi^2\partial^4/\partial\xi^2\partial\eta^2 + \Phi^4\xi^4/\partial\xi^4$. The functions W_1 and W_2 satisfy homogenous boundary conditions, which correspond to attachments of the plate edges (simply supported, clamped or free edges). The conditions can be written symbolically in the following form:

$$\mathbf{V}_i[\mathbf{W}_i]_{\mathrm{B}} = 0, \quad i = 1, 2. \tag{6}$$

For the determination of the vibration frequencies of the compound system, Green's functions G_i of the corresponding differential problems has been applied. The functions are solutions of the differential equation

$$\mathscr{L}G_i(\xi, \eta, \zeta, \theta) - \Phi^2\lambda_1^4 G_i(\xi, \eta, \zeta, \theta) = \delta(\xi - \zeta)\delta(\eta - \theta), \quad i = 1, 2, \tag{7}$$

where $\delta(\cdot)$ denotes the Dirac delta function. These functions with respect to variables ξ and η satisfies boundary conditions (6). The Green's functions for Levy plates are given in reference [7].

Using the Green's functions G_1 and G_2 the differential problem (4)–(6) is replaced by a corresponding integral problem, which consists in finding the eigenfrequencies ω, for which there exists a non-trivial solution of the system of Fredholm equations of the second kind:

$$W_1(\xi, \eta) = \int_0^1\int_0^1 K(\zeta, \theta)G_1(\zeta, \theta, \xi, \eta)[W_2(\zeta, \theta) - W_1(\zeta, \theta)\,\mathrm{d}\zeta\,\mathrm{d}\theta, \tag{8}$$

$$W_2(\xi, \eta) = -\mu\int_0^1\int_0^1 K(\zeta, \theta)G_2(\zeta, \theta, \xi, \eta)[W_2(\zeta, \theta) - W_1(\zeta, \theta)\,\mathrm{d}\zeta\,\mathrm{d}\theta, \tag{9}$$

Assuming that the functions W_1 and W_2 are not identical in the square $0 \leqslant \xi \leqslant 1$, $0 \leqslant \eta \leqslant 1$, by subtraction both sides of equations (8) and (9), are obtains one equation

$$\tilde{W}(\xi, \eta) = -\int_0^1 \int_0^1 K(\xi, \theta)[G_1(\zeta, \theta, \xi, \eta) + \mu G_2(\zeta, \theta, \xi, \eta)]\, \tilde{W}(\zeta, \theta)\, \mathrm{d}\zeta\, \mathrm{d}\theta, \quad (10)$$

where $\tilde{W}(\xi, \eta) = W_2(\xi, \eta) - W_1(\xi, \eta)$.

The case, when the function $K(\xi, \eta)$ has the form

$$K(\xi, \eta) = \sum_{j=1}^{m} K_j \delta(\xi - \xi_j)\delta(\eta - \eta_j) \quad (11)$$

corresponds to a system of two plates whose members are connected at points (ξ_j, η_j), $j = 1, 2, \ldots, m$, by m translational springs. Equation (10) in this case assumes the form

$$\tilde{W}(\xi, \eta) = -\sum_{j=1}^{m} K_j [G_1(\xi_j, \eta_j, \xi, \eta) + \mu G_2(\xi_j, \eta_j, \xi, \eta)]\, \tilde{W}(\xi_j, \eta_j). \quad (12)$$

By substituting $(\xi, \eta) = (\xi_i, \eta_i)$ for $i = 1, 2, \ldots, m$, successively into equation (12), one obtains a set of m equations with unknown $\tilde{W}(\xi_j, \eta_j)$, $j = 1, 2, \ldots, m$. For a non-trivial solution of this set of equations the determinant of the coefficient matrix of the system of equations must vanish, yielding the frequency equation. The case of this plate system was the subject of reference [5].

