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SOUND RADIATION FROM PERIODICALLY SPRING-SUPPORTED BEAMS UNDER THE ACTION OF A CONVECTED UNIFORM HARMONIC LOADING

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The main purpose of this paper is to investigate the effects of periodic spring supports on the acoustic response of an infinite, fluid-loaded beam subject to a harmonic moving line force. The mechanics of a fluid-loaded beam with periodic spring supports was formulated based on the Timoshenko beam theory. The main focus is to examine the effects of the spring support spacing and the travelling line force speed on the radiated sound power. The effects of the spring supports on the radiated sound power decrease as the excitation frequency and the Mach number of the travelling line force increase. When the external line force is stationary and acoustically small, it is interesting to find that the radiated sound power is seen to exhibit peaks at certain low wavenumber ratios except when the length of the external force is an integral multiple of the spring support spacing. It is shown that the locations of these peaks coincide with the lower bounding wavenumber ratios of the odd number of propagation zones. Under the circumstances, the incomplete pressure equalization is responsible for the fluctuations in radiated sound power. However, when the line force is moving, the radiated sound power fluctuates more rapidly at low wavenumber ratios even through the length of the external line force is an integral multiple of the support spacing.

1. INTRODUCTION

The vibroacoustic response of periodic structures, such as beam on equally spaced supports or plates with equally spaced rib stiffeners has received much attention in the past years [1–11]. The propagation constant [1] is used to define the so-called propagation zones and attenuation zones by examining the phase and amplitude variations between two adjacent sections. If the external forces have frequency and wavenumber components that coincide with those of the free propagating waves, the contributions from those components to the response will be greatest [3]. For different types of supports, Mead [5] has shown that the beam on flexible supports has two propagation constants for each frequency instead of one on fixed supports. He also demonstrated that the subsonic convected pressure field could generate supersonic flexural waves that radiate sound. For a plate with periodic rib stiffeners, Mead [10] further indicated that the acoustic radiation is fundamentally

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determined by the periodic geometry. Cray [11] has studied the nearfield and farfield sound radiation from a line-driven fluid-loaded infinite flat plate having periodic and non-periodic attached rib stiffeners, and shown that excitation frequencies below coincidence can generate large magnitude of supersonic wavenumber components, which imply better sound radiation. He has also shown that the radiated sound pressure depends not only on the rib sizes but also on the rib spacing. It is the rib spacing that determines whether the stiffened plate radiates sound strongly or weakly.

Although the vibroacoustic responses of periodic structures have received much attention for many years, excitation to the structures in most research has however been restricted to the stationary, harmonic forces and convected pressure. This paper examines the effects of periodic spring supports on the radiated sound power of an infinite, fluid-loaded beam subject to a harmonic line force moving at subsonic speed. This paper extends the research performed by Keltie [12], who solved the same specific model but for a beam without periodic supports. The sound power is formulated first as a function of the support spacing and Mach number. Then the relation between the radiates sound power and support spacing and the relation between the radiated sound power and Mach number is discussed.

2. FORMULATION

Consider an infinite, homogeneous, elastic beam lying on the plane ($y = 0$) with periodically, equally spaced spring supports attached to the beam as shown in Figure 1. An acoustic medium is filled above the beam ($y > 0$), and there is a vacuum under the beam ($y < 0$). This beam is excited by a uniform harmonic loading over the length $2L$, frequency ω, moving with subsonic speed, V. The vibration equation for a Timoshenko beam is given by Junger and Feit [13]:

$$\bar{E}I\frac{\partial^4 u(x,t)}{\partial x^4} + \rho_v h\frac{\partial^2 u(x,t)}{\partial t^2} - \left(\rho_v I + \frac{\bar{E}I\rho_v}{\kappa^2\bar{G}}\right)\frac{\partial^4 u(x,t)}{\partial x^2\partial t^2} + \rho_v I\frac{\rho_v}{\kappa^2\bar{\bar{G}}}\frac{\partial^4 u(x,t)}{\partial t^4}$$

$$= \left(1 - \frac{\bar{E}I}{\kappa^2\bar{G}h}\frac{\partial^2}{\partial x^2} + \frac{\rho_v h^2}{12\kappa^2\bar{G}}\frac{\partial^2}{\partial t^2}\right)[f(x,t) - p(x, y=0, t) - p_1(x,t)]. \qquad (1)$$

In Equation (1), $u(x, t)$ represents the displacement of the beam, I the cross-sectional second moment of area per unit width, ρ_v the density of the beam, κ^2 the cross-sectional shape factor, $f(x, t)$ the external moving line force, $p(x, y = 0, t)$ the acoustic pressure acting on the beam surface and $p_1(x, t)$ the force from the spring supports. $\bar{E}$ and $\bar{G}$ are the complex elastic and shear modulus, respectively,

$$\bar{E} = E(1 + \mathrm{j}\eta),$$

$$\bar{G} = \frac{\bar{E}}{2(1+v)},$$

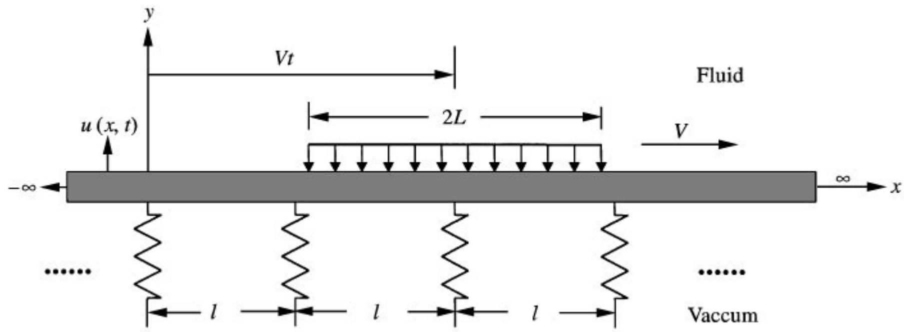

Figure 1. Schematic representation of problem geometry

where η is the structural damping and v is the Poisson ratio. The moving line force $f(x, t)$ is given by

$$f(x,t) = \frac{F_0}{2L}\left[H(x - Vt + L) - H(x - Vt - L)\right] \mathrm{e}^{\mathrm{j}\omega t}, \tag{2}$$

where $H(x)$ is Heavyside step function and F_0 is the external force strength per unit width. The force $p_1(x, t)$ from the periodic spring supports is given by

$$P_1(x,t) = \sum_{n=-\infty}^{\infty} k_s u(x,t)\delta(x - nl), \tag{3}$$

where k_s and l are the stiffness of the spring support per unit width and the spacing between two adjacent supports, respectively. The pressure $p(x, y = 0, t)$ acting on the beam surface satisfies the Helmholtz equation which is given by

