Journal of Sound and Vibration (1999) **227**(2), 409–425
Article No. jsvi.1999.2352, available online at http://www.idealibrary.com on IDEAL®

ON THE DEBOSSING, ANNEALING AND MOUNTING OF BELLS

R. PERRIN*, G. M. SWALLOWE, T. CHARNLEY AND C. MARSHALL
Physics Department, Loughborough University, Loughborough, Leicestershire, LE11 3TU, England

(*Received* 16 *September* 1998, *and in final form* 29 *April* 1999)

Changes in the frequencies of the musical partials of various types of bells following debossing dismounting/mounting and annealing/quench annealing are reported. Debossing, dismounting and quench annealing lead to frequency drops, while mounting gives rises. Annealing can lead to frequency increases or decreases depending upon the maximum temperature employed and the initial residual stress. Qualitative explanations of these phenomena are given in terms of changes in crown stiffness, internal stress and alloy phase structure. These are supported by the results of X-ray diffraction measurements. Although the effects are all small they can be large enough to be detected by a reasonably musical car. This, together with the fact that the effects cannot be controlled, gives a plausible explanation of why modern bellfounders use vertical lathes for tuning, even with small carillon bells, and do not anneal bells when trying to control warble.

1. INTRODUCTION

1.1. PARTIAL SOUNDS AND NORMAL MODES OF BELLS

Analysis of the sound from a representative medium-sized church bell reveals contributions from perhaps 60 different frequency components or "partials" [1]. At least the five lowest in frequency, often called the "musical" partials, are of sufficient importance to the acoustics to merit individual tuning by the founder. Details of their properties, and those of the associated normal modes, for a modern tuned church bell are given in Table 1, where the traditional English names are used. With such a bell the "strike note" is often considered to coincide with the fifth musical partial, or Nominal [2], although some workers consider it to coincide with the second partial, or Fundamental, which is an octave lower. With smaller bells fewer partials contribute significantly, depending on size and profile shapes [2]. It can be as few as two or even only one with small (thick) carillon bells or with small (thin)

---

* Also at Institute of Fundamental Studies, Massey University, Palmerston North, New Zealand.

0022-460X/99/420409 + 17 $30.00/0

Table 1

Properties of the five "musical" partials of a modern church bell

Name	Frequency ratio*	Musical interval	Nodal pattern	
			Meridians ($2m$)	Circles (n)
Hum	1	Tonic	4	0
Fundamental	2	Octave	4	1
Tierce	2·378	Minor 3rd	6	1 (waist)
Quint	2·997	Perfect 5th	6	1 (rim)
Nominal	4	2nd Octave	8	1 (waist)

* Based on the bell being "well tempered". In the literature the ratios for Tierce and Quint are often rounded up to 2·4 and 3 respectively.

handbells. Such bells usually have a strike note coinciding with their lowest frequency partial.

1.2. DEGENERACY STRUCTURE AND WARBLE

Group-theoretical arguments show that the basic axial symmetry of a bell requires most normal modes to occur in degenerate pairs or "doublets". Their modal functions vary like $\sin(m\theta)$ and $\cos(m\theta)$, respectively where θ is the age of rotation about the symmetry axis [3]. The values of m must be integers to make these functions single valued. The only non-degenerate modes are the axisymmetric ones. These have $m = 0$ and are of no acoustical importance. Thus, the nodal patterns contain $2m$ evenly spaced meridians, with those of one doublet member lying midway between those of its partner, and n circles parallel to the rim. These meridians run up the sides of the bell and all intersect at the point where the symmetry axis cuts the exact centre of the crown. This point is therefore always at rest in any mode with $m > 0$. For modes with $m > 2$ the bell is effectively clamped here. If the bell were truly axisymmetric the absolute locations of the meridians would be θ-wise indeterminate until fixed by initial conditions imposed by clapper impact. However, small symmetry breakings inevitably occur in real bells, fixing the absolute locations of the meridians and producing small frequency differences between doublet members. What is heard as a single partial in the bell's sound is a tone between these two frequencies with beating superimposed at their frequency difference. The beat amplitude depends on the clapper's strike point. This phenomenon is known as "warble". While a certain amount of it is desirable, at least in a church bell since it contributes to its individual character, an excess is to be avoided [4].

