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STUDY ON DAMPING IN LAYERED AND JOINTED STRUCTURES WITH UNIFORM PRESSURE DISTRIBUTION AT THE INTERFACES

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The present work aims at studying the mechanism of damping and its theoretical evaluation for layered and jointed cantilever beams with a number of equispaced connecting bolts resulting in uniform distribution of pressure at the interfaces. Experiments have been conducted on a number of specimens for comparison with numerical results. Intensity of interface pressure, its distribution characteristics, relative spacing of the connecting bolts, dynamic slip ratio, frequency and amplitude of vibration are found to play a vital role in the damping capacity of such layered and jointed structures. It is established that the damping capacity of structures jointed with connecting bolts can be improved substantially with an increase in number of layers maintaining uniform intensity of pressure distribution at the interfaces.

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1. INTRODUCTION

The study of damping and its importance in structures has become increasingly significant for controlling the undesirable effects of vibration. Damping capacity of structures can be increased considerably with the use of constrained, unconstrained and sandwich viscoelastic layers, elastic inserts in the parent structure, spaced damping technique, and fabricating layered and jointed structures with welded/riveted/bolted joints. Although these techniques are used in actual practice depending on the situation and environmental condition, the last one, i.e., the layered construction jointed with connecting bolts can be used effectively in machine tool structures. However, in such cases, attention has to be focused on proper orientation and spacing of the connecting bolts in order to maximize the damping capacity.

The logarithmic damping decrement, a measure of damping capacity of layered and jointed structures, is usually determined by the energy principle taking into account the relative dynamic slip and the interfacial pressure between the contacting layers. These two vital influencing parameters are to be assessed accurately for correct evaluation of damping capacity, of such structures. Earlier investigators, e.g. Masuko *et al.* [1], Nishiwaki *et al.* [2, 3], and Motosh [4] have assumed

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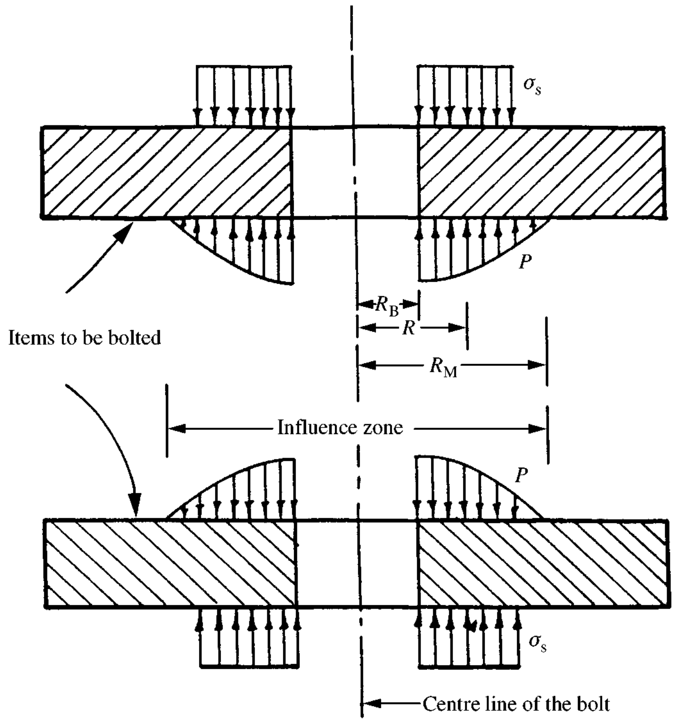

Figure 1. Free-body diagram of a bolted joint showing the influence zone.

uniform intensity of pressure distribution at the interfaces of the layered structures without referring to the actual pattern but by using Rötschar's pressure cone [1]. A lot of work has been done by Fernlund [5], Greenwood [6], Lardner [7], Bradly [8], Sidorov [9], El-Zahry [10], Kobayashi and Matsubayashi [11, 12], Tsai *et al.* [13], Shin *et al.* [14] and Song *et al.* [15] on the pattern, intensity of pressure distribution at the interfaces of bolted joints and damping characteristics. Gould and Mikic [16] and Ziada and Abd [17] have shown that the pressure distribution at the interfaces of a bolted joint is parabolic in nature and there exists an influence zone in the form of a circle with 3·5 times the diameter of the connecting bolt which is independent of the tightening load applied on it as shown in Figure 1. In the present study, a theoretical expression for pressure distribution at the interfaces has been determined using the numerical data of Ziada and Abd [17]. The spacing of the connecting bolts have also been found using the above expression in order to obtain uniform intensity of pressure distribution at the interfaces. The damping capacity of such layered and jointed structures have been evaluated both numerically and experimentally considering various parameters.

2. THEORETICAL ANALYSIS

In case of layered structures jointed with connecting bolts, the intensity of interface pressure distribution under the bolt in a non-dimensional form has been assumed to be a polynomial, i.e.,

$$p/\sigma_s = A_1 + A_2(R/R_B)^2 + A_3(R/R_B)^4 + A_4(R/R_B)^6 + A_5(R/R_B)^8 + A_6(R/R_B)^{10}, \tag{1}$$

where A_1, A_2, A_3, A_4, A_5 and A_6 are constants of the polynomial. Taking the numerical data of Ziada and Abd [17] and using Dunn's curve fitting software package in the computer, the values of these constants are found to be 0·68517E + 00, −0·10122E + 00, 0·94205E − 02, −0·23895E − 02, 0·29487E − 03 and − 0·11262E − 04, respectively.

In order to obtain a uniform intensity of pressure distribution at the interfaces, the consecutive influencing zones are to be superimposed by varying the spacing of the consecutive connecting bolts on the structure. This spacing has been determined with the help of a suitable software package and is found to be 2·00211 times the diameter of the connecting bolts, which is independent of the tightening torque on the connecting bolts. The magnitude of the uniform intensity of pressure distribution with the above spacing has been determined as shown in Figure 2 and found to be

$$p = 0{\cdot}671P/3\pi R_B^2 \tag{2}$$

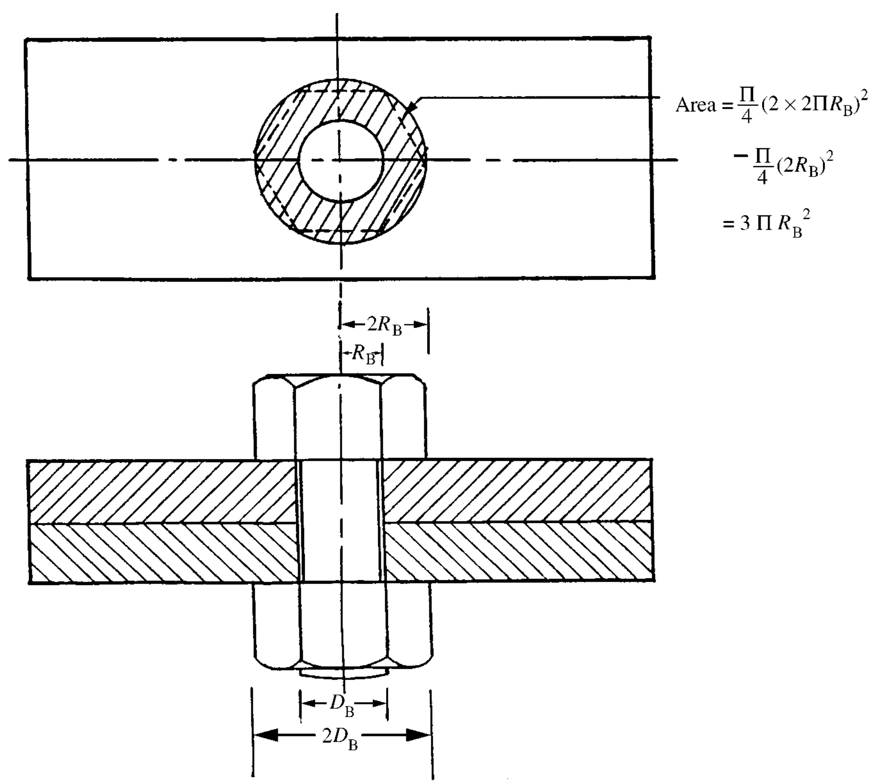

Figure 2. Influence area under a connecting bolt head.