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{C_0^2}\frac{\partial^2}{\partial t^2}\right) p(x,y,t) = 0, \tag{4}$$

where C_0 is the speed of the sound in the acoustic medium. The boundary condition at $y = 0$ is given by

$$\rho_0 \frac{\partial^2 u}{\partial t^2} = -\left.\frac{\partial p}{\partial y}\right|_{y=0}, \tag{5}$$

where ρ_0 is the density of the acoustic medium. After applying the spatial Fourier transformation to the external moving line force, equation (2) becomes

$$\tilde{F}(\xi,t) = F(\xi)\mathrm{e}^{\mathrm{j}(\xi V + \omega)t}, \tag{6}$$

where

$$F(\xi) = F_0 \frac{\sin(\xi L)}{\xi L}.$$

For a steady-state response, equation (6) implies that the transformed displacement $\tilde{U}(\xi, t)$, sound pressure $\tilde{P}(\xi, y, t)$ and periodic spring force $\tilde{P}_1(\xi, t)$ will have the common factor, $e^{j(\xi V+\omega)t}$ in a wavenumber domain:

$$\tilde{U}(\xi, t) = U(\xi)e^{j(\xi V+\omega)t}, \tag{7a}$$

$$\tilde{P}(\xi, y, t) = P(\xi, y)e^{j(\xi V+\omega)t}, \tag{7b}$$

$$\tilde{P}_1(\xi, t) = P_1(\xi)e^{j(\xi V+\omega)t}. \tag{7c}$$

By applying the spatial Fourier transformation to equation (3),

$$\tilde{P}_1(\xi, t) = k_s \sum_{n=-\infty}^{\infty} u(nl, t)\, e^{j(nl)\xi}, \tag{8}$$

Introduce a new transformation in terms of the variable ζ defined as

$$u(nl, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{u}(\zeta, t)\, e^{-j\zeta(nl)}\, d\zeta. \tag{9}$$

Substituting equation (9) into equation (8) yields the spring force:

$$\tilde{P}_1(\xi, t) = \frac{k_s}{2\pi}\int_{-\infty}^{\infty} \tilde{u}(\zeta, t) \sum_{n=-\infty}^{\infty} e^{jnl(\xi-\zeta)}\, d\zeta. \tag{10a}$$

Applying the Poisson summation formula [14] to equation (10a), the latter is rewritten as

$$\tilde{P}_1(\xi, t) = \frac{k_s}{l}\sum_{n=-\infty}^{\infty} U\left(\xi + \frac{2\pi n}{l}\right) e^{i[(\xi+2n\pi/l)V+\omega]t}. \tag{10b}$$

For the special case when $Vt = nl$, the spring force can be expressed by

$$\tilde{P}_1(\xi, t) = \frac{k_s}{l}\sum_{n=-\infty}^{\infty} U\left(\xi + \frac{2\pi n}{l}\right) e^{i[(\xi)V+\omega]t}. \tag{11}$$

After applying spatial Fourier transformation to equations (1) and (4), and substituting equation (7) into transformed equations (1) and (4), the transformed equation (1) is rewritten as

$$Z_m U(\xi) = Z_F F(\xi) - Z_F Z_a U(\xi) - Z_F \frac{k_s}{l}\sum_{n=-\infty}^{\infty} U\left(\xi + \frac{2\pi n}{l}\right), \tag{12}$$

where

$$Z_m = \bar{E}I\xi^4 - \rho_v h(\xi V+\omega)^2 - \left(\rho_v I + \frac{\bar{E}I\rho_v}{\kappa^2\bar{G}}\right)\xi^2(\xi V+\omega)^2 + \rho_v I\frac{\rho_v}{\kappa^2\bar{G}}(\xi V+\omega)^4,$$

$$Z_F = 1 + \frac{\bar{E}I}{\kappa^2 h\bar{G}}\xi^2 - \frac{\rho_v h^2}{12\kappa^2\bar{G}}(\xi V+\omega)^2,$$

$$Z_a = \frac{j\rho_0(\xi V+\omega)^2}{K_y},$$

$$K_y = \begin{cases} \sqrt{(\xi M + k_0)^2 - \xi^2} & \text{for } \xi^2 < (\xi M + k_0)^2, \\ -j\sqrt{\xi^2 - (\xi M + k_0)^2} & \text{for } \xi^2 > (\xi M + k_0)^2, \end{cases}$$

with $k_0\ (=w/c_0)$ is the acoustic wavenumber and $M\ (=V/C_0)$ is the Mach number. Equation (12) is expressed in a similar form given by Keltie [12] except for the last term that results from the equally spaced spring supports. Rewrite equation (12) using notations as

$$U(\xi) = R(\xi) - K(\xi) \sum_{n=-\infty}^{\infty} U\left(\xi + \frac{2\pi n}{l}\right), \tag{13}$$

where

$$R(\xi) = \frac{F(\xi)}{Z_B + Z_a},$$

$$K(\xi) = \frac{1}{Z_B + Z_a} \frac{k_s}{l},$$

$$Z_B = Z_m / Z_F.$$

In order to find the explicit solution of the displacement response in equation (13), three notations are defined as follows:

$$K_0(\xi) \equiv \sum_{n=-\infty}^{\infty} K\left(\xi + \frac{2\pi n}{l}\right), \tag{14a}$$

$$R_0(\xi) \equiv \sum_{n=-\infty}^{\infty} R\left(\xi + \frac{2\pi n}{l}\right), \tag{14b}$$

$$U_0(\xi) \equiv \sum_{n=-\infty}^{\infty} U\left(\xi + \frac{2\pi n}{l}\right), \tag{14c}$$