For the Levy-type plates, the functions W_i and G_i ($i = 1, 2$), can be written in the form (the plate is simply supported at the boundary $\xi = 0$ and $\xi = 1$)

$$W_i(\xi, \eta) = 2 \sum_{n=1}^{\infty} Y_n^i(\eta) \sin n\pi\xi, \quad (13)$$

$$G_i(\xi, \eta, \zeta, \theta) = 2 \sum_{n=1}^{\infty} g_n^i(\eta, \theta) \sin n\pi\xi \sin n\pi\zeta. \quad (14)$$

If the stiffness modulus depends on the variable η only: $K(\xi, \eta) = K(\eta)$ for $0 \leqslant \xi \leqslant 1, 0 \leqslant \eta \leqslant 1$, then after the use of equations (13) and (14), one obtains from equation (10) the integral equations in the form

$$\tilde{Y}_n(\eta) = -\int_0^1 K(\theta)[g_n^1(\theta, \eta) + \mu g_n^2(\theta, \eta)]\, \tilde{Y}_n(\theta)\, \mathrm{d}\theta, \quad n = 1, 2, \ldots, \quad (15)$$

where $\tilde{Y}_n(\eta) = Y_n^2(\eta) - Y_n^1(\eta)$.

Consider now the case of the function $k(\eta)$ in the form

$$K(\eta) = \sum_{j=1}^{m} \bar{K}_j \delta(\eta - \eta_j). \tag{16}$$

The case corresponds to the system of line connected plates. Equation (10) can be written now as follows:

$$\tilde{Y}_n(\eta) = -\sum_{j=1}^{m} \bar{K}_j [g_n^1(\eta_j, \eta) + \mu g_n^2(\eta_j, \eta)]\, \bar{Y}_n(\eta_j). \tag{17}$$

By substituting $\eta = \eta_i$ for $i = 1, 2, \ldots, m$, successively into equation (17) one obtains for every $n = 1, 2, \ldots,$ a set of m equations with unknown $\tilde{Y}_n(\eta_j)$, $j = 1, 2, \ldots, m$. Similarly as in the case of point connected plates the non-trivial solution of this set of equations exist, when the determinant of the coefficient matrix of the system vanishes, yielding the frequency equations for $n = 1, 2, \ldots$. The vibration problem of this system of plates was the subject of reference [6].

In the general case of the function $k(\eta)$ which occurs in equation (15), the frequency values are determined by the application of an approximate method. For this purpose the integral in equation (15) is replaced by a sum which follows from use of a quadrature rule (for instance of a Newton–Cotes type [8]). In this case, from equation (15) the following equations arise:

$$\tilde{Y}_n(\eta) = -\sum_{i=0}^{m} A_i K(\theta_i)[g_n^1(\theta_i, n) + \mu g_n^2(\theta_i, \eta)]\, \tilde{Y}_n(\theta_i), \; n = 1, 2, \ldots, \tag{18}$$

where A_i are the weighting coefficients and θ_i are the knots of the quadrature. Assuming that $\eta = \theta_j$ in equation (18), successively for $j = 0, 1, \ldots, m$, one obtains a set of linear homogeneous equations with unknown $Y_n^i = \tilde{Y}_n(\theta_i)$. For the non-trivial solution to exist, the determinant of the coefficient matrix **A** must vanish. It yields the frequency equation

$$\det \mathbf{A} = 0. \tag{19}$$

This equation is then solved numerically.

3. A SYSTEM OF TWO SIMPLY SUPPORTED RECTANGULAR PLATES

In the particular case of the system of rectangular simply supported plates, which are connected by a homogeneous elastic layer, the exact solution of the problem can be obtained. The functions W_i and G_i corresponding to the plates simply supported at all edges, can be written in the form of double series:

$$W_1(\xi, \eta) = 4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin m\pi\xi \sin m\pi\eta, \quad W_2(\xi, \eta) = 4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \sin m\pi\xi \sin m\pi\eta, \tag{20}$$

$$G_i(\xi, \eta, \zeta, \theta) = 4 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} g^i_{mn} \sin m\pi\xi \sin \pi\zeta \sin n\pi\eta \sin n\pi\theta, \tag{21}$$