If T is the tightening torque applied on the connecting bolt, the axial load on the bolt (P) is given by Shigley [18] as

$$P = [T/0{\cdot}2D_{\mathrm{B}}] \tag{3}$$

Putting this value of axial load in the expression (2) we get

$$p = 0{\cdot}17799T/R_{\mathrm{B}}^2 \tag{4}$$

For a layered and jointed cantilever beam with uniform pressure distribution at the interfaces as shown in Figure 3, an initial deflection at the free end will show a relative dynamic slip at the interfaces as shown in Figure 4. As given by Masuko *et al.* [1] the relative dynamic slip $u(x, t)$ at a particular position and time

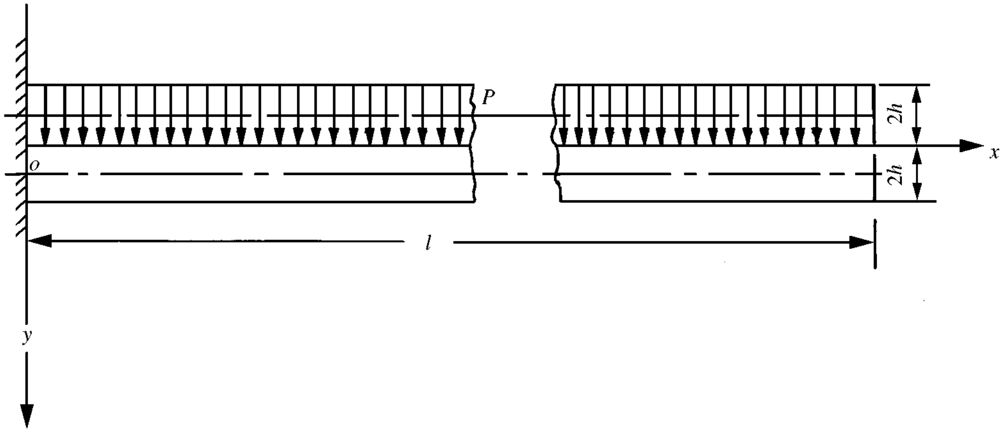

Figure 3. Layered and jointed cantilever beam with uniform pressure distributioin at the interfaces.

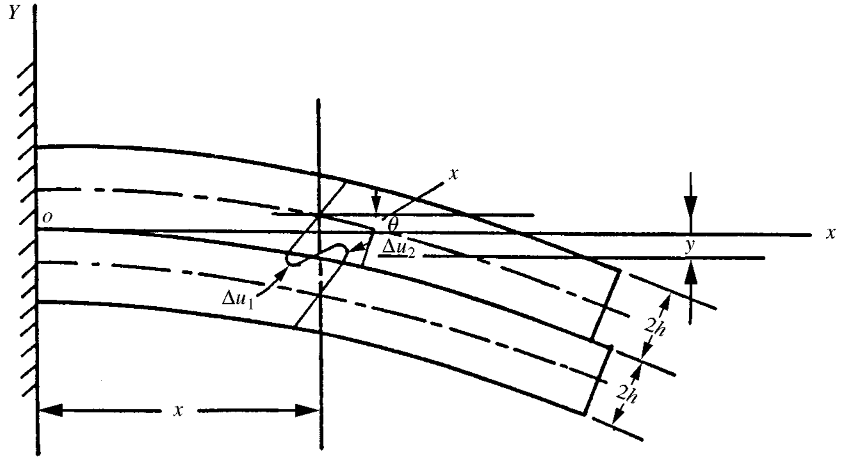

Figure 4. Mechanism of slip at the interfaces.

is given by

$$u(x, t) = \Delta u_1 + \Delta u_2 = 2h \tan[\partial y(x, t)/\partial x], \tag{5}$$

where $y(x, t)$ is the bending deflection of the layered and jointed cantilever beam in the y direction. However, the actual relative dynamic slip $u_r(x, t)$ between the interfaces during the vibration will be less and may be written as

$$u_r(x, t) = \alpha u(x, t) = 2\alpha h \tan[\partial y(x, t)/\partial x], \tag{6}$$

where α is the dynamic slip ratio.

For a layered and jointed structural member, the damping ratio, ψ, is expressed as the ratio between the loss energy dissipated due to the relative dynamic slip between the interfaces and the total energy introduced into the system and is expressed as

$$\psi = [E_{\text{loss}}/(E_{\text{loss}} + E_{\text{ne}})], \tag{7}$$

where E_{loss} and E_{ne} are the energy loss due to interface friction and energy to be introduced during the unloading process as shown in Figure 5. For the above cantilever beam with uniform pressure distribution at the interface, p, the energy loss due to the above frictional force at the interface per half-cycle of vibration may be written as

$$E_{\text{loss}} = \int_0^{\pi/\omega_n} \int_0^l \mu p b[\{\partial u_r(x, t)/\partial t\} \, dx \, dt]. \tag{8}$$

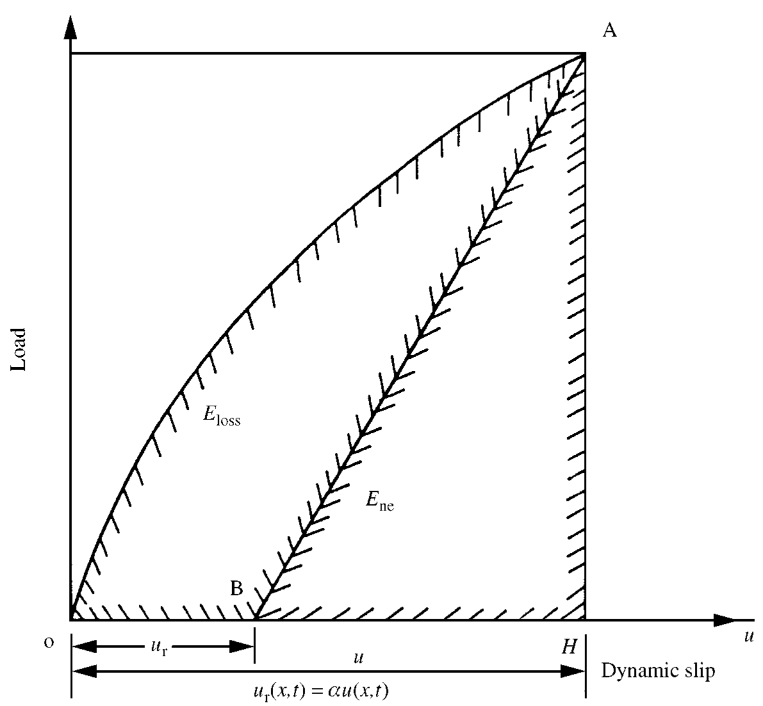

Figure 5. Relationship between u_r and u.