If the range of summation index n in equations (14a–c) is from negative infinite to positive infinite, each of the equations has the key property [11]

$$G_0\left(\xi + \frac{2\pi n}{l}\right) = G_0(\xi). \tag{14d}$$

Using the relations shown in equations (14a–c), equation (13) is rewritten as

$$U(\xi) = R(\xi) - K(\xi) U_0(\xi). \tag{15}$$

Letting $\xi = \xi + 2\pi m/l$ in equation (15) and summing both sides of this equation over all m, equation (15) becomes

$$\sum_{m=-\infty}^{\infty} U\left(\xi + \frac{2\pi m}{l}\right) = \sum_{m=-\infty}^{\infty} R\left(\xi + \frac{2\pi m}{l}\right) - \sum_{m=-\infty}^{\infty} K\left(\xi + \frac{2\pi m}{l}\right) U_0\left(\xi + \frac{2\pi m}{l}\right). \tag{16}$$

Upon substituting equation (14d) into equation (16), equation (16) can be simplified as

$$U_0(\xi) = \frac{R_0(\xi)}{[1 + K_0(\xi)]}. \tag{17}$$

Substituting Equation (17) into Equation (15) yields the explicit solution of the displacement response

$$U(\xi) = R(\xi) - K(\xi)\frac{R_0(\xi)}{1 + K_0(\xi)}. \tag{18}$$

From equation (18), the displacement response is made of two parts. The first term represents the displacement contribution from the beam without periodic spring supports. The second term results from the addition of periodically elastic supports. By integrating the surface acoustic intensity over the entire beam, the radiated sound power per unit width, II, is given by [12]

$$\prod = \frac{\rho_0}{4\pi}\int_{\xi_1}^{\xi_2} \frac{(\xi V + \omega)^3}{K_y} |U(\xi)|^2 \, d\xi, \tag{19}$$

where

$$\xi_1 = \frac{-k_0}{1+M} \leqslant \xi \leqslant \frac{k_0}{1-M} = \xi_2.$$

It is convenient to present numerical results in terms of dimensionless parameters. The dimensionless radiated sound power W per unit width is defined as [12]

$$W = \frac{4\pi\omega(\rho_v h)^2}{\rho_0 F_0^2} \prod. \tag{20a}$$

Free bending wavenumber k_B:

$$k_B = \left[\frac{12\rho_v \omega^2}{Eh^2}\right]^{1/4}. \tag{20b}$$

Longitudinal wave speed C_L:

$$C_L = \sqrt{\frac{E}{\rho_v}}. \tag{20c}$$

Fluid loading factor α_0:

$$\alpha_0 = \frac{\rho_0 C_L}{\sqrt{12}\rho_v C_0}. \tag{20d}$$

Wavenumber ratio γ:

$$\gamma = \frac{k_0}{k_B}. \tag{20e}$$

Stiffness ratio S:

$$S = \frac{k_s}{E}. \tag{20f}$$

Substituting equation (20) into equation (19), the dimensionless radiated sound power per unit width is expressed as

$$W = \int_{\xi_1}^{\xi_2} \alpha^3 \beta \left| Z_f \frac{\sin(\zeta k_0 L)}{\zeta k_0 L} - Z_f \frac{R'_0(\zeta)}{1 + K'_0(\zeta)} \right|^2 |D|^{-2} \, \mathrm{d}\zeta, \tag{21a}$$

where

$$\zeta_1 = \frac{-1}{1+M} \leqslant \zeta \leqslant \frac{1}{1-M} = \zeta_2,$$

$$\alpha = 1 + M\zeta,$$

$$\beta = \sqrt{\alpha^2 - \zeta^2},$$

$$Z_f = 1 + \frac{2(1+v)\gamma^4}{\kappa^2} \left(\frac{C_0}{C_L}\right)^2 \left[\zeta^2 - \frac{1}{(1+\eta \mathrm{j})} \left(\frac{C_0}{C_L}\right)^2 \alpha^2\right],$$

$$D = \beta(D_1 - D_2 + D_3) + \mathrm{j} D_4,$$

$$D_1 = \gamma^4 \zeta^4 (1 + \eta \mathrm{j}),$$

$$D_2 = \alpha^2 \left\{1 + \left[1 + \frac{2(1+v)}{\kappa^2}\right] \gamma^4 \zeta^2 \left(\frac{C_0}{C_L}\right)^2\right\},$$

$$D_3 = \frac{2(1+v)}{\kappa^2 (1+\eta \mathrm{j})} \alpha^2 \gamma^4 \left(\frac{C_0}{C_L}\right)^4,$$

$$D_4 = Z_f \frac{\alpha_0 \alpha^2}{\gamma^2},$$

$$R'_0(\zeta) = \frac{S}{4\sqrt{3}\gamma^2} \left(\frac{C_L}{C_0}\right)^3 \frac{(2L/l)}{(k_0 L)} \times \sum_{n=-\infty}^{\infty} \frac{Z_f(\zeta + (n\pi/k_0 L)(2L/l))\, \beta(\zeta + (n\pi/k_0 L)(2L/l))}{D(\zeta + (n\pi/k_0 L)(2L/l))} \times \frac{\sin[(k_0 L \zeta + n\pi (2L/l))]}{(k_0 L \zeta + n\pi (2L/l))}, \tag{21b}$$

$$K'_0(\zeta) = \frac{S}{4\sqrt{3}\gamma^2} \left(\frac{C_L}{C_0}\right)^3 \frac{(2L/l)}{(k_0 L)} \sum_{n=-\infty}^{\infty} \frac{Z_f(\zeta + (n\pi/k_0 L)(2L/l))\, \beta(\zeta + (n\pi/k_0 L)(2L/l)}{D(\zeta + (n\pi/k_0 L)(2L/l))} \tag{21c}$$

3. ANALYSIS PARAMETERS

Numerical examples will now be presented to illustrate some features of the theoretical results. Properties of the specific beam model analyzed are as follows: $E = 20 \times 10^{10}\,\mathrm{Nt/m^2}$, $\rho_v = 7800\,\mathrm{kg/m^3}$, $h = 2.54 \times 10^{-2}$ m, $v = 0.3$, $\kappa^2 = 0.85$ and $\eta = 0.01$. The beam is assumed to be submerged in water ($C_0 = 1481\, m/s$, $\rho_0 = 1000$ kg/m) or in air ($C_0 = 343$ m/s, $\rho_0 = 1.27$ kg/m). The spatial extent of the moving line force is assumed to be acoustically small and fixed at the value $k_0L = 0.1$ and the stiffness ratio S is assumed to be 1. In order to demonstrate the effects of the speed of the travelling force on radiated sound power, three different Mach numbers, $M = 0$, 0.25 and 0.5, were chosen.