1.3. USE OF VERTICAL LATHES IN BELL TUNING

Church and carillon bells are tuned after casting by turning metal off their insides in an axially symmetric way. Much of the founder's skill lies in knowing how much metal to remove and from where [5–7]. Special lathes, which have the unusual feature of holding the bell with its axis vertical, are dedicated to this task. This is partly because such bells can be very massive indeed, weighing up to many tons. If placed in a conventional horizontal lathe, the chuck would be subjected to huge stresses making it impossible to keep the bell's symmetry axis accurately aligned during turning. Vertical lathes are also used elsewhere in industry when very heavy objects are worked.

A second advantage of using a vertical lathe is that there is no need to cast a boss onto the head of the bell for a chuck to grasp. With a horizontal lathe such a boss is needed and would have to be cut off after tuning was complete to allow the bell to be mounted in one of the conventional ways. Handbell makers always use horizontal lathes and cast their bells with some form of boss. Some of them use a normal cylindrical boss which is cut off after tuning, while others use a tapering boss or "tang" which is not. Needing to cut off a boss from a larger church or carillon bell and leave a flat enough surface might be a problem but should only be a minor inconvenience for smaller bells. Finite-element calculations predict and vibration measurements confirm that the central part of the crown of a bell takes very little part in the motions of the modes of any of the musical partials [6] with the exact centre always being at rest in line with symmetry requirements . This is exemplified in Figure 1 for a modern 214 kg D_5 Taylor church bell. This figure shows one half of a cross-section of the bell in the reference plane $\theta = 0$ containing the symmetry axis. It also shows the "in-plane" parts of the modal functions for each of the musical partials. Rotating the diagram about this axis, while letting the mode shapes vary like $\cos(m\theta)$. will generate the complete bell shape and standing wave pattern for one doublet member of the partial. Given the forms of these modal functions in the central crown region one would expect the influence of a central boss on these partials to be very small. This would also be in line with the empirical external bell tuning functions of van Heuven [7] and of Perrin *et al.* [6]. We were

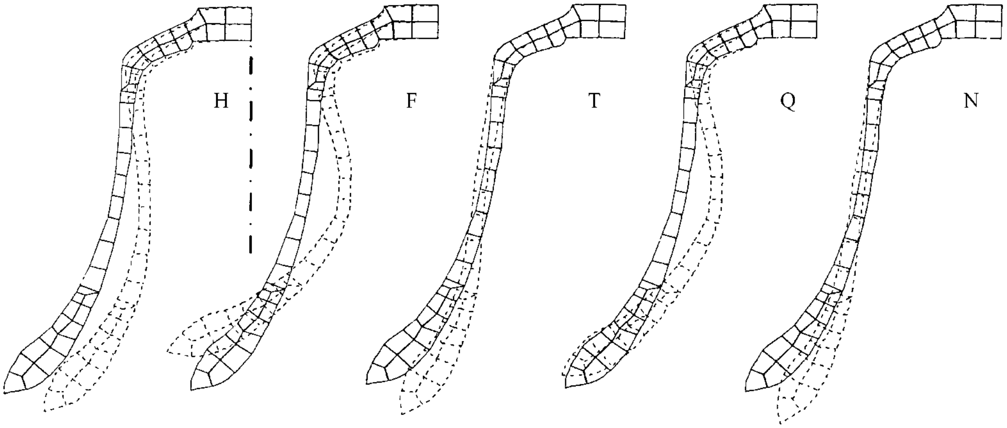

Figure 1. Modal shapes for the musical partials of a D_5 Taylor church bell predicted by finite-element calculations. Full lines represent the equilibrium configurations and dotted lines the modal shapes for the Hum (H), Fundamental (F), Tierce (T), Quint (Q) and Nominal (N). The structure shown for the crown ends on the bell's symmetry axis which is indicated by –· –· –· –· –·

therefore surprised to be told [8] that one reason why founders avoid horizontal lathes, even for tuning smaller carillon bells where the mass to be supported is not a problem, is that debossing can cause the frequencies of different partials to fall by *different* and sometimes *musically significant* amounts. Naturally, the debossed bell cannot then be fitted back into the horizontal lathe to be retuned. It would be useless to try to retune even if it could be so refitted because turning metal off a bell cannot *raise* the frequencies of the individual partials separately [6].

1.4. RATIONALE OF THE PRESENT WORK

One of the authors (RP) has had the experience of helping to tune an 80 kg church bell on a conventional horizontal lathe at a bell foundry in Australia. Rechecking the tuning after removal of the boss gave results which were not entirely as we expected but confirmed the bellfounder's traditional wisdom by yielding drops in all the musical partials, with a musically significant drop in the Hum. In an attempt to understand this phenomenon a number of experiments have since been performed on various types of bells. These have encompassed not only debossing but also other conceivably related phenomena arising when bells are mounted/dismounted using bolts passing through their crowns and when they are heat treated. In some of the later experiments we have been able to supplement vibrational/acoustical measurements with X-ray diffraction studies.