However, the energy introduced into the layered and jointed cantilever beam in the form of strain energy per half-cycle of vibration is given by

$$E_{\mathrm{ne}} = (3EI/l^3)y^2(l, 0). \tag{9}$$

From the above expressions (8) and (9) we get

$$E_{\mathrm{loss}}/E_{\mathrm{ne}} = \int_0^{\pi/\omega_n} \int_0^l [\mu p b\{\partial u_{\mathrm{r}}(x, t)/\partial t\}\, \mathrm{d}x\, \mathrm{d}t]/[3EI/l^3)y^2(l, 0)]. \tag{10}$$

Considering uniform pressure distribution throughout the contact area of the interfaces and assuming dynamic slip ratio, α, to be independent of the distance from the fixed end of the cantilever beam and time, the above expression (10) can be modified using expression (6) as

$$E_{\mathrm{loss}}/E_{\mathrm{ne}} = [2\mu bhp\alpha/\{(3EI/l^3)y^2(l, 0)\}] \int_0^{\pi/\omega_n} \int_0^l [\partial\{\tan \partial y(x, t)/\partial x\}\, \mathrm{d}x\, \mathrm{d}t]/\partial t. \tag{11}$$

Moreover, the slope of the cantilever beam $\partial y(x, t)/\partial x$ being quite small, $[\tan \partial y(x, t)/ \partial x\}] \simeq \partial y(x, t)/\partial x$.

Therefore, expression (11) is modified to

$$E_{\mathrm{loss}}/E_{\mathrm{ne}} = [2\mu bhp\alpha/\{(3EI/l^3)y^2(l, 0)\}] \int_0^{\pi/\omega_n} \int_0^l [\{\partial^2 y(x, t)/\partial x \partial t\}\, \mathrm{d}x\, \mathrm{d}t]. \tag{12}$$

Considering the boundary and the initial conditions of the cantilever beam as $y(l, 0) = y_0$ (positive downward deflection) and $\partial y(l, 0)/\partial t = 0$ (no initial velocity), respectively, the bending deflection of the beam under vibration can be expressed as

$$y(x, t) = Y(x)\{y_0/Y(l)\} \cos \omega_n t, \tag{13}$$

where $Y(x)$ is the space function and the rest is the time function.

Using the above expression (13) in equation (12) and changing the limits of the time interval from 0 and π/ω_n to 0 and $\pi/2\omega_n$ and multiplying the expression by two for yielding definite solution we get

$$E_{\mathrm{loss}}/E_{\mathrm{ne}} = \frac{4\mu bhp\alpha}{(3EI/l^3)y^2(l, 0)} \int_0^{\pi/2\omega_n} \int_0^l \partial^2[Y(x)\{y_0/Y(l)\} \cos \omega_n t]\mathrm{d}x\, \mathrm{d}t/[\partial x/\partial t]. \tag{14}$$

Using expression (13) and (14)

$$E_{\mathrm{loss}}/E_{\mathrm{ne}} = [4\mu bhp\alpha y(l, 0)]/[3(EI/l^3)y^2(l, 0)]. \tag{15}$$

Replacing $3EI/l^3 = k$, i.e., the equivalent spring constant (static bending stiffness) of the layered and jointed beam, the above expression reduces to

$$E_{\mathrm{loss}}/E_{\mathrm{ne}} = [4\mu bhp\alpha]/[ky(l, 0)]. \tag{16}$$

Expression (7) is modified as

$$\psi = [E_{\mathrm{loss}}/(E_{\mathrm{loss}} + E_{\mathrm{ne}})] = 1/[1 + E_{\mathrm{ne}}/E_{\mathrm{loss}}]. \tag{17}$$

Putting the values of $E_{\text{loss}}/E_{\text{ne}}$ from expression (16) in expression (17) we get

$$\psi = \frac{1}{1 + [ky(l, 0)]/[4\mu bhp\alpha]}. \tag{18}$$

The logarithmic damping decrement, δ, is usually expressed as, $\delta = \ln(a_n/a_{n+1})$. Assuming that the energy stored in the system is proportional to the square of the corresponding amplitude, the relationship between logarithmic damping decrement and damping ratio can be written as

$$\delta = \ln(E_{\text{n}}/E_{\text{n}+1})^{1/2} = [\ln\{1/(1 - \psi)\}]/2. \tag{19}$$

In case of $\psi \leqslant 1$, the Maclaurin expression of the above expression (19) will yield

$$\delta = [\psi + (\psi^2/2)]/2. \tag{20}$$

Similarly, in order to find out the logarithmic damping decrement for multi-layered cantilever beams, the number of interfacial layers are to be taken into consideration. If "m" number of layers are jointed together with connecting bolts to construct the multi-layered cantilever beams so as to have uniform interface pressure, the damping ratio for such beams is given by:

$$\psi = \frac{1}{1 + [ky(l, 0)]/[4(m - 1)\mu bhp\alpha]}. \tag{21}$$

3. EXPERIMENTAL TECHNIQUES AND EXPERIMENTS

In order to find experimentally the logarithmic damping decrement of layered and jointed beams and to compare it with the numerical ones, an experimental set-up with a number of specimens has been fabricated. The experimental set-up with detailed instrumentation is shown in Figure 6. The specimens are prepared from commercial mild steel flats of the sizes as presented in Table 1 by joining two as well as a larger number of layers with the help of equispaced connecting bolts of uniform tightening torque on them.

The distance between the consecutive connecting bolts have been kept at 2·00211 times the diameter of the connecting bolts in order to ensure uniform intensity of pressure distribution at the interfaces of the layered and jointed cantilever beams. The cantilever lengths of the specimens have been varied accordingly in order to accommodate the corresponding number of connecting bolts as presented in Table 1.

The experiment has been conducted in three phases.

First of all, the tests are conducted to determine the modulus of elasticity of the specimen materials through a static bending test. Solid cantilever specimens out of the same stock of commercial flats are held rigidly at the fixed end and its free end deflection (Δ) is measured by applying static loads (W). From these static loads and corresponding deflections, average static bending stiffness (W/Δ) is determined. The bending modulus for the specimen material is then evaluated from the expression $E = [(W/\Delta)(l^3/3I)]$. The value of "E" for mild steel specimens is found to be 172·7 GN/m^2.

