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LETTERS TO THE EDITOR

ELASTOMER MODELLING FOR USE IN PREDICTING HELICOPTER LAG DAMPER BEHAVIOR

D. L. KUNZ

Department of Aerospace Engineering, Old Dominion University, Norfolk, VA 23529-0247, U.S.A.

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1. INTRODUCTION

As the use of elastomeric materials in helicopter lag dampers has become more prevalent, the need for better methods for modelling these materials has become evident [1, 2]. The two principal criteria that drive the need for fidelity in damper models are: (1) accurate prediction of damper energy dissipation; and (2) accurate prediction of blade loading. The prediction of energy dissipation by the damper is critical to the design of the rotor system, since the principal function of the damper is to provide sufficient damping to prevent aeromechanical instabilities, such as ground and air resonance. Accurate predictions of blade loads require that the forces and moments in the lag damper load path are modelled adequately. This is of particular importance when calculating rotor startup and aircraft maneuver loads, since the damper motion is not simple harmonic under those conditions.

It is apparent that the behavior of the majority of elastomeric materials is inherently non-linear, and dependent on displacement, deformation rate, and temperature. This non-linear behavior has motivated analysts to devise various methods of modelling these materials. Some investigators [1–4] have taken the approach of using a simple Kelvin Solid model, then replacing the linear stiffness and damping constants with non-linear functions. This approach retains the simplicity of the Kelvin Solid, while including some non-linear effects. Others [5–7] have introduced increasingly complex solid models, which have a greater number of stiffness and/or damping components, and additionally have implemented non-linear stiffness and damping functions. This approach increases the complexity of the damper representation, but improves the potential for enhanced modelling fidelity.

Typically, the suppliers of helicopter lag dampers document the properties of the dampers they deliver by applying a simple harmonic displacement to the damper and recording the output force [4]. This type of testing is usually performed at a frequency typical of the operational environment that will be seen by the damper, and at amplitudes that encompass the range of displacements that the damper will see in service. The question that arises when one considers implementing solid models with increased complexity is: "Is harmonic testing sufficient for the calculation of the additional parameters required for complex, non-linear models?"

2. APPROACH

The approach taken herein is to investigate the testing requirements for three linear solid models, then extend the results to non-linear damper models. First, it will be shown that the data from simple test procedures can be used to calculate unique stiffness and damping parameters for the Kelvin Solid. Then, identical methodology will be used on the Standard Solid to see if unique stiffness and damping constants can be calculated from data obtained by the same procedures. Finally, the methodology will be applied to a Generalized Kelvin Solid.

As noted above, one of the principal characteristics of elastomeric materials is that they exhibit non-linear behavior. Clearly, an investigation of linear damper models is insufficient for establishing a methodology that is applicable to the calculation of non-linear damper properties. However, following the example of reference [4], it will be shown that a connection can be made between the linear and non-linear models.

3. LINEAR SOLID MODELS

For the following solid models, it will be assumed that the material properties are linear. The equations of motion for each solid model will be solved using Laplace transforms. For the Kelvin Solid, this solution procedure is somewhat unnecessary since the solutions can be obtained by simple substitution. However, for the more complex models, the Laplace transform solution allows closed-form solutions to be obtained for each type of excitation.

Each of the damper models will be subjected to three different testing procedures: creep (constant applied force), ramp displacement (constant displacement rate), and single-frequency harmonic displacement. These three test procedures were chosen because they are commonly used, and because they are relatively easy to run in a laboratory environment. The force applied for the creep simulation is $f = \bar{f}$, and the corresponding Laplace transform of the force is $F = \bar{f}/s$. For the ramp displacement simulation, the specified displacement and its Laplace transform are $u = \bar{v}t$ and $U = \bar{v}/s^2$, respectively. The displacement applied for the harmonic displacement simulation is specified to be $u = \bar{u}\sin \Omega t$, where $\bar{u}$ is the displacement amplitude and Ω is the frequency. Then, the Laplace transform of the displacement can be written as $U = \Omega\bar{u}/(s^2 + \Omega^2)$. In all of the simulations, it is assumed that the initial displacements and displacement rates are zero.

3.1. KELVIN SOLID

The Kelvin Solid is the simplest of the viscoelastic material models, and is the traditional model used to represent the properties of an elastomeric material. Its mechanical equivalent consists of a spring and dashpot in parallel [Figure 1], where the stiffness and damping parameters are constants.

