2. Y. KUBOTA, S. SEKIMOTO and E. H. DOWEL 1990 *Journal of Sound and Vibration* **138**, 321–330. The high-frequency response of a plate carrying a concentrated mass.
3. CREMER L., HECKL M. and UNGAR E. E. 1988 *Structure-borne Sound*. Berlin: Springer-Verlag, second edition.
4. R. H. LYON 1975 *Statistical Energy Analysis of Dynamical Systems*. Cambridge, MA:MIT Press.
5. C. H. HODGES and J. WOODHOUSE 1986 *Rep. Progress Physics* **49**, 107–170. Theories of noise and vibration transmission in complex structures.
6. F. J. FAHY 1994 *Transaction of the Royal Society of London, Series A* **346**, 431–447. Statistical energy analysis: a critical overview.
7. P. W. SMITH Jr. 1979 *Journal of the Acoustical Society of America* **65**, 695–698. Statistical models of coupled dynamical systems and the transition from weak to strong coupling.
8. B. R. MACE 1993 *Journal of Sound and Vibration* **166**, 429–461. The statistical energy analysis of two continuous one-dimensional subsystems.
9. S. FINNVEDEN 1995 *Journal of Sound and Vibration* **187**, 495–529. Ensemble averaged vibration energy flows in a three element structure.
10. A. J. KEANE and W. G. PRICE 1987 *Journal of Sound and Vibration* **117**, 363–386. Statistical energy analysis of strongly coupled systems.
11. R. H. LYON 1969 *Journal of the Acoustical Society of America* **45**, 545–565. Statistical analysis of power injection in structures and rooms.

AUTHOR'S REPLY

EARL H. DOWELL AND DEMAN TANG

Duke University, Durham, NC 27708-0300, *U.S.A.*

(*Received* 16 *February* 1999)

The authors appreciate the opportunity to comment on the interesting work described in the letter by Dr. Finnveden [1].

First of all, we would like to note the areas of agreement between the results of Dr. Finnveden and ours [2, 3]. Both, the analysis of Dr. Finnveden and ours deduce the same formula and hence the same results for the spatially average mean-square response of the plate *per se* and find that it does not depend on the characteristics of the spring/mass attachment no matter what the mass or stiffness of the attachment. This is a remarkable result which has also been confirmed by experiment for the special case of a very stiff spring in reference [3].

The principal, indeed the only, disagreement between the results of Dr. Finnveden and our own is regarding the response of the spring/mass attachment itself. This issue is a subtle one for both AMA and SEA, for it is clear that the basic premise of many oscillatory modes in the frequency interval of interest (say 1/3 octave) cannot be satisfied by the single-degree-of-freedom spring/mass oscillator *per se*. The plate itself, of course, can and usually does have several modes in the frequency interval of interest.

It is for this reason, perhaps, that our two approaches diverge. Dr. Finnveden in fact derives two distinct SEA models to describe the spring/mass response. In the first, what he calls a two-element model, he assumed that the spring/mass as well as the plate can be treated as separate distinct SEA element. He then models their coupling, though he notes that

The conditions for the validity of the hypothesis (10) [*regarding the coupling between the plate and the spring/mass*] *and hence SEA, are not fully understood yet. However, much research and experience have given guidelines. Some of these, being relevant for the present discussion, are*:

(1) *Each substructure must have resonances within the considered frequency band since SEA describes resonant and free wave motion.*
(2) *The modal overlap factor, i.e., the ratio of resonance bandwidth to the average frequency spacing of resonances, must not be too small. For reverberant systems, coupling power is substantial only when modes in connected systems have roughly the same frequency. The probability of resonant interaction may be limited when the modal overlap is small and hence there may be large deviation between the SEA expectation of coupling power and the actual value for a particular structure.*
(3) *A structure suitable for SEA is irregular and randomly excited because 'the essential condition is incoherence between different components of response—either the modal response or, in ray theoretical formulations, components that have travelled different paths to the same point'* [7].
(4) *Coupling must not be too strong. It is believed that coupling is weak if the modal behavior of a substructure is not much altered when it is connected to the rest of the structure* [10].