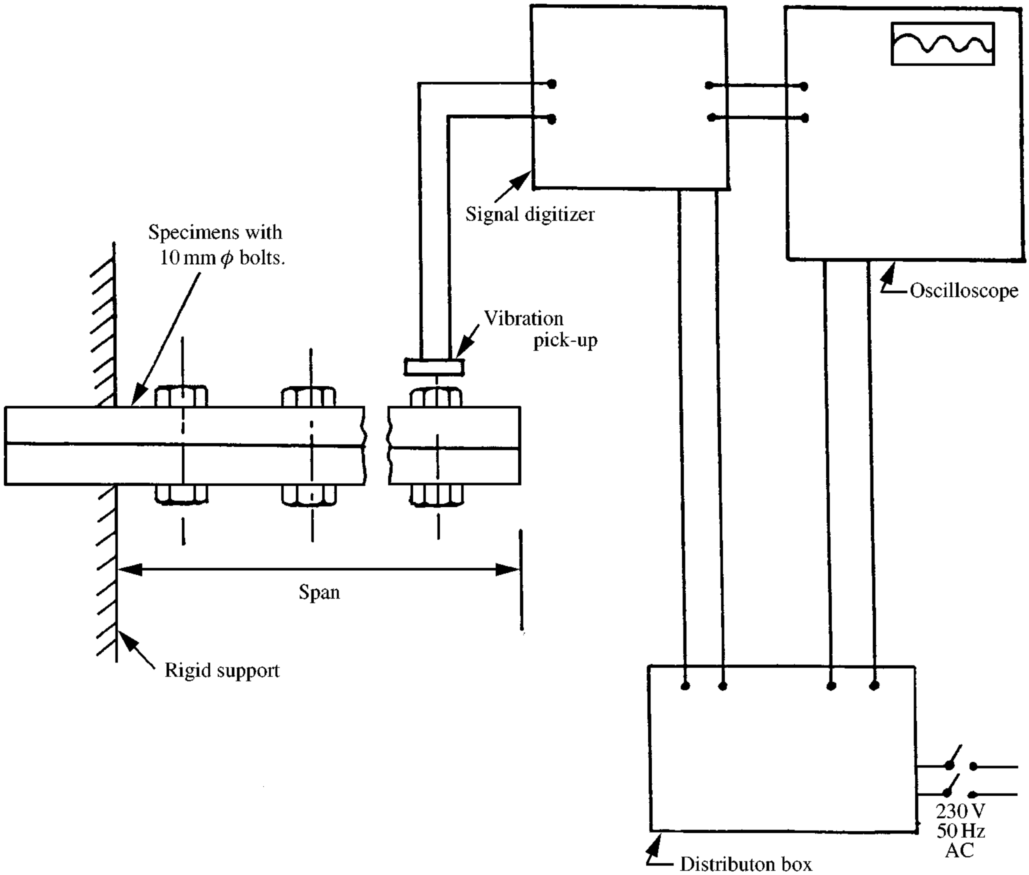

Figure 6. Schematic diagram of the experimental set-up with detailed instrumentation.

TABLE 1

Details of mild steel specimens used for layered and jointed beams with uniform intensity of pressure at the interfaces

Thickness of the specimen (mm)	Width of the specimen (mm)	Diameter of the connecting bolt (mm)	Number of layers	Number of columns of bolts	Cantilever length (mm)
3·0	40·04			18	360·38
5·4	40·04	10	2	17	340·36
12·4	38·00			16	320·34
4·5	40·04			18	360·38
		10	3	17	340·36
8·1	40·04			16	320·34
6·0	40·04			18	360·38
		10	4	17	340·36
10·8	40·04			16	320·34
6·0	40·04	Solid beam			360·38

Secondly, the equivalent spring constant (static bending stiffness) k of the specimens are determined as per the above procedure. While performing this test, the tightening torque on connecting bolts have been varied but kept the same for all the bolts while taking observations on different specimens. It is observed during experimentation that the equivalent spring constant (static bending stiffness) of the layered and jointed beam is always less than that of an equivalent solid one and increases with an increase in tightening torque on the connecting bolts and remains constant after a limiting of tightening torque 10·370 N m (7·50 lb ft). The ratio of this equivalent spring constant (bending stiffness) at the limiting tightening torque condition with the equivalent spring constant (static bending stiffness) of a solid one (α') are found for all specimens (see Figure 7). The average value of α' for each group of specimens have been utilized in the numerical analysis.

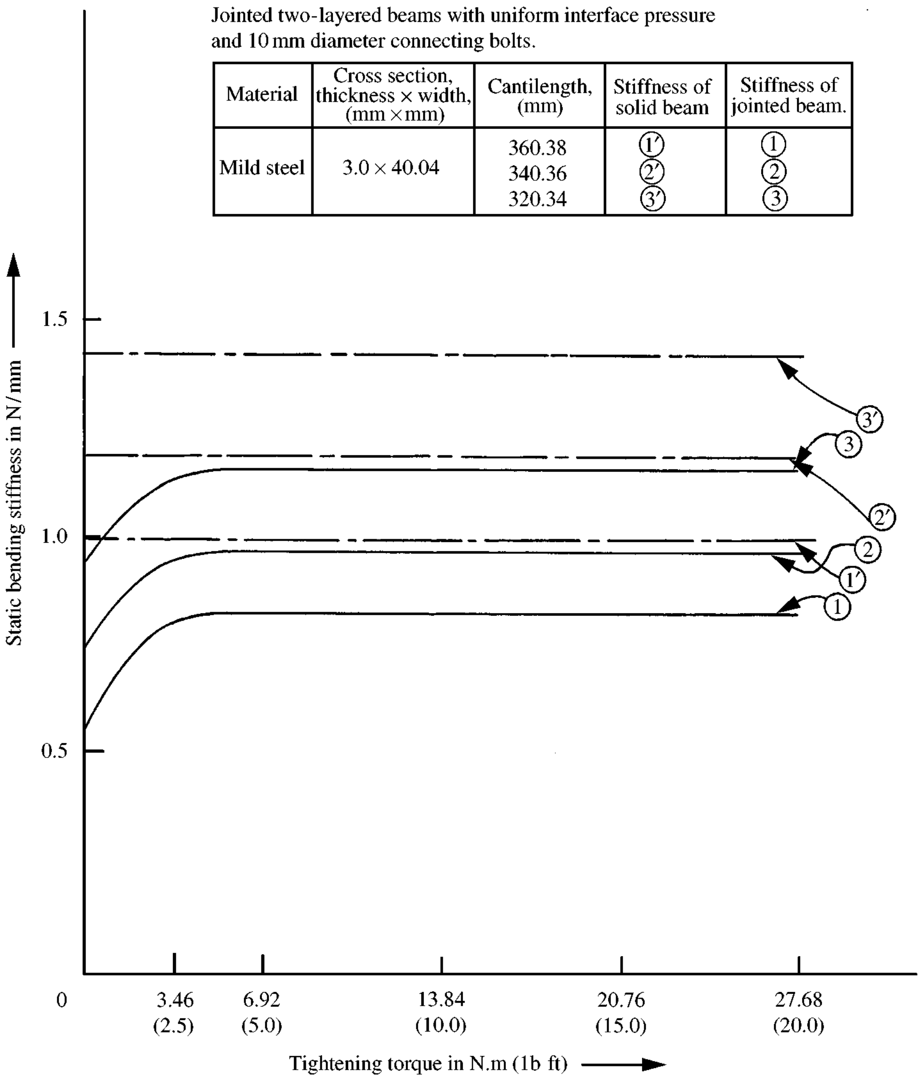

Figure 7. Variation of static bending stiffness with applied tightening torque on the connecting bolts.