where on the basis of equation (7), the coefficients g^i_{mn} are

$$g^i_{mn} = \frac{1}{k^4_{mn} - \Phi^4 \lambda_i^4}, \tag{22}$$

and where $k^4_{mn} = \pi^4[\Phi^2 m^2 + n^2]^2$. Assuming that $K(\xi, \eta) \equiv K = \text{const}$, and taking into account equations (20) and (21) in equations (8) and (9), two equations are obtained:

$$(1 + Kg^1_{mn})A_{mn} - Kg^1_{mn}B_{mn} = 0, \tag{23}$$

$$-\mu Kg^2_{mn}A_{mn} + (1 + \mu Kg^2_{mn})B_{mn} = 0. \tag{24}$$

From the above equations one obtains the frequency equation of the compound system:

$$1 + K[g^1_{mn} + \mu g^2_{mn}] = 0. \tag{25}$$

Equation (25) may be rearranged in the form

$$r\Omega^4 - [(1 + r)k^4_{mn} + (1 + \mu r)K]\Omega^2 + k^4_{mn}[k^4_{mn} + (1 + \mu)K] = 0, \tag{26}$$

where $r = \rho_1 D_2/(\rho_2 D_1)$, $\Omega = \Phi^4 \lambda_2^2$. The roots of equation (26) for each pair of m, n, are the frequency values of the compound system. Accordingly there are two sets of frequencies:

$$(\Omega^{\pm}_{mn})^2 = \frac{1}{2r}\left[(1 + r)k^4_{mn} + (1 + \mu r)K \pm \sqrt{[(1 - r)k^4_{mn} + (1 - \mu r)K]^2 + 4\mu r K^2}\right]. \tag{27}$$

The mode shape functions $Y_{i,mn}$ corresponding to the eigenfrequencies $\Omega^{\pm}_{mn}$, obtained from equations (20) and (23), are

$$Y_{1,mn} = \sin m\pi\xi \sin n\pi\eta, \qquad Y_{2,mn} = Q^{\pm}_{mn} \sin m\pi\xi \sin n\pi\eta, \tag{28}$$

where $Q^{\pm}_{mn} = (1/K)[k^4_{mn} - (\Omega^{\pm}_{mn})^2 r + K]$. Taking into account equation (27), one has

$$Q^{\pm}_{mn} = \frac{1}{2K}\left[(1 - r)k^4_{mn} + (1 - \mu r)K \pm \sqrt{[(1 - r)k^4_{mn} + (1 - \mu r)K]^2 + 4\mu r K^2}\right]. \tag{29}$$

However, $Q_{mn}^{+} > 0$ and $Q_{mn}^{-} < 0$ for all pairs of m, n. On the basis of equations (28) it can be said that during the free vibration with the frequency values Ω_{mn}^{+} the plates of the system move in the same direction. Similarly, during the free vibration with the frequency values Ω_{mn}^{-} the plates of the system move in the opposite directions.

4. RESULTS AND DISCUSSION

Consider a system of two identical plates (the same boundary conditions and physical properties) connected by an elastic layer. If $W_1(\xi, \eta) = W_2(\xi, \eta)$ for $0 \leqslant \xi \leqslant 1, 0 \leqslant \eta \leqslant 1$, then the right-hand side of equations (4) and (5) are zero, the eigenvalues of the compound system are equal to the eigenvalues corresponding to a single plate (the degenerate frequencies of the system [9]). These eigenfrequencies are independent of the stiffness of the connecting elastic layer. In the case of $W_1(\xi, \eta) \neq W_2(\xi, \eta)$, subtraction of both sides of equations (4) and (5), gives an equation, which has the form of the differential equation of a single plate on an elastic Winkler foundation. Therefore, the non-degenerate frequencies of the system of two identical plates connected by an elastic layer with stiffness modulus K are the same as for a single plate on the elastic foundation with stiffness coefficient $2K$. The transverse vibration of single rectangular plates on inhomogeneous elastic foundation has been investigated by using the Rayleigh–Ritz method in reference [1] and by means of the modal constraint method in reference [2].