In Figure 1, the force applied to the solid is f, and the stiffness and damping of the model are represented by k_1 and c_1, respectively. The equation of motion for the

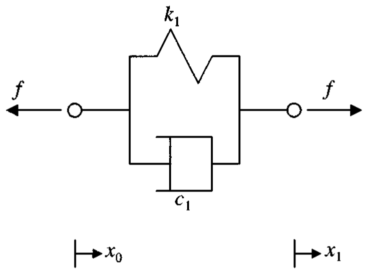

Figure 1. Kelvin Solid mechanical analogy.

mechanical analogy of the Kelvin Solid shown in Figure 1 is

$$f = c_1\dot{u} + k_1 u, \tag{1}$$

where $u = x_1 - x_0$. If it is assumed that the stiffness and damping are constants, then equation (1) can be transformed to

$$F = (c_1 s + k_1)U - c_1\hat{u}, \tag{2}$$

where F and U are the Laplace transforms of the force f, and the displacement u, respectively, and $\hat{u}$ is the initial displacement.

Consider the creep test, where a constant force $\bar{f}$ is applied to the damper ($\hat{u} = 0$) and the resulting displacement is measured. Substituting into equation (2) and solving for u,

$$u = \frac{\bar{f}}{k_1}(1 - \mathrm{e}^{-\lambda_1 t}), \tag{3}$$

where $\lambda_1 = k_1/c_1$. In its relaxed state, when t is large and the transient term is therefore small, only the constant k_1 can be calculated from the measured displacement and the known force input. In order to calculate c_1, the relaxation λ_1 must be measured while the material is reaching equilibrium, using k_1 which has been calculated previously.

The test in which a constant displacement rate is maintained can also be used to calculate both the stiffness and damping constants. Substituting the Laplace transform of a constant displacement rate into equation (2), solving for F, and taking the inverse Laplace transform,

$$f = (c_1 + k_1 t)\bar{v}. \tag{4}$$

The stiffness constant k_1 can be determined from the slope of the measured force curve. Since at time zero the force and displacement rate must both be zero in an actual test (as opposed to a simulation), the damping constant c_1 is calculated by projecting the constant slope back to time zero.

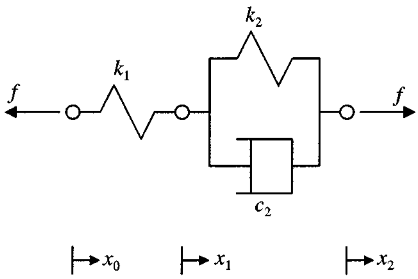

Figure 2. Standard Solid mechanical analogy.

The harmonic displacement test is the technique that is most commonly used to measure damper characteristics. If the Laplace transform of the sinusoidal displacement is substituted into equation (2), the resulting equation can be solved for F. The inverse transform of F can then be obtained, giving the force resultant from the applied displacement:

$$f = \bar{u}(c_1 \Omega \cos \Omega t + k_1 \sin \Omega t). \tag{5}$$

Since the measured force due to the harmonic displacement of the damper can be Fourier transformed into its sine and cosine components (as described in reference [4]), and since $\bar{u}$ and Ω are known, both the constants c_1 and k_1 can be easily and uniquely determined for a particular input amplitude and frequency. Therefore, for a Kelvin Solid model of a linear material, the values of both constants c_1 and k_1 can be calculated by using either harmonic excitation, creep, or a ramp displacement testing.

3.2. STANDARD SOLID

The Standard Solid is obtained by putting a spring in series with the Kelvin Solid, described above. As compared to the Kelvin Solid, which is characterized by two parameters, the Standard Solid is modelled by three parameters. Figure 2 shows the mechanical equivalent of the model, which is schematically the same as those used in references [5, 7]. However, in reference [5], only the k_1 stiffness was assumed to be non-linear; while in reference [7] the damping and both stiffnesses were modelled as being non-linear.

Following the same procedure as was used for the Kelvin Solid, the equations of motion for the Standard Solid can be obtained:

$$f = k_1 u_1, \qquad f = c_2(\dot{u} - \dot{u}_1) + k_2(u - u_1), \tag{6}$$

where $u = x_2 - x_0$ and $u_1 = x_1 - x_0$. The Laplace transforms of the equations of motion are then

$$F = k_1 U_1, \qquad F = (c_2 s + k_2)(U - U_1) - c_2(\hat{u} - \hat{u}_1), \tag{7}$$

where $\hat{u}$ and $\hat{u}_1$ are the initial displacements of u and u_1.