Finally, the logarithmic damping decrement and natural frequency of vibration of all the specimens at their first mode of free vibration are found experimentally. The tightening torque on all the connecting bolts of the specimens are kept equal for each set of observations and varied in steps as 3·46, 6·92, 13·84, 20·76, and 27·68 N m (i.e., 2·50, 5·00, 10·00, 15·00 and 20·00 lb ft respectively). The lengths of these specimens during experimentation are also varied. In order to excite the specimens at their free ends as considered in theory, a spring loaded exciter is used and the amplitudes of excitation is varied in steps and maintained as 0·1, 0·2, 0·3,

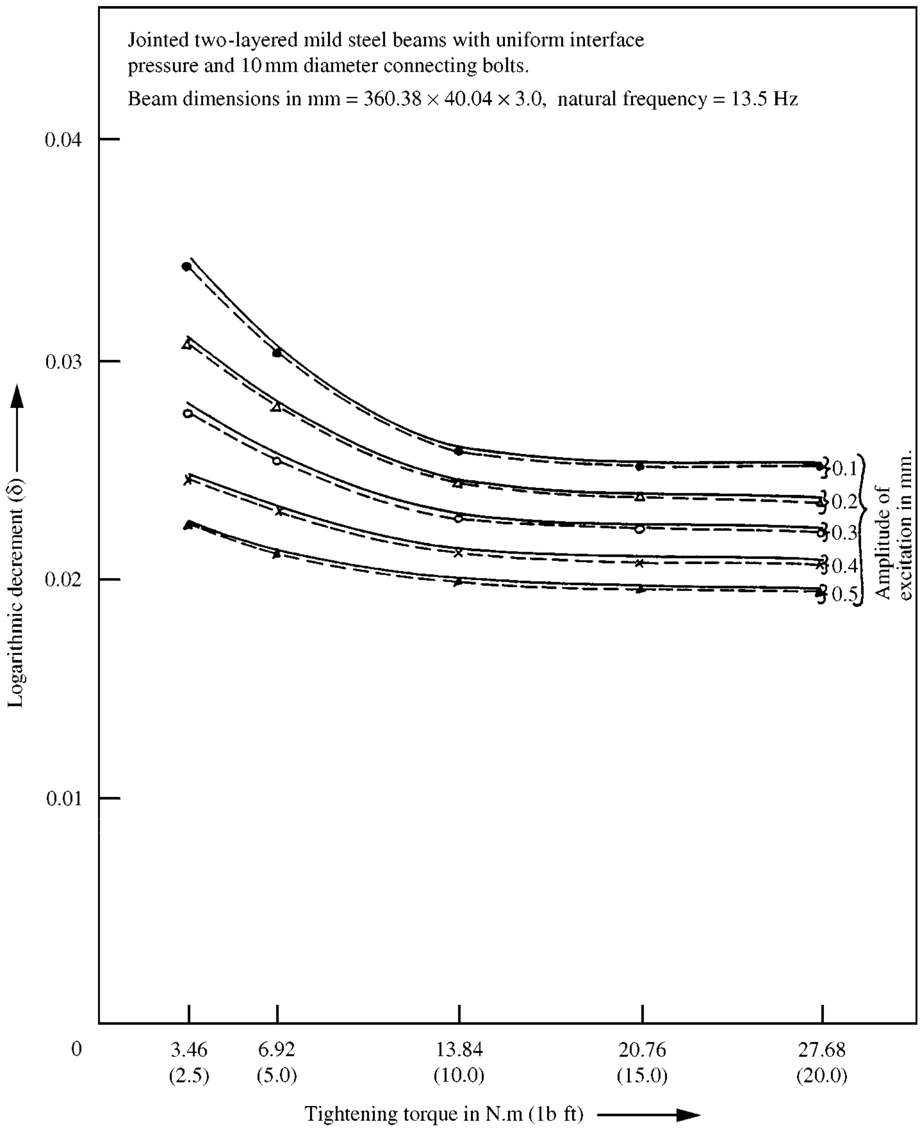

Figure 8. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts. —— Theoretical; – – – Experimental.

0·4, and 0·5 mm for all the specimens under different conditions of tightening torque on the connecting bolts. The free vibration is sensed with a non-contact-type vibration pick-up and the corresponding signal is fed to a C.R.O. through the digitizer. Both logarithmic damping decrement and natural frequency of first-mode vibration are evaluated from the signals recorded on the C.R.O. Similar tests are also conducted for equivalent solid cantilever beams in order to compare their logarithmic decrement values with that of jointed cantilever beams.

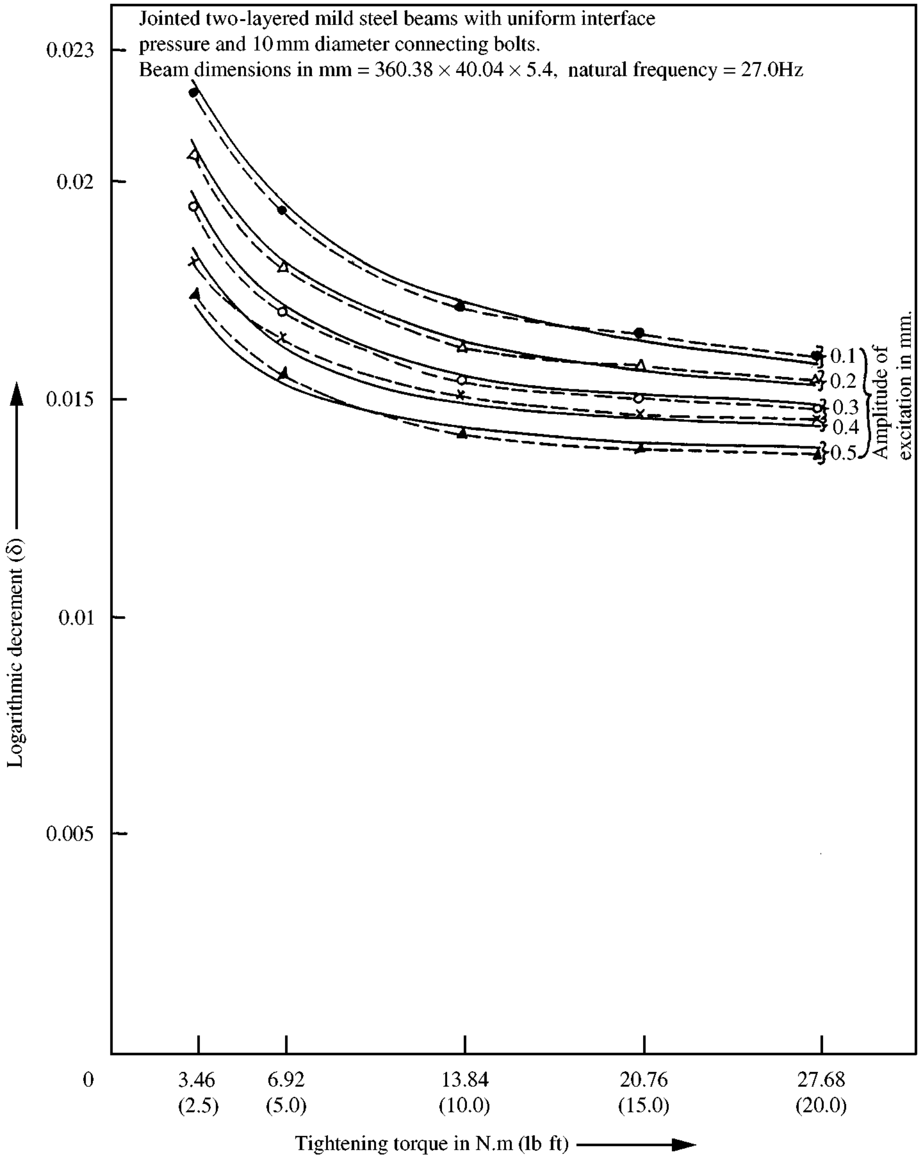

Figure 9. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts. —— Theoretical; – – – Experimental.