2. DEBOSSING OF BELLS

2.1. INITIAL EXPERIMENTAL RESULTS

The 80 kg bell with which we originally worked was cast for Bagot Bellfoundries in Adelaide using conventional bell metal (roughly 20% tin to 80% copper). It had a boss and was tuned using a horizontal lathe, its Hum note being C_5. In this bell the Hum was sufficiently dominant over the other partials for the strike note to be at its frequency, when struck by its clapper in the normal way on the sound bow, rather than the two octaves higher which would be expected in a larger church bell. We shall therefore refer to it as a "C_5" bell. Although this bell had some minor profile differences from the Taylor bell of Figure 1, the forms of their corresponding musical modes will be very similar indeed.

The five musical partials were measured in turn, after removal from the lathe, by tapping the bell at appropriate points with its clapper and using a Korg II tone tester to identify the dominant acoustic frequency. The doublets were all sufficiently split for us to measure the components' tones separately, by tapping at nodal meridians of each in turn. The results are listed in Table 2 in cents relative to ideal conventional values. Also shown are doublet means and splittings. Using the convention that $A_4 = 440$ Hz, this bell would ideally have its Hum at 523·25 Hz with its other musical partials' frequencies given by the ratios in Table 1. The measurements were repeated after the boss had been cut off: the new values are also given in the table. All the tones fell but the Hum fell far more than the others. Doublet splittings were almost unchanged, signifying that the degree of axial

TABLE 2

Musical partials of an 80 kg "C_5" bell before and after debossing; Errors estimated at ± 1/2 cent; all figures in cents relative to equitempered ideal values

Partial	With boss			Without boss			Change		$\Delta f/f$
	Upper	Lower	Mean	Upper	Lower	Mean	Mean	Splitting	
Hum	+ 6	+ 2	+ 4	− 11	− 15	− 13	− 17	0	0·0099
Fundamental	− 1	− 7	− 4	− 5	− 11	− 8	− 4	0	0·0023
Tierce	+ 10	+ 8	+ 9	+ 9	+ 7	+ 8	− 1	0	0·0006
Quint	− 5	− 7	− 6	− 7	− 9	− 8	− 2	0	0·0012
Nominal	− 2	− 4	− 3	− 2	− 5	− 3·5	− 0·5	+ 1	0·0003

symmetry was unaltered by boss removal. For readers unfamiliar with cents we also include in the table the changes in frequency as a fraction of each doublet's initial mean frequency: i.e., $\Delta f/f$.

The minimum interval change the ear can discern with a pure tone depends upon both its frequency and intensity. There is some disagreement between authors about the exact values of this "just noticeable difference" [9–11]. When several frequencies are present, as with larger bells and most musical instruments, the situation is even more complicated. However, the minimum interval an average musician can discern is often reckoned to be about 10 cents ($\Delta f/f = 0{\cdot}0058$) [9, 12], so it seems reasonable to use this as the criterion when considering bell partials. On this basis a 17 cents fall in the Hum of the C_5 Bagot bell is of practical significance, but the changes in the other partials are not.

2.2. QUALITATIVE EXPLANATION

Since 17 cents represents a 1% frequency change there is evidently no major conflict between our original expectation of very small frequency changes and the founders' reports of musical significance. However, some explanation of the relative sizes of these small drops is still required.

Continuity and differentiability of the modal functions, plus the symmetry-based requirements that they vanish at the exact centre of the crown for $m > 0$, means that the in-plane parts of the functions must fall monotonically to zero as one moves inwards across the crown in any fixed vertical plane. It is also clear, upon bearing in mind that the $2m$ antimodal meridians also intersect at the same point, that the greater the value of m the more rapid the fall to zero will be: i.e., the greater the effective stiffness of the central crown region. Thus, for all the musical partials the exact centre of the crown is rigidly fixed while the rest of the crown region will be extremely stiff with the stiffness getting greater as m increases. All this is true for any profile of bell, whether it has a boss or not, provided the crown is complete. Figure 1 shows that all the in-plane parts of the modal function behave in this way. If we were to add a boss to the central crown region now it would increase the stiffness and so increase all the frequencies. However these changes would all be small, or even zero, because adding the boss is merely increasing somewhat the stiffness of region which indeed is already very stiff. Starting with a boss on the bell and then removing it would obviously have the inverse effect and produce small falls in frequency.

