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AN ADDITIONAL CONTRIBUTION ON THE TRANSVERSE VIBRATION OF A UNIFORM CIRCULAR THICK BEAM WITH NON-CLASSICAL BOUNDARY CONDITIONS

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It is well known that the determination of the normal modes and natural frequencies of a vibrating uniform beam initially loaded with an axial compressive (or tensile) load is a classical problem, and its various refinements have received considerable attention in important theoretical investigations that are concerned with this area.

The recent letter by Karunendiran *et al.* [1] presents exact solutions of frequency equations and mode shapes, and also investigates the effects of axial loads on the natural frequencies of a uniform circular Timoshenko beam with non-classical boundary conditions.

It is interesting to indicate that Farghaly [2] has derived an exact expression for the natural frequencies of restrained beams subjected to a constant axial compressive load considering the rotatory inertia and the shear deformation effects. In addition, the functional relation of the recent note [1] offers an excellent opportunity for verification by comparison with the equation derived by Saito and Otomi [3]. This paper was concerned with the problem of the vibration and stability of an elastically supported Timoshenko beam carrying an attached mass under axial and tangential loads, and the vibration characteristics of such beams and the influence of translational and rotational spring supports on the critical load were clarified.

Furthermore, when both ends of a Timoshenko beam subjected to an axial load are laterally immovable, the results obtained by Kunukkasseril and Arumugan [4] should be taken into account.

More recently, the formulation described by Maurizi *et al.* [5] was used by Bellés and Maurizi [6,7] to study the first five natural frequencies of unconstrained short beams for four different boundary conditions: free–free, free–guided, free–hinged and guided–guided. The methodology employed in reference [5] has been followed in the investigation reported for a limited number of values of shear coefficient k and for different values of r/L (radius of gyration of the cross-section/beam span).

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The frequency parameters, wherein the effects of the shear deformation and rotatory inertia were considered, have been listed here in Tables 1–3 with the previously stated boundary conditions, for $r/L = 0{\cdot}1, 0{\cdot}04, 0{\cdot}02, 0{\cdot}01$ and $0{\cdot}002$ and for $k = 0{\cdot}833$ (rectangular section), 0·851 (circular section) and 0·439 (standard I beam section, PN16). The value of the shear coefficient of the circular section was found by the theory of elasticity [8, 9] and the one corresponding to the standard I section was determined by using the approximated expression eh/A, where e is the web thickness, h is the web depth and A is the area [8]. On the other hand, the Poisson ratio μ is taken equal to 0·30.

The commonly accepted notation $\lambda = L(m\omega^2/EI)^{\frac{1}{4}}$ is adopted for the frequency parameters transcribed in Tables 1–3. Here L is the length of the beam, m is the mass per unit length, ω is the radian frequency, E is Young's modulus and I is the moment of inertia.

The following conclusions can be drawn from the results presented in the Tables:

(1) In all cases, the frequency parameter λ increases with the increase in the magnitude of the shear coefficient k. At the limit, where k tends to infinity, only the effect of rotatory inertia is considered. Obviously, $r/L \rightarrow 0$ represents the Bernoulli–Euler model.

TABLE 1

Frequency parameter λ—rectangular section ($k = 0{\cdot}833$)

Boundary conditions	Mode number	r/L 1/10	1/25	1/50	1/100	1/500
Free–free	1	0	0	0	0	0
	2	0	0	0	0	0
	3	4·097799	4·581156	4·689689	4·719729	4·729625
	4	5·815055	7·241058	7·669415	7·804596	7·851223
	5	7·177848	9·589881	10·52298	10·86460	10·99017
Free–guided	1	0	0	0	0	0
	2	2·254851	2·344845	2·359864	2·363724	2·364969
	3	4·559438	5·261491	5·432302	5·480948	5·497123
	4	6·240542	7·858847	8·397820	8·574776	8·636735
	5	7·457466	10·12780	11·20886	11·62028	11·77427
Free–hinged	1	0	0	0	0	0
	2	3·497402	3·834708	3·902298	3·920435	3·926354
	3	5·461376	6·603298	6·932510	7·032943	7·067135
	4	6·905944	9·031614	9·824144	10·10448	10·20581
	5	7·639887	11·15447	12·55059	13·12059	13·34201
Guided–guided	1	0	0	0	0	0
	2	2·896094	3·093007	3·128983	3·138409	3·141465
	3	5·034469	5·941931	6·186013	6·257967	6·282164
	4	6·642783	8·453893	9·115181	9·341004	9·421333
	5	7·943233	10·65025	11·88386	12·37203	12·55821

TABLE 2
Frequency parameter λ—circular section ($k = 0{\cdot}851$)