4. NUMERICAL ANALYSIS

The numerical values of logarithmic damping decrement are evaluated using expressions (4), (18) and (20) as well as (21). The values of static stiffness of different specimens have been found taking into account α' found in the experimental analysis along with the static stiffness of the equivalent solid beams. The value of kinematic coefficient of friction has been taken as 0·009 as has been used by Masuko *et al.* [1]. The dynamic slip raito (α) as found by Nanda [19] has been utilized for the numerical analysis. All these numerical results have been plotted along with the corresponding experimental ones for comparison in Figures 8–14.

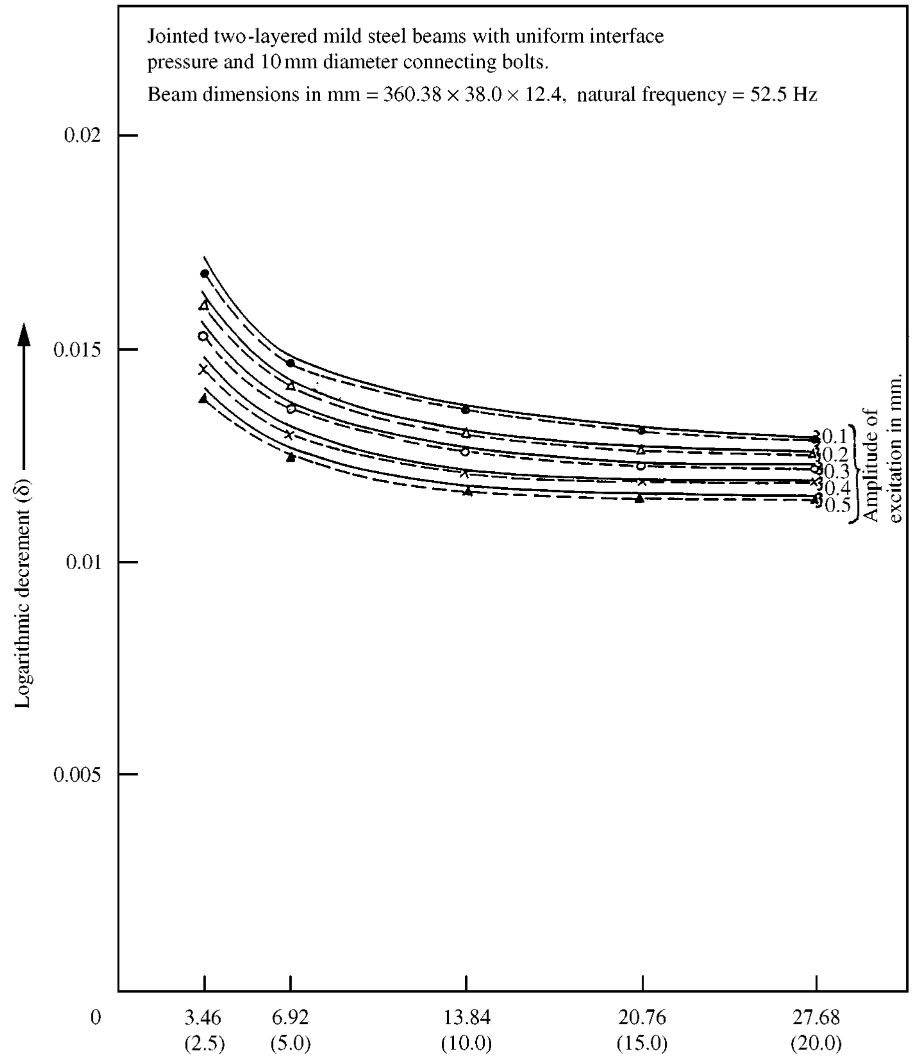

Figure 10. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts. —— Theoretical; – – – Experimental.

These figures show the variation of logarithmic damping decrement with applied tightening torque on the connecting bolts. It is observed from the curves that both numerical and experimental results are very close to each other with a maximum variation of 2·4%.

5. DISCUSSION AND CONCLUSION

From the theoretical analysis as well as numerical and experimental results, the following salient points have been observed. They are discussed below and conclusions have been drawn.