4. NUMERICAL RESULTS AND DISCUSSION

For a stationary, external line force with the acoustic length $k_0L = 0.1$, the non-dimensional radiated sound power is plotted in Figure 2 for an unsupported beam and beams with two different spring support spacings, $2L/l = 2/\pi$ and 1, respectively. From equations (21b, c) and Figure 2, it is easily shown that the influence of spring supports on the radiated sound power decreases while the wavenumber ratio γ increases. Note that the non-dimensional frequency Ω is defined as

$$\Omega = \left(\frac{\omega^2 \rho_v A l^4}{EI}\right)^{1/2}.$$

The relation between wavenumber ratio γ and non-dimensional frequency Ω is given by

$$\gamma = \frac{2k_0L}{(2L/l)\sqrt{\Omega}}.$$

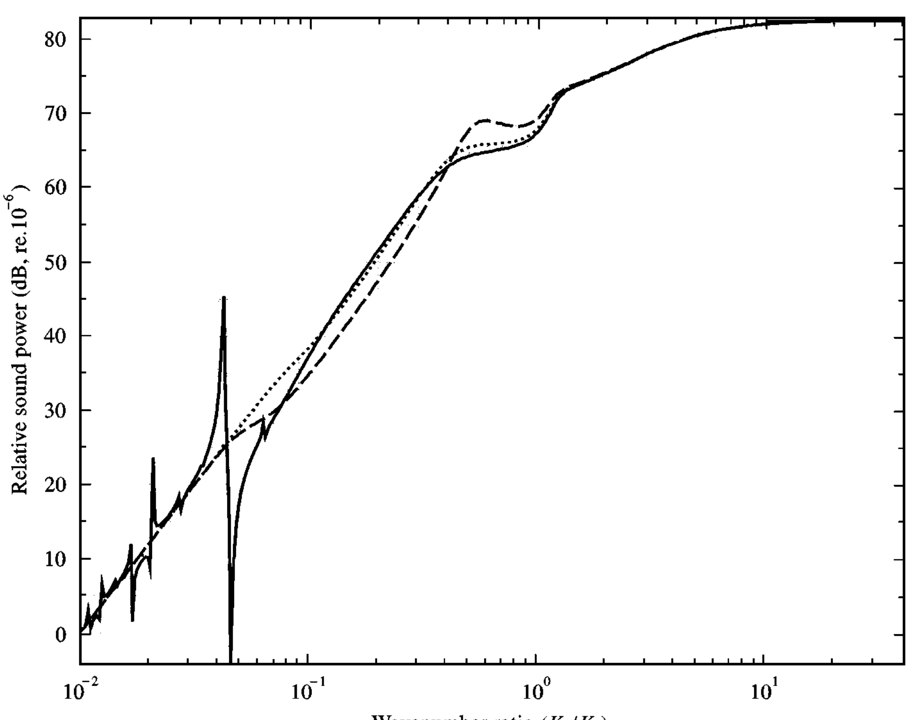

Figure 2. Relative sound power level versus wavenumber ratio for a range of $2L/l$ value, $K_0L = 0.1$, $M = 0$, in water. —— $2L/l = 2/\pi$; .... $2L/l = 1$; ------ unsupported.

For the beam with spring support spacing $2L/l = 2/\pi$, it is interesting to find that the radiated sound power shows pronounced peaks at certain wavenumber ratios. Nonetheless, all these peaks occur at low wavenumber ratios (i.e. $\gamma < 0.1$). Note that there is no peak in the radiated sound power for either an unsupported beam or a beam with spring support spacing $2L/l = 1$. These peaks are related to the nature of the wave propagation and attenuation in a periodically supported structure. Explanations for the reported phenomenon will be discussed later.

The non-dimensional radiated sound power is plotted in Figure 3 for another three different spring support spacings, $2L/l = 2$, 4 and 24. From the general trend indicated by these results, it is clear that there is no fluctuation in radiated sound power at the wavenumber ratio of less than 0.1 whenever $2L/l$ is an integer.

Figure 3 is typical of the results where the addition of spring supports increases the total structural impedance that results in less vibroacoustic response. This is true especially at low wavenumber ratios. Comparing Figure 2 with Figure 3, the peak near the wavenumber ratio $k_0/k_B = 1$ emerges when the ratio $2L/l$ increases from $2L/l = 2$ to 24. With this ratio $2L/l$ increases the system behaves more like an infinite beam with elastic foundation, and the emerging peak near $k_0/k_B = 1$ shifts towards the higher wavenumber ratio. A discussion of this phenomenon will follow.