Now, consider the results of using creep testing on the Standard Solid. Substituting the Laplace transform of a constant force $\bar{f}$ into equation (7), and solving for the displacement

$$u = \bar{f}\left(\frac{k_1 + k_2}{k_1 k_2} - \frac{1}{k_2}\,\mathrm{e}^{-\lambda_2 t}\right), \tag{8}$$

where $\lambda_2 = k_2/c_2$. When the damper reaches its relaxed state ($t = \infty$), the value of a single expression containing both k_1 and k_2 can be determined. During the period of transient response, the transient term usually is dominant, and both the coefficient of the exponential term and the exponent λ_2 can be determined by a curve fit to the transient response. Thus, three independent measurements are available, from which the three model constants can be calculated uniquely. From the simulation of a ramp displacement, the measured force shown in equation (9) is obtained:

$$f = \bar{v}\left[\frac{k_1 k_2}{k_1 + k_2}\,t + \frac{k_1^2 c_2}{(k_1 + k_2)^2}\,(1 - \mathrm{e}^{-\lambda_{12} t})\right], \tag{9}$$

where $\lambda_{12} = (k_1 + k_2)/c_2$. There are three distinct terms in the force response to the displacement: (1) a term that is linearly dependent on time; (2) a constant term; and (3) a transient term. Note that at time zero, the force is identically zero. This model is therefore a significant improvement over the Kelvin Solid model, which requires an artificial axis shift in order to account for the initial force transient.

Calculation of the stiffness and damping parameters follows a procedure similar to that used for the Kelvin Solid. Once the transients have died out ($t = \infty$), the coefficient of the linear term in time can be determined from the slope of the response. The projection of the slope back to time zero fixes the value of the constant term. Then, the relaxation λ_{12} is calculated from the transient portion of the response measurement. Again, three independent measurements are available to permit calculation of unique values of the model constants.

To simulate the imposition of a harmonic displacement on the Standard Solid, the Laplace transformed sinusoidal displacement with amplitude $\bar{u}$, frequency Ω, and $\hat{u} = \hat{u}_1 = 0$ is substituted into equations (7), and the resulting equations are solved simultaneously for F. Then, performing the inverse transform on F, the force resultant is obtained:

$$f = \frac{k_1 \bar{u}}{c_2^2 \Omega^2 + (k_1 + k_2)^2}\,\{[c_2^2 \Omega^2 + (k_1 + k_2) k_2] \sin \Omega t + k_1 c_2 \Omega (\cos \Omega t - \mathrm{e}^{-\lambda_{12} t})\}, \tag{10}$$

where again $\lambda_{12} = (k_1 + k_2)/c_2$. After the initial transients have died out ($t = \infty$), the steady state response can be measured. Fourier analysis of the steady state response permits the sine and cosine components of the force to be calculated. However, since this procedure yields only two independent measurements, only two, unique, independent parameters can be determined. In order to calculate

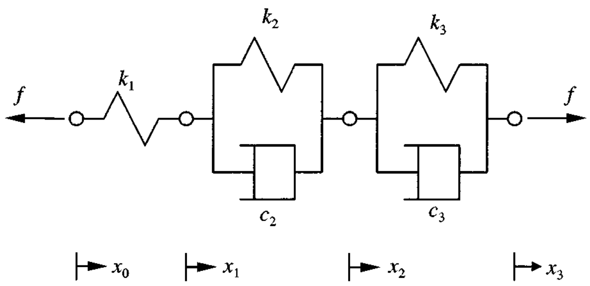

Figure 3. Generalized Kelvin Solid mechanical analogy.

a third independent parameter, an additional measurement must be made. The only remaining measurement available is to calculate λ_{12} from the transient portion of the response. Based on the relative magnitudes of the terms in equation (10), that calculation could be very difficult, because of the contributions from the harmonic terms.

3.3. GENERALIZED KELVIN SOLID

Another possible representation of an elastomeric damper is the Generalized Kelvin Solid shown in Figure 3. The mechanical analogy looks like the Standard Solid (Figure 2) with an additional Kelvin Solid added in series.