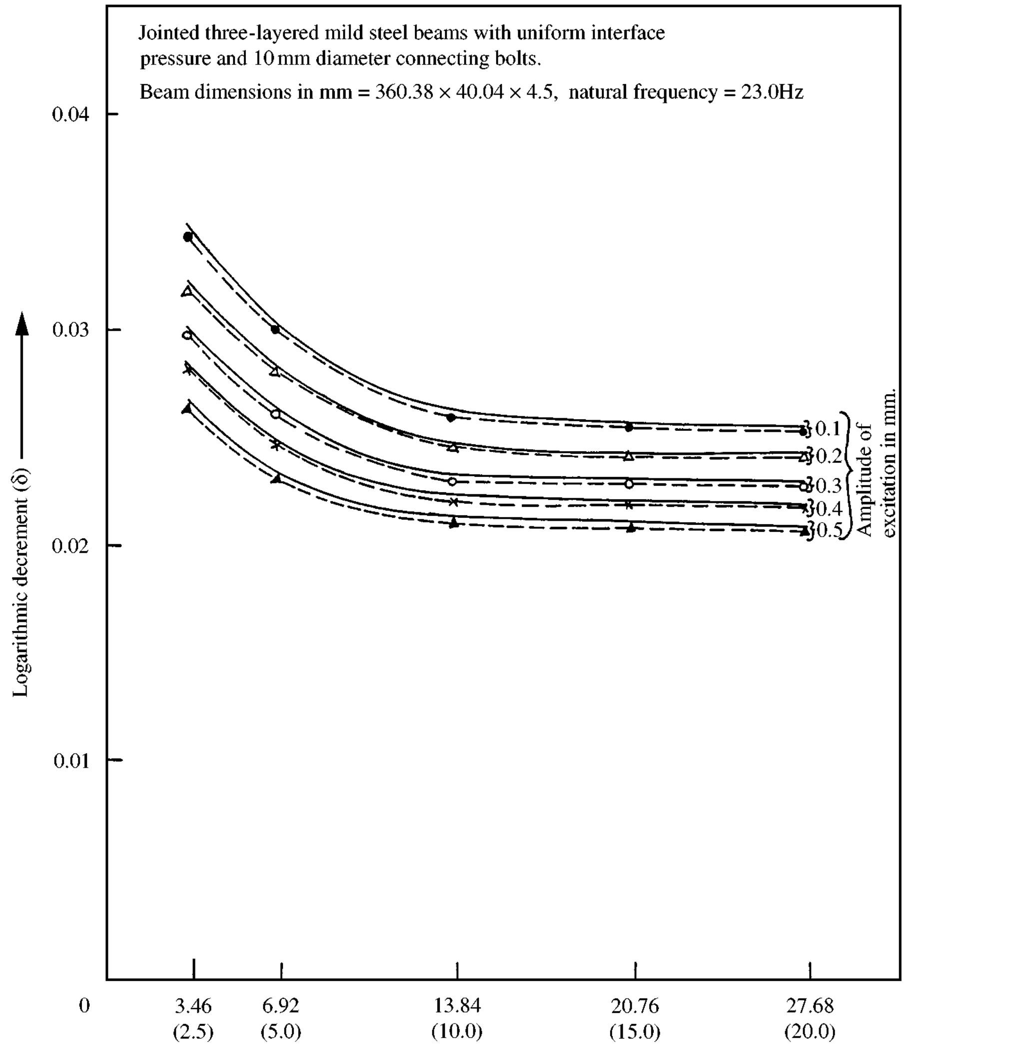

Figure 11. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts. —— Theoretical; – – – Experimental.

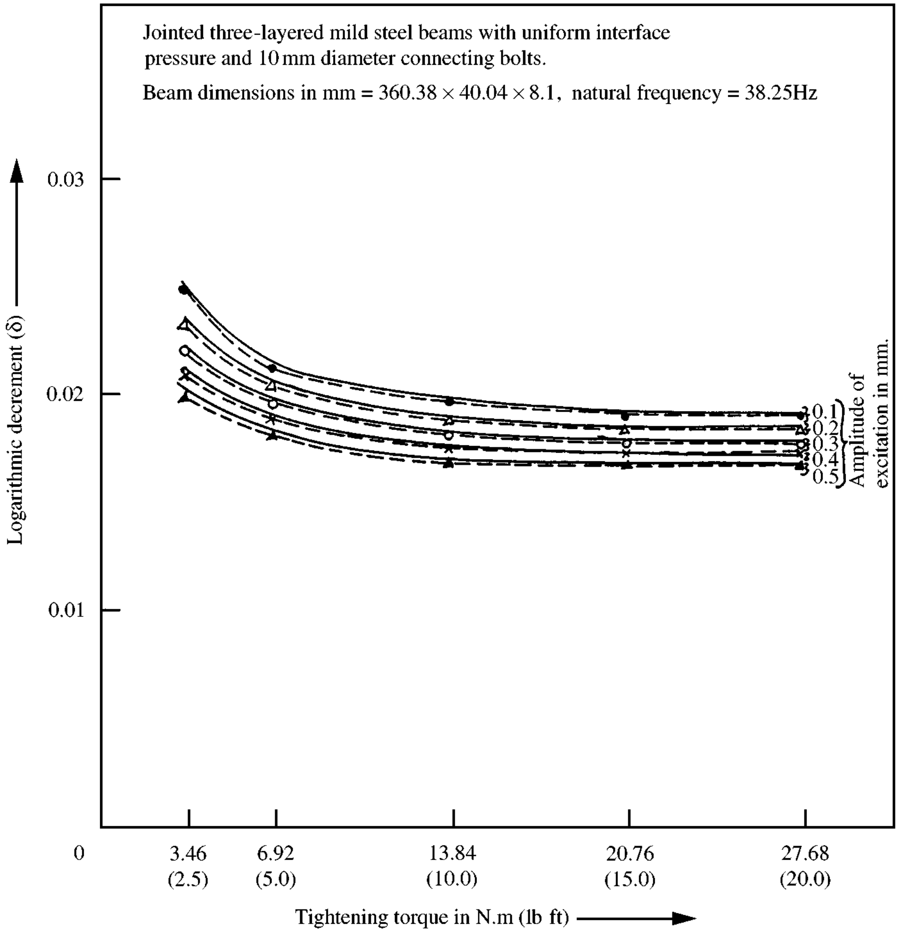

Figure 12. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts. —— Theoretical; – – – Experimental.

(1) It is established that the interface pressure between the contacting layers jointed by connecting bolts can attain uniformity throughout the contacting surfaces for a particular spacing of the connecting bolts which has been found to be 2·00211 times the diameter of the bolt.

(2) In the lower range of the tightening torque on the connecting bolts, the equivalent spring constant (static bending stiffness) of the layered and jointed structure is smaller than that of the equivalent solid one and increases with an increase in the tightening torque and remains constant beyond a particular value, i.e., 10·370 N m (7·50 lb ft).

(3) The natural frequency of first mode vibration of any layered and jointed structure does not vary appreciably with an increase in the tightening torque on the connecting bolts.