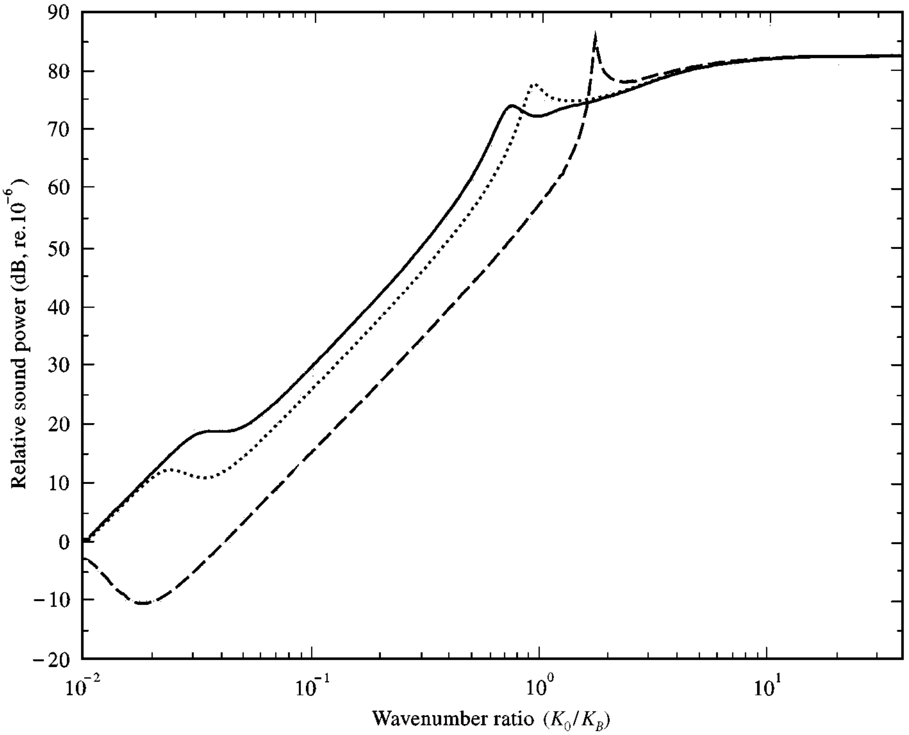

Figure 3. Relative sound power level versus wavenumber ratio for a range of $2L/l$ values, $K_0L = 0.1$, $M = 0$, in water. —— $2L/l = 2$; .... $2L/l = 1$; – – – – – – $2L/l = 24$.

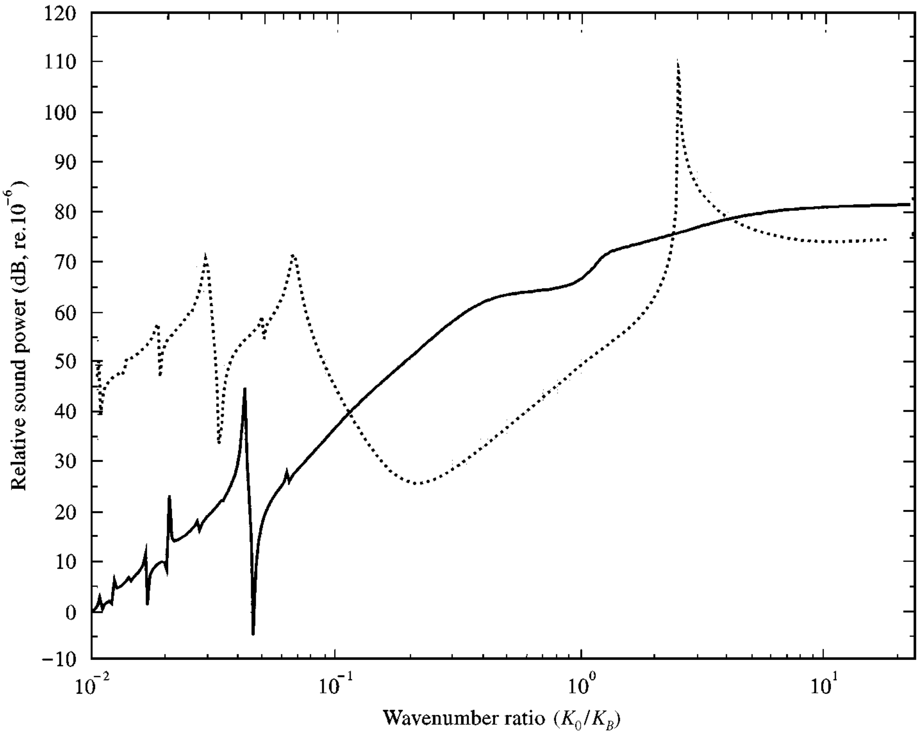

Figure 4. Relative sound power level versus wavenumber ratio $2L/l = 2/\pi$, $K_0L = 0.1$, $M = 0$. —— In water; .... in air.

In order to demonstrate the fluid loading effects on the acoustic radiation, Figure 4 shows the non-dimensional-radiated sound power for a water-loaded and an air-loaded, periodically spring-supported beam excited by a stationary, harmonic line force with $k_0L = 0.1$ and $2L/l = 2/\pi$, respectively. When compared with Figure 2, both cases show sound power fluctuations at low wavenumber ratios. Another immediately noticeable feature is the pronounced peak at wavenumber ratio $\gamma = 2.3$ for the air-loaded, periodic spring-supported beam. Note that there is no pronounced peak for the case of water-loaded beam near wavenumber ratio $\gamma = 1$. This would be expected as the dense medium has a very large damping action on the beam at this wavenumber ratio. It greatly attenuates the peak response.

In an attempt to clearly explain why there is no fluctuation in radiated sound power at low wavenumber ratios whenever the length of external distributed line force is an integral multiple of the support spacing, some simplifications for equations (21a–c) are derived. For the case of the length of external distributed line force being equal to an integral multiple of the support spacing, equation (21a) is rewritten as

$$W \sim \int_{\zeta_1}^{\zeta_2} \alpha^3 \beta \left| Z_f \sin(\zeta k_0 L) \left[\frac{1}{\zeta k_0 L} - \frac{N_0'(\zeta)}{1 + K_0'(\zeta)} \right] \right|^2 |D|^{-2} \, d\zeta \qquad (22)$$

where

$$N'_0(\zeta) = \frac{S}{4\sqrt{3}\gamma^2}\left(\frac{C_L}{C_0}\right)^3 \frac{m}{(k_0L)} \sum_{n=-\infty}^{\infty} (-1)^{nm} \frac{Z_f(\zeta + (nm\pi/k_0L))\beta(\zeta + (nm\pi/k_0L))}{D(\zeta + (nm\pi/k_0L)(k_0L\zeta + nm\pi)},$$

$2L/l = m$, m is a integer.