The present method is tested numerically for a system of two identical, rectangular plates connected by homogeneous elastic layer with dimensionless stiffness coefficient $K = 1000$. The plates of the system are assumed as simply supported at all edges (S-S-S-S) or simply supported at two opposite edges and free at the others (S-F-S-F) with $\Phi = 0{\cdot}5$. The eigenfrequencies of a system of identical S-S-S-S plates, on the basis of equation (27), are expressed by $\Omega_{mn}^{+} = \sqrt{k_{mn}^4 + 2K}$ and $\Omega_{mn}^{-} = k_{mn}^2$. The free vibration frequencies of a rectangular plate on a homogeneous foundation may be directly obtained from the frequencies corresponding to the plate without foundation [10]. The frequency equation for the rectangular S-F-S-F plate, is given in references [10–12]. The comparison of the non-dimensional frequency parameters obtained on the basis of equation (19) with the exact solution is given below in Table 1. The approximate results obtained for different numbers of subintervals ($m = 10$; 20) have shown the high agreement with the exact solution.

The first example concerns the compound system of the S-S-S-S and S-F-S-F square plates connected by an elastic layer which occupy a band of the plates. The scheme of the system is shown in Figure 1 (line areas of the plates are connected by the elastic layer, dashed line denotes simply supported edge). The connecting layer is defined by

$$k(\eta) = \begin{cases} k & \text{for } \eta_1 \leqslant \eta \leqslant 1 - \eta_1, \\ 0 & \text{for other } \eta. \end{cases} \tag{30}$$

TABLE 1

Comparison of the non-degenerate frequency values of the systems of two identical plates ($\Phi = 0{\cdot}5$) connected by a homogeneous elastic layer ($K = 1000$) for the exact and presented method

	S-S-S-S plates			S-F-S-F plates		
		The presented method			The presented method	
Mode (m,n)	Exact solution	Number of elements 10	20	Exact solution	Number of elements 10	20
(1,1)	46·3918	46·3857	46·3914	44·7845	44·7821	44·7879
(1,2)	61·3144	61·3094	61·3141	45·2476	45·5693	45·3443
(1,3)	101·6591	101·6557	101·6589	51·9182	52·8384	52·1784
(2,1)	48·8839	48·8781	48·8835	45·7467	45·7800	45·7595
(2,2)	66·5975	66·5929	66·5972	47·5429	47·8590	47·6367
(2,3)	108·3555	108·3523	108·3553	57·8686	58·6469	58·0886
(3,1)	55·0353	55·0302	55·0350	49·7611	49·8260	49·7819
(3,2)	76·1908	76·1869	76·1906	53·4145	53·7154	53·5026
(3,3)	119·7010	119·6982	119·7009	68·3172	68·9369	68·4924

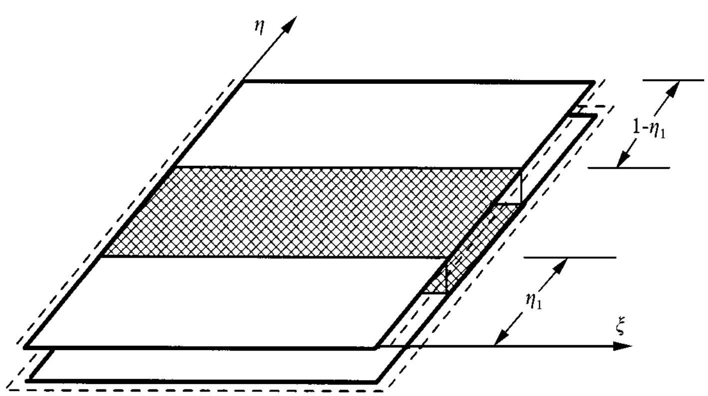

Figure 1. A scheme of the system of S-S-S-S and S-F-S-F plates connected by an elastic layer occupying the area: $0 \leqslant \xi \leqslant 1$, $\eta_1 \leqslant \eta \leqslant 1 - \eta_1$.