The equations of motion for the mechanical analogy of the Generalized Kelvin Solid are

$$f = k_1 u_1, \qquad f = c_2 \dot{u}_2 + k_2 u_2, \qquad f = c_3(\dot{u} - \dot{u}_2 - \dot{u}_1) + k_3(u - u_2 - u_1). \tag{11}$$

These equations of motion can be Laplace transformed, resulting in

$$F = k_1 U_1, \qquad F = (c_2 s + k_2) U_2 - c_2 \hat{u}_2,$$

$$F = (c_3 s + k_3)(U - U_2 - U_1) - c_3(\hat{u} - \hat{u}_2 - \hat{u}_1). \tag{12}$$

The creep result in equation (13) is very similar in appearance to the simulations of the Standard Solid, equation (8):

$$u = \bar{f}\left(\frac{k_2 k_3 + k_1 k_3 + k_1 k_2}{k_1 k_2 k_3} - \frac{1}{k_2}\,\mathrm{e}^{-\lambda_2 t} - \frac{1}{k_3}\,\mathrm{e}^{-\lambda_3 t}\right), \tag{13}$$

where $\lambda_2 = k_2/c_2$ and $\lambda_3 = k_3/c_3$. The major difference in the displacements is that the Generalized Kelvin Solid has two transient functions with difference relaxation values. Herein lies the principal reason for adding the second Kelvin link. If the values of λ_2 and λ_3 are sufficiently different, the material will change its response behavior with time (and, as a result, displacement). Calculation of all five stiffness and damping element properties from this single test relies on being able to calculate where each of the relaxation parameters is dominant. While not impossible, accurate calculation would appear to be highly dependent on the values of λ_2 and λ_3.

Equation (14) shows the resultant force when the model is subjected to a ramp displacement

$$f = \frac{k_1 D_1}{(k_2k_3 + k_3k_1 + k_1k_2)^2}\,\bar{v} + \frac{k_1k_2k_3}{k_2k_3 + k_3k_1 + k_1k_2}\,\bar{v}t$$
$$- \frac{k_1\bar{v}}{(k_2k_3 + k_3k_1 + k_1k_2)^2}\left[D_1\cosh\left(\frac{\sqrt{B}}{2c_2c_3}\,t\right)\right.$$
$$\left. + \frac{D_2}{\sqrt{B}}\sinh\left(\frac{\sqrt{B}}{2c_2c_3}\,t\right)\right]\mathrm{e}^{-((k_1/c_3 + \lambda_3 + \lambda_{12})/2)t} \tag{14}$$

where

$$B = [(k_1 + k_3)c_2 + (k_1 + k_2)c_3]^2 - 4(k_2k_3 + k_3k_1 + k_1k_2)c_2c_3,$$
$$D_1 = k_1(k_3^2c_2 + k_2^2c_3),$$
$$D_2 = k_1k_3^2(k_1 + k_3)c_2^2 + k_1k_2^2(k_1 + k_2)c_3^2$$
$$+ k_1[k_2^2(k_1 + k_3) + k_3^2(k_1 + k_2) + 4k_1k_2k_3]c_2c_3.$$

The response of the Generalized Kelvin Solid is similar in form to the ramp simulations of the other models, except that the coefficient of the transient term now contains transcendental functions. Identification of that coefficient could be quite a formidable task. Note that equation (14) is only a valid solution when B is positive. Fortunately, when all of the stiffness and damping parameters are positive (as they must always be), B will always be positive.

The harmonic displacement response for the Generalized Kelvin Solid is

$$f = \frac{k_1\Omega\bar{u}}{W_{ts}}(H_1\Omega^2 + D_1)\cos(\Omega t) + \frac{k_1\bar{u}}{W_{ts}}\{c_2^2c_3^2\Omega^4 + [k_3(k_1 + k_2)c_2^2 + k_2(k_1 + k_3)c_3^2]\Omega^2$$
$$+ (k_2k_3 + k_3k_1 + k_1k_2)^2\}\sin(\Omega t)$$
$$- \frac{k_1\Omega\bar{u}}{W_{ts}}\left[(H_1\Omega^2 + D_1)\cosh\left(\frac{\sqrt{B}}{2c_2c_3}\,t\right)\right.$$
$$\left. + \frac{H_2\Omega^2 + D_2}{\sqrt{B}}\sinh\left(\frac{\sqrt{B}}{2c_2c_3}\,t\right)\right]\mathrm{e}^{-(k_1/c_3 + \lambda_3 + \lambda_{12})2)t)}, \tag{15}$$