For a great value of m and an acoustically small line force ($k_0L = 0.1$), it may be easily shown that $N'_0(\zeta)$ and $K'_0(\zeta)$ are dominated by the terms near zeroth summation value ($n = 0$) due to the great difference in magnitude of ζ between the denominator and numerator. It implies that influences of the spring support on the vibration of the beam come mainly from those close to the excitation. Under the circumstances, $N'_0(\zeta)$ and $K'_0(\zeta)$ can be approximated by the terms at $n = 0$:

$$N_0(\zeta) \sim \frac{S}{4\sqrt{3}\gamma^2}\left(\frac{C_L}{C_0}\right)^3 \frac{m}{(k_0L)} \frac{Z_f(\zeta)\beta(\zeta)}{D(\zeta)[k_0L\zeta]},$$

$$K'_0(\zeta) \sim \frac{S}{4\sqrt{3}\gamma^2}\left(\frac{C_L}{C_0}\right)^3 \frac{m}{(k_0L)} \frac{Z_f(\zeta)\beta(\zeta)}{D(\zeta)}, \quad m \geqslant 2.$$

Equation (22) can then be rewritten as

$$W \sim \int_{\zeta_1}^{\zeta_2} \alpha^3\beta \left|\frac{Z_f \sin(\zeta k_0L)}{\zeta k_0L}\right|^2 \left|D + \frac{Z_f S}{4\sqrt{3}\gamma^2}\left(\frac{C_L}{C_0}\right)^3 \frac{m}{k_0L}\sqrt{\alpha^2 - \zeta^2}\right|^{-2} \mathrm{d}\zeta \tag{23}$$

Furthermore, equation (23) can be simplified as

$$W \sim \int_{\zeta_1}^{\zeta_2} \alpha^3\beta \left|\frac{Z_f \sin(\zeta k_0L)}{D'\zeta k_0L}\right|^2 \mathrm{d}\zeta, \tag{24}$$

where

$$D' = \beta(D_1 - D_2 + D_3 + D_s) + \mathrm{j}D_4$$

and

$$D_s = \frac{Z_f S}{4\sqrt{3}\gamma^2}\left(\frac{C_L}{C_0}\right)^3 \frac{m}{k_0L}.$$

From equation (24), it shows that the radiated power is governed in the same form as that of a fluid-loaded beam with additional stiffness D_s. Hence, like a fluid-loaded beam without periodic supports, there will be no fluctuation of the radiated sound power at low wavenumber ratios. Under the circumstance, the impedance due to the fluid loading is much less than that of the structure, so a radiation peak emerges near $\gamma = 1.0$ as observed in Figure 3. It is also seen that this peak occurs at the wavenumber ratio $\gamma = 2.3$ instead of $\gamma = 1$ due to the additional stiffness and mass effect introduced by the periodic spring supports and fluid, respectively.

In order to seek a greater understanding of the effects of the spring support on the radiated sound power, the upper and lower bounding non-dimensional frequencies Ω of the first five propagation zones of a periodically simply supported beam are calculated and listed in Table 1. It should be noted that the upper and

TABLE 1

Bounding frequencies for the first five propagating zones of a periodically supported beam

Propagation zone no.	Non-dimensional frequency Ω	Wavenumber ratio γ
1	9·87	0·1
	22·4	0·066
2	39·5	0·05
	61·8	0·04
3	89	0·033
	121	0·0286
4	158	0·025
	199	0·022
5	247	0·02
	298	0·018

lower bounding frequencies for each propagation zone are obtained while the periodically simply supported beam is in a vacuum. The corresponding wavenumber ratios for the bounding frequencies of each propagation zone are also listed in Table 1. Notice that the wavenumber ratio γ is inversely proportional to the square root of non-dimensional frequency Ω. In Figure 4, the relative sound power is plotted versus the wavenumber ratio instead of the non-dimensional frequency.

As discussed previously, the total radiated acoustic power fluctuates at low wavenumber ratios, while the length of external distributed line force is acoustically small and is not an integral multiple of the support spacing. For the case of the air-loaded beam shown in dotted line in Figure 4, the wavenumber ratios at which peaks are located nearly coincide with the lower bounding wavenumber ratios of the odd number of propagation zones as listed in Table 1. The wavenumber ratio of the largest peak is $\gamma = 0.066$ which corresponds to the upper bounding frequency of the first propagation zone, i.e. $\Omega = 22.4$. The non-dimensional frequency $\Omega = 22.4$ corresponding to the wavenumber ratio $\gamma = 0.066$ is identical to the fundamental natural frequency of a single-bay beam element with its ends fully clamped. At this frequency, each single-bay element of the periodically spring-supported system moves in phase. There is neither wave propagation nor wave attenuation from bay to bay. Although the surface motion of the beam vibrates slowly below coincidence, large acoustic power will be generated due to incomplete hydrodynamic cancellation [13]. The second peak occurs at $\gamma = 0.029$ corresponding to the upper bounding frequency of the third propagation zone, i.e. $\Omega = 121$. The local pressure equalization takes place inside a single-bay element, but not between two adjacent bays due to in-phase wave motion from bay to bay. The sound power is radiated solely from the flexural near field at discontinuities, i.e. locations near spring support. At the upper bounding frequency of the second propagating zone, i.e. $\Omega = 61.8$ (or $\gamma = 0.04$), wave motions are antisymmetric about the midspan of a single-bay element so less acoustic power is generated due to pressure equalization.

If the excitation frequency is inside an attenuation zone, the free flexural waves tend to decay as they move away from the excitation to the next bay. The radiated acoustic power is generated from the bays close to the excitation. On the other hand, more bays are involved in vibration and are responsible for acoustic radiation when the wavenumber ratio is inside a propagation zone. Above the coincidence, when the surface motion is so far that the local pressure cannot be equalized, the excess pressure radiates away from the surface as sound.

The magnitude of the relative sound power at wavenumber ratio $\gamma = 0.0423$ seen in Figure 2 exceeds that from an unsupported beam by approximately 20 dB. In order to examine this enhanced radiation, the wavenumber spectrum of the periodically spring-supported beam at wavenumber ratio $\gamma = 0.0423$ with $M = 0$, $k_0 L = 0.1$, $2L/l = 2/\pi$ is displayed in Figure 5.