The calculations are performed for $k = 100$, 500 and 1000. The effect of the width of the elastic layer on the eigenfrequencies of the compound system is presented in Figure 2. The free vibration frequencies of the system $\eta_1 \to 0{\cdot}5$ correspond to the eigenfrequencies of the isolated plates. The stiffness of the elastic layer as well the width of the layer effect the frequencies of the compound system significantly.

The next example concerns the system consisting of two square, identical S-F-S-F plates connected by two elastic bands (Figure 3). The changes of the frequency values of the compound system versus the width of the bands are presented in

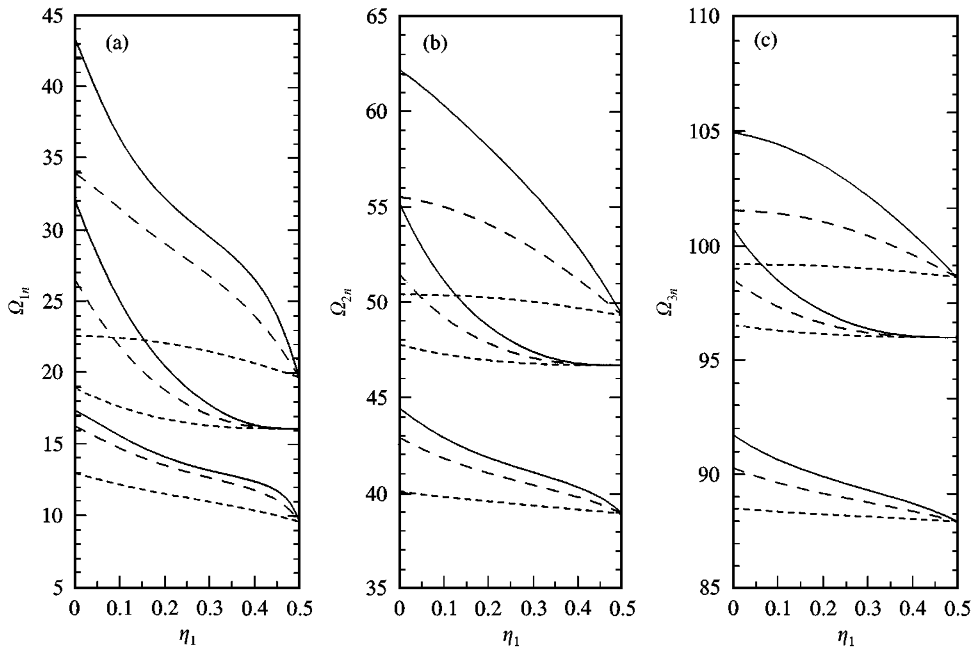

Figure 2. Frequency parameter values $\Omega_{mn} = \omega_{mn} a^2 \sqrt{\rho/D}$ as functions of η_1 for the system of two square plates shown in Figure 1: - - - - $K = 100$, – – – – $K = 500$, —— $K = 1000$.

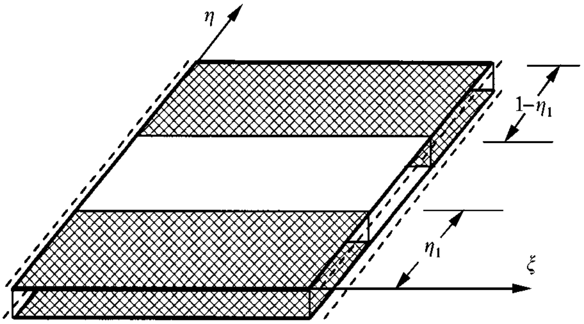

Figure 3. A scheme of the system of two S-F-S-F plates connected by an elastic layer occupying the areas: $0 \leqslant \xi \leqslant 1$, $0 \leqslant \eta \leqslant \eta_1$ and $0 \leqslant \xi \leqslant 1$, $\eta_1 \leqslant \eta \leqslant 1 - \eta_1$.