Boundary conditions	Mode number	r/L 1/10	1/25	1/50	1/100	1/500
Free–free	1	0	0	0	0	0
	2	0	0	0	0	0
	3	4·101365	4·582363	4·690045	4·719822	4·729629
	4	5.828460	7·246877	7·671447	7·805162	7·851246
	5	7.199261	9·602850	10·52841	10·86625	10·99024
Free–guided	1	0	0	0	0	0
	2	2·255697	2·345022	2·359911	2·363736	2·364969
	3	4·567246	5·264207	5·433123	5·481165	5·497132
	4	6.257796	7·867447	8.400949	8·575662	8·636771
	5	7.486045	10·14455	11·21613	11·62252	11·77436
Free–hinged	1	0	0	0	0	0
	2	3·501167	3·835724	3·902581	3·920507	3·926357
	3	5·473841	6·608588	6·934264	7·033422	7·067154
	4	6·928642	9·044089	9·829115	10·10595	10·20587
	5	7·677477	11·17576	12·56058	13·121381	13·34215
Guided–guided	1	0	0	0	0	0
	2	2·899267	3·093742	3·129180	3·138459	3·141467
	3	5·046681	5·946608	6·187485	6·258360	6·282180
	4	6·664666	8·465717	9·119661	9·342295	9·421387
	5	7·973839	10·67106	11·89322	12·37497	12·55834

TABLE 3
Frequency parameter λ—standard I section ($k = 0{\cdot}439$)

Boundary conditions	Mode number	r/L 1/10	1/25	1/50	1/100	1/500
Free–free	1	0	0	0	0	0
	2	0	0	0	0	0
	3	3·950689	4·530263	4·674436	4·715715	4·729462
	4	5·322174	7·006639	7·583927	7·780357	7·850209
	5	7·523835	9·090979	10·29991	10·79478	10·98713
Free–guided	1	0	0	0	0	0
	2	2·219475	2·337218	2·357858	2·363216	2·364948
	3	4·266903	5·149954	5·397393	5·471624	5·496742
	4	5·647652	7·522671	8.267683	8·536945	8·635138
	5	7·735342	9·500959	10·91434	11·52548	11·77008
Free–hinged	1	0	0	0	0	0
	2	3·348442	3·791960	3·890178	3·917298	3·926228
	3	5·013876	6·391438	6·858752	7·012454	7·066285
	4	7·129822	8·554923	9·620069	10·04214	10·20312
	5	7·568628	10·37277	12·15083	12·98484	13·33586
Guided–guided	1	0	0	0	0	0
	2	2·772437	3·062022	3·120540	3·136250	3·141378
	3	4·604027	5·754850	6·124043	6·241079	6·281468
	4	5·919505	8·004125	8·931103	8·286076	9·418988
	5	7·877266	9·891233	11·50970	12·24809	12·55267

(2) Comparing the results corresponding to rectangular and circular sections (for the same values of r/L and identical boundary conditions) one can conclude that the influence of the above-mentioned cross-section types on the first five natural frequencies is very small from a practical viewpoint.

(3) It should be pointed out that a remarkable good agreement is observed between the values so computed and the ones obtained by Shastry and Venkateswara Rao* —via a finite element formulation—in reference [10].

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REFERENCES

1. S. Karunendiran, W. L. Cleghorn and J. W. Zu 1999 *Journal of Sound and Vibration* **220**, 186–191. Transverse vibration of a uniform circular thick beam with non-classical boundary conditions.
2. S. H. Farghaly 1993 *Journal of Sound and Vibration* **161**, 181-183. Comments on "The general equation of frequencies for vibrating uniform one-span beam under compressive axial loads".
3. H. Saito and K. Otomi 1979 *Journal of Sound and Vibration* **62**, 257-266. Vibration and stability of elastically supported beams carrying an attached mass under axial and tangential loads.
4. V. X. Kunukkasseril and M. Arumugan 1975 *Journal of the Acoustical Society of America* **57**, 89-94. Transverse vibration of constrained rods with axial force fields.
5. M. J. Maurizi, R. E. Rossi and P. M. Bellés 1990 *Journal of Sound and Vibration* **141**, 359-362. Free vibration of uniform Timoshenko beams with ends elastically restrained against rotation and translation.
6. P. M. Bellés and M. J. Maurizi 1994 *Internal Report* 1/94, *Department of Engineering*, *Universidad Nacional del Sur*, *Argentina*. Free vibrations of unconstrained Timoshenko beams.
7. M. J. Maurizi, N. M. Auciello and M. A. De rosa 1995 *Pubblicazione* No 1/95, 86-92, *Facoltà di Ingegneria*, *Università degli Studi della Basilicata*, *Italia*. Le travi di, Timoshenko: Sommari di contributi scientifici.
8. O. Belluzzi 1963 *Scienza delle Costruzioni*, Vol. **1**, 247 Bologna, Italy: Zanichelli editore.
9. G. R. Cowper 1966 *Journal of Applied Mehanics* **33**, 335-340. The shear coefficient in Timoshenko's beam theory.
10. B. P. Shastry and G. Venkateswara rao 1985 *Journal of Sound and Vibration* **102**, 442-444. Free vibrations of short unconstrained beams.

*A 16 element idealization of the beam was used.