Nulls at $k/k_0 = -1$ and $k/k_0 = 1$ in the magnitude of the displacement response are due to the infinite acoustic impedance caused by the presence of fluid. The addition of spring supports results in coherent interference of the reflection that causes the displacement fluctuations. The locations of the peaks in the wavenumber spectrum are associated with the spring support-to-support spacing. The sound radiation at $\gamma = 0.0423$ results solely from the supersonic wavenumber components lying between $k/k_0 = -1$ and $k/k_0 = 1$, the so-called supersonic region. As can

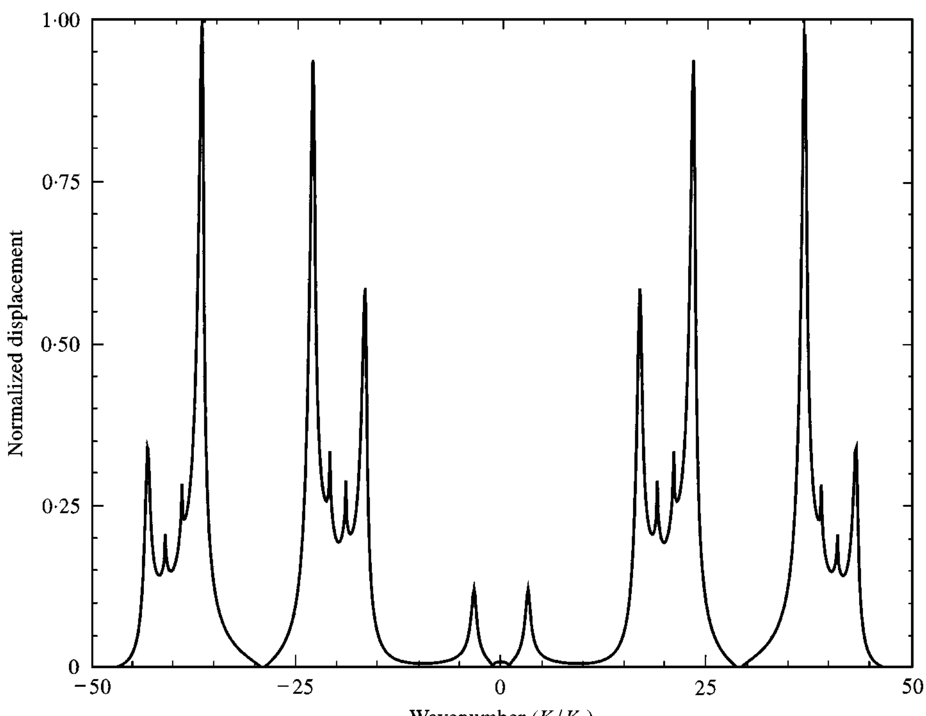

Figure 5. Normalized wavenumber spectrum at $\gamma = 0.0423$, $K_0 L = 0.1$, $2L/l = 2/\pi$, $M = 0$.

be observed from this figure, there exists an energy concentration zone inside this region.

In order to present differences in the magnitude of sound radiation due to the addition of spring supports, the normalized displacement response in the wavenumber spectrum is shown in Figure 6. In this figure, the wavenumber spectrum of the periodically supported beam has been divided by that of an unsupported beam. It clearly shows that the displacement response of the periodically supported beam in the supersonic region has been increased by a magnitude nearly an order over that of an unsupported beam. These enhanced supersonic components are responsible for the increase in radiated sound power as noted in Figure 2.

Fixing M at the values of 0, 0.25 and 0.5, the 0.5, the non-dimensional radiated sound power is plotted in Figure 7 for $k_0L = 0.1$, $2L/l = 2/\pi$. Figure 7 shows that the radiated sound power exhibits fluctuations at certain low wavenumber ratios. The amplitude of the fluctuation in radiated sound power decreases as the Mach number increases.

The non-dimensional sound power is plotted in Figure 8 for the ratio $2L/l = 2$ and three different Mach numbers, $M = 0$, 0.25 and 0.5. Compared with Figure 2, Figure 8 shows that there are fluctuations in the radiated sound power even though $2L/l$ is an integer. In addition, amplitudes of the fluctuations in the radiated sound power decrease as the Mach number increases. Locations of the peaks are different

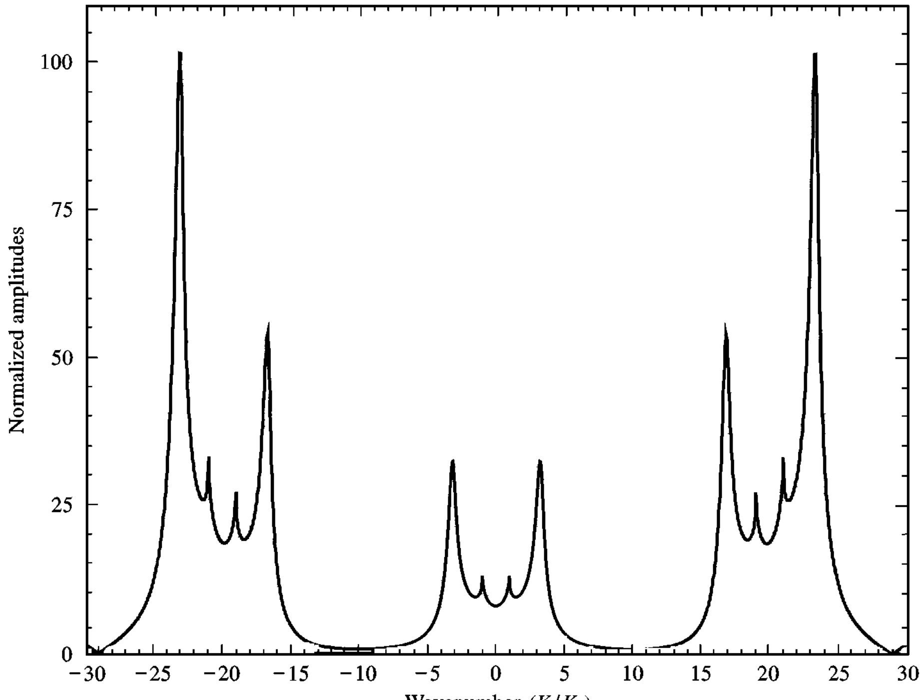

Figure 6. Ratio of wavenumber spectrum at $\gamma = 0.0423$, $K_0L = 0.1$, $2L/l = 2/\pi$, $M = 0$.