Figure 4. Because the plates are identical, there exist the frequencies independent of the stiffness of elastic layer connecting the plates and width of the elastic bands. These degenerate frequencies of the system are equal to the frequencies of a single S-F-S-F plate. The non-degenerate frequencies are equal to the frequencies of a single S-F-S-F plate on an elastic foundation which is distributed in the same way as the elastic layer for the plate system.

5. CONCLUSIONS

The solution of the free vibration problem of a system of two rectangular plates was obtained by applying the Green's function method. In the given examples the

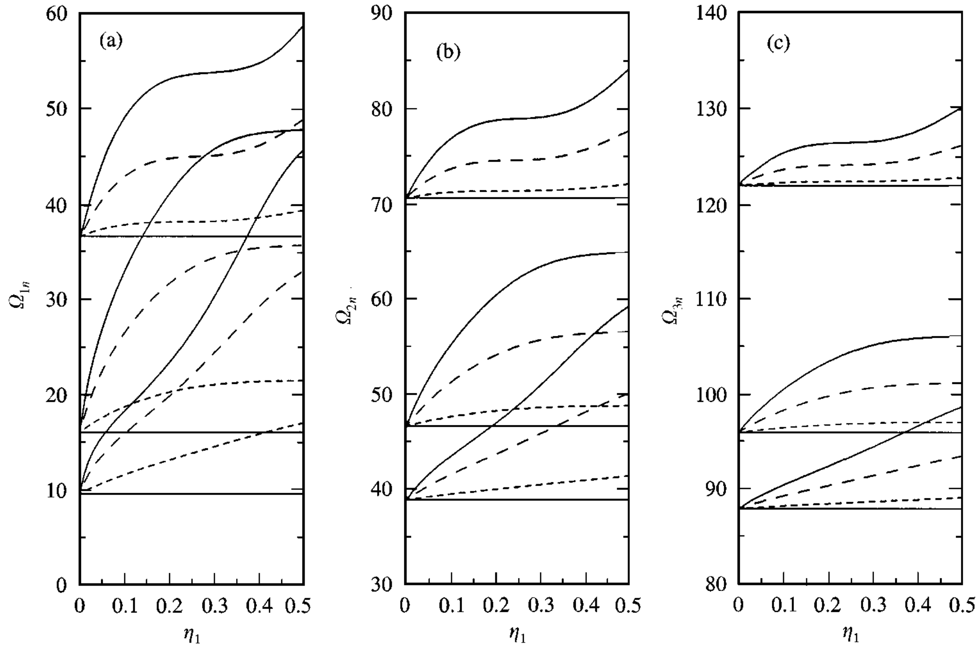

Figure 4. Frequency parameter values $\Omega_{mn} = \omega_{mn} a^2 \sqrt{\rho/D}$ as functions of η_1 for the system of two square plates shown in Figure 3; - - - - $K = 100$, – – – – – $K = 500$, —— $K = 1000$.

plates of the system are connected by means of an elastic layer which occupies designated bands of the component plates. The presented approach may be applied to the system of plates connected by an arbitrary non-homogeneous elastic layer.

In the case of two identical plates the spectrum of the combined system included the eigenfrequencies of the isolated plates. The degenerate frequencies do not depend on the stiffness of the connecting layer. The plates of the system vibrating with such frequency move in the same direction. The non-degenerate frequencies of the system of identical plates are the same as for single plate supported on an elastic Winkler foundation.

The numerical examples have shown that the stiffness and the area size of the elastic layer connecting the plates significantly affect the vibration frequencies of the combined system, as it was expected.

ACKNOWLEDGMENT

This paper was supported by the State Committee for Scientific Research under the grant No. 7T07A01211.

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