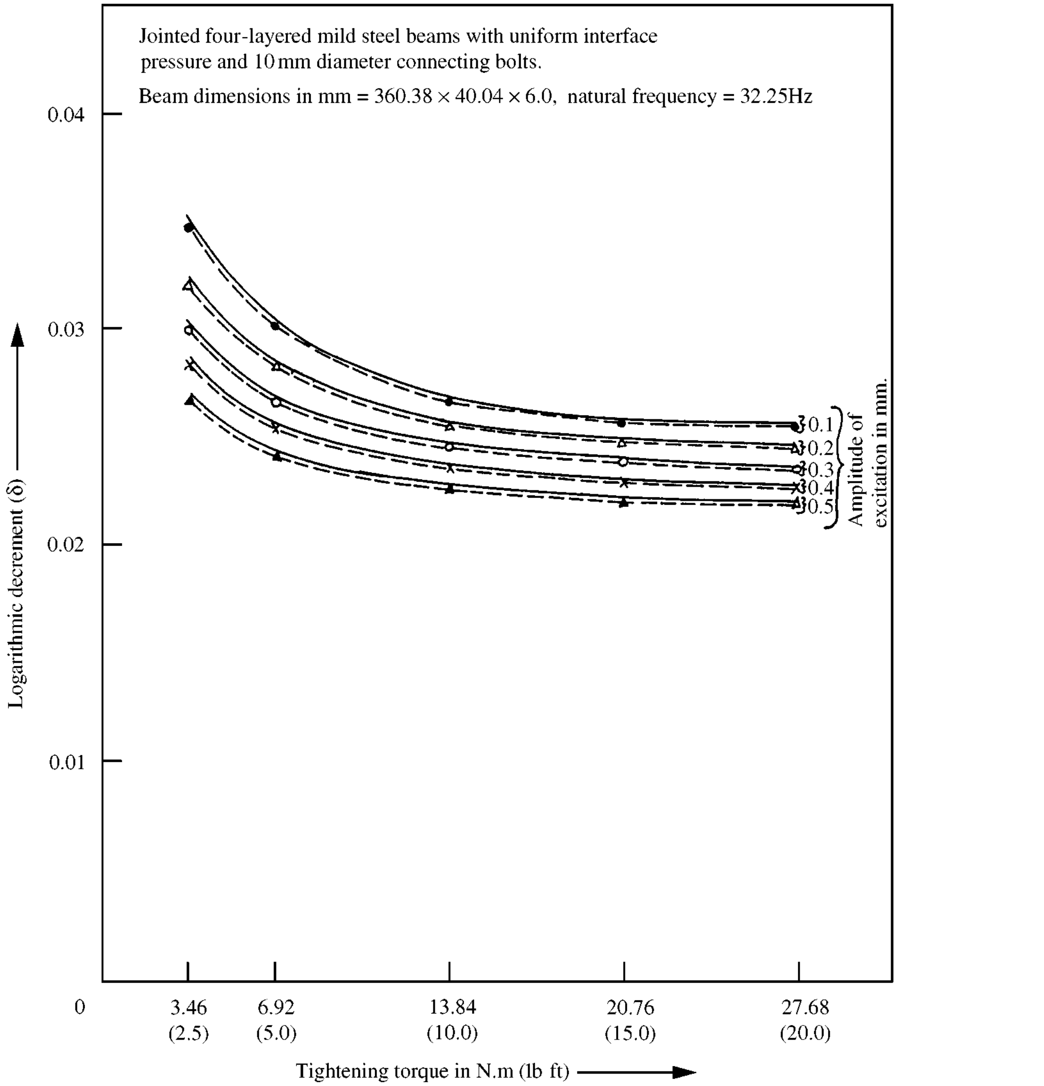

Figure 13. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts. —— Theoretical; – – – Experimental.

(4) It is found that the logarithmic damping decrement decreases with an increase in tightening torque on the connecting bolts owing to higher interface pressure with lower dynamic slip ratio at the interfaces which tend to behave like a solid beam. However, the logarithmic damping decrement increases with an increase in tightening torque in the lower range as established by Masuko *et al.* [1] which is not viable in real applications.

(5) Logarithmic damping decrement also decreases with an increase in initial amplitude of excitation due to introduction of higher energy into the system compared to that of the dissipated energy due to interface friction.

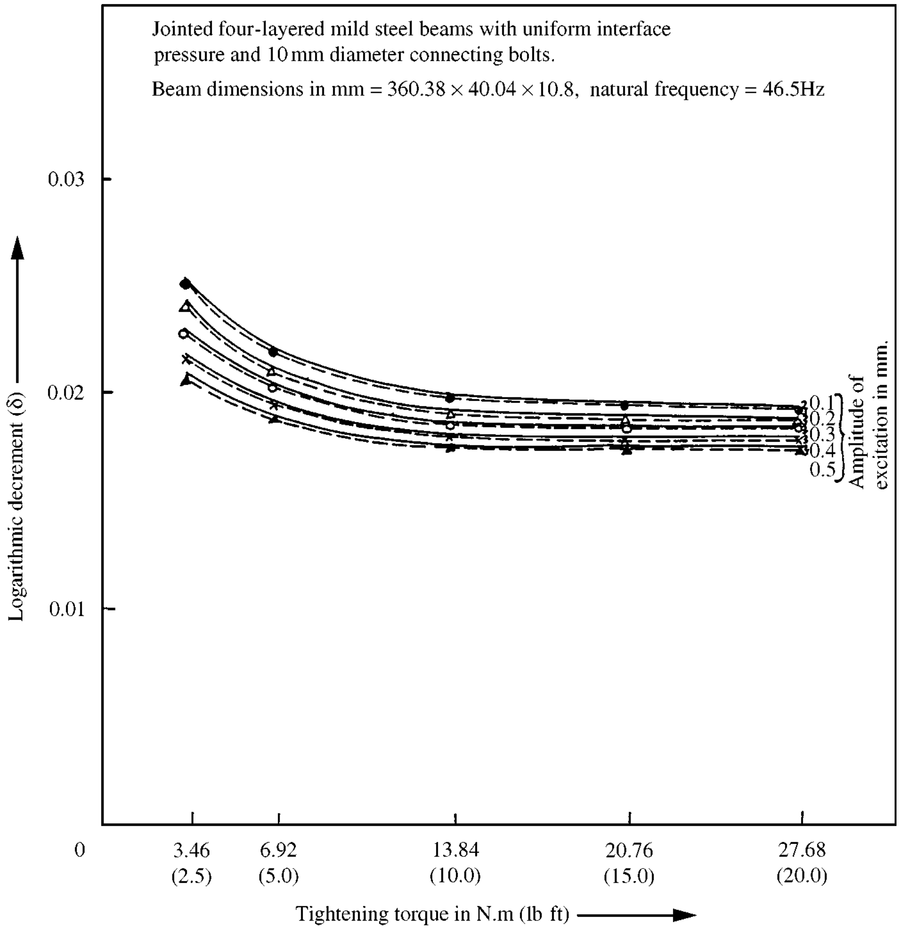

Figure 14. Variation of logarithmic decrement (δ) with applied tightening torque on the connecting bolts. —— Theoretical; – – – Experimental.

(6) The arrangement of the connecting bolts has an influence on the logarithmic damping decrement of layered and jointed structures. The logarithmic damping decrement increases with a decrease in distance between consecutive bolts due to an increase in average interface pressure and attains maximum under the condition of uniform intensity of pressure distribution at the interfaces. It is not practically possible to reduce the distance beyond this value.

(7) Logarithmic damping decrement increases with an increase in the number of layers in a layered and jointed structure due to an increase in the interface friction layers which causes an increases in the energy loss due to interface friction.

Finally, it is established that the damping capacity of the layered and jointed structure can be improved under uniform interface pressure condition by using connecting bolts with minimum possible tightening torque on them as well as with a larger number of layers. This increase in logarithmic damping decrement may go even up to 489·6%.

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APPENDIX: NOMENCLATURE

a_1	amplitude of vibration of the first cycle
a_{n+1}	amplitude of vibration of the $(n+1)$th cycle
b	width of the specimen
D_{B}	diameter of the connecting bolt
E_{loss}	energy loss due to interface friction per cycle
E_n	energy stored in the system with amplitude of vibration a_1
E_{n+1}	energy stored in the system with amplitude of vibration a_{n+1}
E_{ne}	energy stored in the system per cycle
E	static bending modulus
$2h$	thickness of each layer of the cantilever specimen
I	second moment of area
k'	equivalent spring constant (static bending stiffness) of the solid cantilever beam
k	equivalent spring constant (static bending stiffness) of the layered and jointed beam
l	free length of the layered and jointed cantilever beam
m	number of layers
n	number of cycles
P	axial load on the connecting bolt due to tightening
p	interface pressure due to tightening load
R_{B}	radius of the connecting bolt
R	any radius within the influencing zone around the connecting bolt
T	tightening torque applied on the connecting bolt
$u(x, t)$	relative dynamic slip between interfaces in the absence of a friction force at a bolted joint
$u_r(x, t)$	relative dynamic slip between interfaces at a bolted joint in the presence of a friction force
W	static load
$y(1, 0)$	initial amplitude of excitation at the free end of the specimen
α'	ratio of equivalent spring constant (static bending stiffness) of the layered and jointed cantilever beam to that of a solid one (k/k')
α	dynamic slip ratio
δ	logarithmic damping decrement
ψ	damping ratio
μ	kinematic coefficient of friction
Δ	deflection due to static load
ω_{n}	natural frequency of vibration
σ_{s}	surface stress on the jointed structure due to tightening load