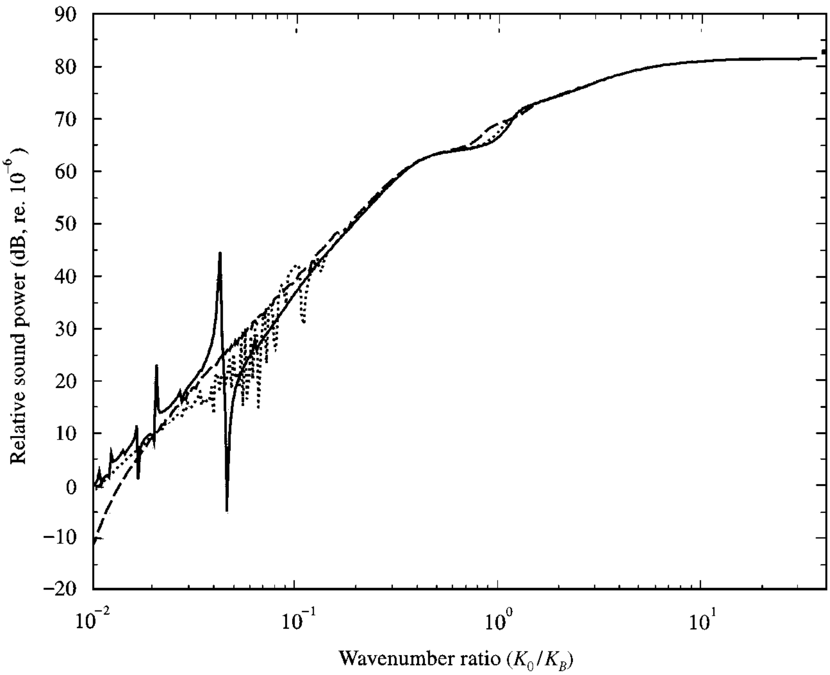

Figure 7. Relative sound power level versus wavenumber ratio for a range of M values, $K_0L = 0.1$, $2L/l = 2/\pi$, in water. —— $M = 0$; .... $M = 0.25$; - - - - - - - $M = 0.5$.

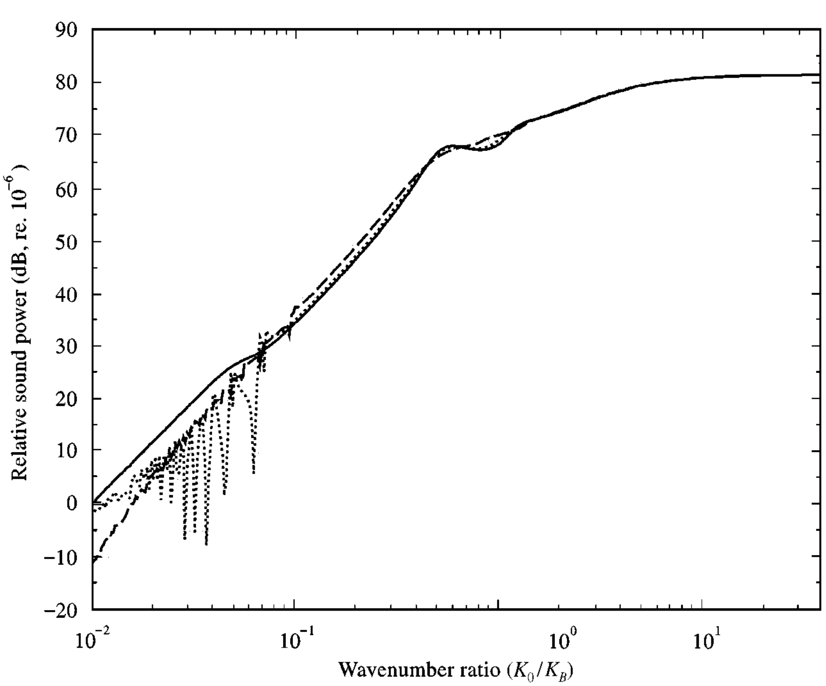

Figure 8. Relative sound power level versus wavenumber ratio for a range of M values, $K_0L = 0.1$, $2L/l = 2$, in water. —— $M = 0$; .... $M = 0.25$; - - - - - - - $M = 0.5$.

from those of a beam subject to a stationary line force. The exact locations of these peaks still remain undetermined. It is believed that differences in locations of these peaks may be attributed to the Doppler effect due to the moving speed of the travelling line force.

5. CONCLUSION

The sound radiation from a fluid-loaded beam with periodic spring supports under the action of moving line forces is formulated and studied. The influences of the spring support spacing and the speed of travelling line force on the acoustic power are discussed. At lower wavenumber ratios, the addition of the spring support increases the total structural impedance, which implies less radiated sound power. The influences of the spring supports on the radiated acoustic power decrease as the wavenumber ratios and travelling speed of the force increases. When the external line force is stationary and acoustically small, the radiated sound power is seen to exhibit peaks at certain low wavenumber ratios except that the length of the external distributed line force is an integral multiple of the support spacing. It is shown that the wavenumber ratios at which peaks are located nearly coincide with the lower bounding wavenumber ratios of the odd number of propagation zones. The incomplete hydrodynamic cancellation is responsible for the fluctuations in radiated sound power. When the line force is moving at subsonic speed, the radiated sound power fluctuates more rapidly even though the length of external distributed line force is an integral multiple of the support spacing.

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APPENDIX: NOMENCLATURE

C_L	longitudinal wave speed
C_0	sound speed in the acoustic medium
$\bar{E}$	complex elastic modulus, $\bar{E} = E(1 + \mathrm{j}\eta)$
F_0	external force strength per unit width
$f(x, t)$	external moving line force
$\bar{G}$	complex shear modulus
h	beam height
$H(x)$	heavyside step function
I	cross-sectional second moment of area per unit width, $I = h^3/12$
j	$\sqrt{-1}$
k_B	free bending wave number
k_0	acoustic wave number
k_s	stiffness of the spring support per unit width
L	line force length
l	spacing between two adjacent supports
M	Mach number
$p(x, y = 0, t)$	acoustic pressure acting on the beam surface
$p_1(x, t)$	spring supports force
S	stiffness ratio
$u(x, t)$	displacement of the beam
V	moving force speed
W	dimensionless radiated sound power per unit width
α_0	fluid loading factor
κ^2	cross-sectional shape factor
γ	wave number ratio
ζ	dimensionless wavenumber variable
ξ	wavenumber variable
η	structural damping
v	Poisson ratio
$\prod$	radiated sound power per unit width
ρ_0	acoustic medium density
ρ_v	beam density
ω	frequency
Ω	non-dimensional frequency