And indeed the two-element model does not perform as well as the alternative single-element model that Dr. Finnveden also derives using what he terms

SEA inspired 'standard' methods.

In this single-element model he takes the plate response as known from the SEA result for the spatially averaged response and implicitly assumes the local response of the plate at which the spring/mass is attached to be the same as that response. With the plate response known, the spring/mass response is determined by standard methods. For details, see Finnveden where he ingeniously invokes a term for equivalent damping or dissipation based upon an ensemble averaging of mobilities inspired by SEA.

In general, the single-element SEA model gives better results than the two-element SEA model and also AMA for the two examples considered by Dr. Finnveden. The first example is the one treated previously by us and the second is the one introduced by Dr. Finnveden which involves a heavier plate. For the first example, AMA gives reasonable results for lower frequencies, but does not appear to do well at all at higher frequencies. In the second example that trend is even more pronounced, so what can the problem be?

First of all, we note there is a prediction of AMA which has no counterpart in the SEA analysis of Dr. Finnveden. Namely that locally, at the point on the plate where the spring/mass is connected with the plate, the plate response is substantially different from the plate response averaged over the plate. More specifically, and asymptotically for many resonances in a frequency interval, the r.m.s. response of the plate at most points becomes essentially the same and our AMA analysis agrees with the SEA analysis on this result. However, for the point where the spring/mass is connected (and also the point of excitation by the way), the response of the plate is different according to AMA. AMA predicts those differences which have been confirmed by experiment [3] for the special case of a stiff spring attachment. SEA as reported by Dr. Finnveden is silent on this matter and by implication assumes the response of all points on the plate to be the same, including the point where the spring/mass is connected. This is nearly true, but not altogether. And one of the exceptional points according to AMA and as verified by experiment and classical modal analysis is precisely where the spring/mass is attached to the plate.

Interestingly, the results for Dr. Finnveden's second example are close to those predicted by AMA (or vice versa) for the ratio of the spring/mass response to the plate response *at the point of connection.*

Indeed AMA predicts two results which correspond to Dr. Finnveden's expectations. These are the following.

(1) The ratio of spring/mass response to plate response at the connection point for the mean-square values does go as ω^4 for $\omega \to \infty$. This is the ratio of equation (27_b) to equation (30_c) in references [2]. More generally, the ratio of the squares of these quantities varies as $1/(1 - \omega^2/\omega_0^2)^2$. Note this is neither a SEA or AMA result *per se*, but is a classical result. In addition, AMA tells us what happens to the plate when many resonant modes are excited including the distinction between the local response at the point of excitation and the point where the spring/mass is attached.

(2) Also, the spring–mass system behaves like a vibration absorber in that the plate response at the point of connection tends to zero when $\omega_c \to \omega_0$. See equation (30_c) of reference [2]. Note however, that at all other points on the plate the response is unaffected by the spring/mass oscillator when the number of resonant modes of the plate is large in the frequency range of interest, i.e., we approach the AMA (and SEA) limit.

All this still leaves unanswered why Dr. Finnveden's results are different from those of ours, but perhaps further study will reveal the reason. We hope our comments here will help in resolving this question.

REFERENCES

1. S. FINNVEDEN *Journal of Sound and Vibration.* Modal the high-frequency response of a plate carrying a concentrated mass/spring system **224**, 783–798.
2. E. H. DOWELL and D. TANG 1998 *Journal of Sound and Vibration* **213**, 843–863. The high-frequency response of a plate carrying a concentrated mass/spring system.
3. Y. KUBOTA and E. H. DOWELL 1986 *Journal of Sound and Vibration* **106**, 203–216. Experimental investigation of asymptotic modal analysis for a rectangular plate.