where

$$H_1 = k_1(c_2 + c_3)c_2c_3,$$

$$H_2 = k_1[(k_1 + k_3)c_2^2 + (k_1 + k_2)c_3^2 + (2k_1 - k_2 - k_3)c_2c_3]c_2c_3,$$

$$W_{ts} = c_2^2c_3^2\Omega^4 + [2(k_2k_3 + k_3k_1 + k_1k_2) + B]\Omega^2 + (k_2k_3 + k_3k_1 + k_1k_2)^2.$$

Like the ramp displacement response, the response function in equation (15) contains transcendental functions and multiple products of the component parameters.

4. EXTENSION TO NON-LINEAR DAMPER MODELLING

Strictly speaking, the responses that were calculated for the models described above are only valid for models of linear damping materials. Since elastomeric materials exhibit non-linear behavior, these results cannot be used for the purposes of material characterization. However, the data from harmonic damper tests [8] have shown that the non-linearities are usually relatively weak, and the stiffness and damping element properties can be modelled quite accurately by replacing the element constants with element functions. This technique was successfully applied in reference [4], where the use of stiffness and damping functions resulted in responses that were virtually identical to the test data.

In the preceding sections, the closed-form responses were shown to become progressively more complex as the complexity of the models was increased. For the Kelvin Solid, replacing the stiffness and damping constants with functions appears to be a reasonable step that one could take to approximate non-linear material properties. Reference [4] demonstrated this technique only for harmonic displacement test data; but if data were available, the same technique could be applied to relaxation and ramp test data.

The closed-form responses calculated for the Standard Solid are significantly more complex than those calculated for the Kelvin Solid. However, despite the problems that result from having to include products of the stiffness and damping functions, it is conceivable that a valid calculation could be performed. The same may be stated for the relaxation responses from the Generalized Kelvin Solid. Products of the stiffness and damping functions could make calculation difficult, but probably not impossible. On the other hand, the responses for the ramp and harmonic displacement tests from the Generalized Kelvin Solid are so complex that calculation of unique stiffness and damping functions would be extraordinarily difficult.

5. CONCLUSIONS

Closed-form responses from relaxation, ramp displacement, and harmonic displacement tests were obtained analytically from three material models. The progression from the Kelvin Solid, which has one stiffness and one damping component, to the Generalized Kelvin Solid, which has three stiffness and two damping components, shows that moderate increases in the number of components rapidly increases the complexity of the response functions. Based on these results, calculating unique stiffness and damping constants for ramp and harmonic displacements of a model with greater than three components could prove to be a formidable challenge.

By replacing stiffness and damping constants with stiffness and damping functions, the linear, closed-form responses can be used to approximate non-linear material properties. However, because of the additional complexity resulting from this substitution, only the simple, two- and three-component models are suitable candidates for direct calculation. Therefore, with respect to the development of improved analytical models of elastomeric materials, analysts would be well advised to consider limiting the complexity of their models.

One potential method for determining model parameters that was not addressed herein is the use of system identification or parameter identification techniques. These techniques would almost certainly be valuable for complex, linear solid models, but it is not clear (to this author) how well these methods would work for complex, non-linear solid models. For example, in reference [4] a fifth order polynomial was used for the stiffness coefficient and a cubic polynomial was used for the damping coefficient of a Kelvin Solid, making a total of 10 coefficients. By extension, a minimum of 26 coefficients would be required for the Generalized Kelvin Solid.

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APPENDIX: NOMENCLATURE

c_1, c_2	damping constants
F	Laplace transformed force
f	applied force
k_1, k_2, k_3	stiffness constants
s	Laplace transform variable
t	time
U	Laplace transformed displacement
u	total damper displacement
u_1, u_2	intermediate damper displacement
v	total damper displacement rate
x_0, x_1, x_2, x_3	damper position variables
λ_1, λ_2, λ_3, λ_{12}	damper relaxation
Ω	harmonic excitation frequency
$(\hat{\ })$	initial value
$(\bar{\ })$	constant value
(*)	derivative with respect to time