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LETTERS TO THE EDITOR

REALISTIC FOLLOWER FORCES

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A cantilevered column subjected to a tangential follower force has been referred to as Beck's column, since the critical force for the column was first found by Beck [1] in 1952. However, his result and its related topics have not been appreciated by structural engineers. For example, Timoshenko [2] comments on Beck's result that "no definite conclusion can be made (as yet) regarding the practical value of this result, since no method has been devised for applying a tangential force to a column during bending". Admiringly, even now, there is a group of scientists who do not believe in the reality of tangential follower forces [3].

The origin of a follower force can be found in an end rocket thrust applied to flexible missiles and spacecraft. In this sense a follower force is a very realistic force in the field of aerospace structures engineering. Free–free beams under a follower force have been intensively studied by many authors [4, 5]. If we assume the origin of fixed co-ordinates to the free–free beam at its tip end as shown in Figure 1, then we can have a cantilevered beam subjected to a tangential follower force at its tip end. The beam on the new fixed co-ordinates is referred to as Beck's column. Thus, it is seen that Beck's column is a simplified version of the free–free beams under a follower force, though the column is not the exact model of the actual structures. The actual flexible structures have variable mass, stiffness and axial compressive stress distributions, while the column model has the corresponding uniform distributions. It is however emphasized that Beck's column keeps basically the same fundamental stability/bending flutter characteristics as the corresponding free–free beams. In order to clarify the dynamic stability of free–free beams, it is a sharp way to work on Beck's column in place of free–free beams, since the experimental verification can be done on the cantilevered column, while it is very difficult to do so on the free–free beams.

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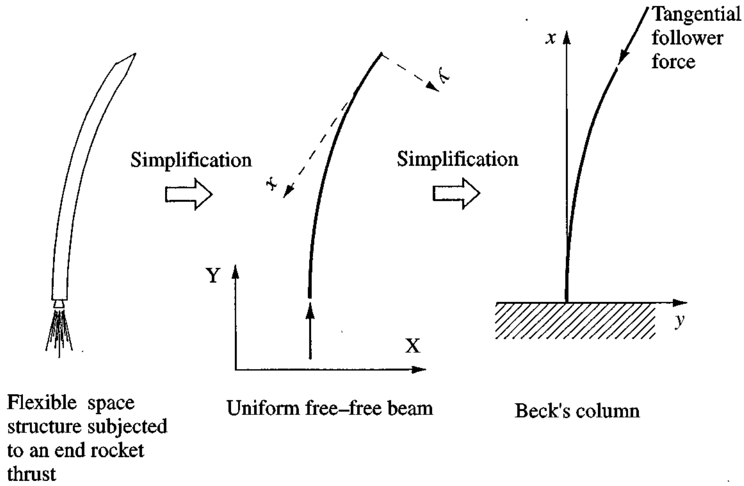

Figure 1. Modeling of a flexible structure under an end rocket thrust.

Beck's column and modified Beck's columns have been investigated by a vast body of researchers. The first review of this branch of applied mechanics has been made in a book form by Bolotin [6]. So far a lot of interesting results on the effect of follower forces have been drawn through analytical studies based on some mathematical models. However, it has been the case that some of the results are confusing because the mathematical models on which the results were brought are greatly and sometimes unduly simplified [7]. Hence, it has been suggested by several authors [8] that the stability problems of non-conservative systems should be studied in connection with the corresponding engineering practice and that theoretical results on non-conservative systems must be checked by experiments.

As to the experimental demonstrations of non-conservative systems, Herrmann and his colleagues [9] made the first systematic demonstrations of the system. First review of the experimental studies on instability of elastic systems subjected to non-conservative forces was reported by Sugiyama and Sekiya [10]. However, because of the recent fashion of computer-oriented way of having many papers, to one's credit, a thorough experimental study of the tangential follower force problem has not been conducted.

Under these circumstances, the intended aim of this letter is to show the scientists who do not believe in the reality of follower forces, the recent advances of non-conservative stability problems. Most importantly, this letter would first of all like to show them the experimental demonstrations of dynamic stability of cantilevered columns subjected to a follower force. The key idea to realize a follower force is to mount a real small-sized solid rocket motor to a cantilevered column as its tip end. Thus, a follower force can be realized as a rocket thrust applied to the cantilevered column. Three experiments have been reported by Sugiyama and his collaborators [11–13]. The first experiment was aimed at showing experimental demonstration of the flutter caused by a rocket thrust [11]. The second was to verify the quantitative theoretical prediction of flutter force [12].

The third was to demonstrate the stabilizing effect of the follower thrust on the dynamics of a vertical cantilevered column [13].

These experimental models have implied that in place of Beck's column a cantilevered column having a rigid body at its tip may be considered as a more realistic/standard non-conservative model [14]. It has been found so far that if the optimum shape of Beck's column for the maximum flutter force is searched, the flutter force of the optimized column can be nearly 9 times the force of the original Beck's column [15]. The former seems unrealistic. On the other hand, if the shape optimization of the Beck-type column having a tip rigid body, i.e., a more realistic non-conservative model, is conducted, a moderate and reasonable maximum flutter force is found [16].

As to the reality of a follower force by itself, it is better left unsaid. However, for scientists who cannot believe in the concept of follower force, it is worthwhile to mention the cantilevered pipe conveying fluid [17, 18]. Momentum flux discharged from the free end of the pipe makes a follower thrust. Even the experiment of flutter caused by a follower force can be conducted easily at a swimming pool when a long hose from a tap is immersed in water and the tap is opened full to supply water to the pool. The long hose can dance in the pool to show flutter motion due to a follower force.

One of the interesting topics in non-conservative stability problems has been the destabilizing effect of damping [6]. It is noted that this effect has not been verified experimentally. The destabilizing effect is obtained when the asymptotic stability condition is applied to non-conservative systems. The asymptotic stability condition implies that the dynamical system is unstable if the amplitude of the disturbed motion of the system becomes infinite as the time goes to infinity. However, it is of vital importance to watch the behavior of complex eigenvalues in case of the destabilizing effect of damping. It is found that the most dangerous eigenvalue mostly runs parallel to the imaginary axis. The growth-rate of the amplitude of the motion is thus very small indeed. Mathematically, the asymptotic stability criterion gives the theoretical flutter limit. However in practice, a follower force caused by a rocket motor can act upon elastic structures only for a finite time interval. In the latter case, the theoretical flutter limit obtained by neglecting damping can predict the experimental flutter limit [11, 12].

Finally, it is also worthwhile to note that the follower force is not caused only by jet or rocket thrust. Considering automobile disk brakes, for example, the "squeezing" force acting on the rotating disk is again a non-conservative follower force. This force is caused by dry friction and the squeal is a flutter-type instability. The onset of brake squeal is completely equivalent to the passage through the stability boundary in Beck's column [19, 20].

It is now obvious that the concept of follower force is very important not only in aerospace engineering, but also in automobile engineering, and certainly also in the area of fluid–structures interaction technologies.

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