The *relative* sizes of the frequency drops in Table 2 can also be understood qualitatively from the modal forms of the musical partials' modes. The Tierce and Nominal have modes with a nodal circle in the waist, little motion above that circle and almost none at all above the shoulder. They would therefore be expected to be almost unaffected by debossing, but with frequency falls being greater for the Tierce ($m = 3$) than for the Nominal ($m = 4$) because of their m values. The other three musical partials all involve more significant motion around the shoulder and up into the crown. The details of their amplitudes in these regions are very similar indeed. However, the Fundamental and Quint both have one nodal circle towards the rim of the bell and consequently are subject to considerable bending in the waist region. The Hum is the only partial to have no nodal circles and, while it has some bending in the waist, this is less than that for the other two partials. Consequently, when bending stiffness in the crown is reduced it is relatively more important for the Hum (which has $m = 2$) than the other two, so its frequency falls most. Since Fundamental and Quint have very similar in-plane modal functions, their falls in cents are roughly equal, but the Fundamental ($m = 2$) falls slightly more than the Quint ($m = 3$).

2.3. TANG REMOVAL

If this proposed "crown bending stiffness" mechanism is correct then cutting off the tang from a handbell should also produce small frequency drops. Falls of "20 or 30 cents" have indeed been reported by Rossing and Sathoff [13] but without giving details or offering an explanation. For a handbell the falls they described must have been in the Hum tone.

2.4. INTERNAL STRESS

Following casting there is always some residual stress. The quicker the cooling the greater this is likely to be. This stress will change local inter-atomic distances and so move equilibrium configurations to slightly different positions on the interatomic force versus distance curve which will change the local values of Young's modulus and hence modal frequencies. Cutting off a boss will relieve some of this residual stress in the region at the top of the crown and so the modal frequencies will move back towards their unstressed values. The arguments used in section 2.2 for the *relative* sizes of the frequency changes in musical partials produced by changes in upper crown stiffness apply equally well here. Thus internal stress relief provides another mechanism which could contribute to the observed frequency changes. While there are theories for some pure metals, there is no satisfactory theory for the change of Young's modulus with strain for alloys [14].

One expects intuitively that the post-casting residual stress in the top of the crown will be compressive and hence cause the modal frequencies to be raised. However, this is contrary to the implications of results reported half a century ago by Jan Arts [15, 16]. He found that quench annealing (which would certainly have increased internal stress) led to *falls* in partial frequencies. Subsequent heating and slow cooling (which would have reduced the internal stress again) sent the frequencies back up. This would appear to exclude local internal stress relief as the main mechanism for producing frequency drops on debossing. The observation that frequencies do in fact drop on debossing leads to the conclusion that falls due to crown bending stiffness reduction must exceed any rises due to internal stress relief.

3. ANNEALING OF BELLS

3.1. THE CONTROL OF WARBLE

The breaking of axial symmetry leading to "warble" can be due to geometrical and/or metallurgical imperfections. With large bells, weighing many tons, casting is usually done in a sand pit in the ground where they remain for many days to cool slowly and uniformly. This ensures axial symmetry in the metallurgy and minimizes internal stress. It has been reported by the present authors [17] that small carillon bells cast in square sand boxes above ground are inclined to warble badly but that this problem can be solved by casting in the ground, as with large bells. Founders sometimes use a technique called "grinding" to try to reduce warble. This requires considerable skill and can usually be applied to only one partial [4]. A second partial can have its warble controlled by a suitable selection of clapper strike point to minimize its beat amplitude. Beyond this little can be done short of recasting the bell.

If the asymmetry were due to internal stress resulting from, say, directionally preferential cooling, then it should be possible to remove it by annealing. Geometrical defects and blow holes would, of course, not be removed by this treatment. Attempts to remove warble by annealing have been reported to us [8] as having been unsuccessful in that, no matter what it might do to the warble, the overall tuning of the bell can be sploiled with different changes in the pitches of different partials. One of the authors (RP) was also involved in a preliminary study of this effect at the same Australian foundry using the raw casting of a modern handbell. Again the results confirmed conventional craft wisdom. However, initially they appeared to contradict the results of Arts.

3.2. RESULTS ON ANNEALING A RAW CASTING

The raw casting used in the experiment was originally destined to be a Malmark C_5 handbell. It had a mass of 1·42 kg and a strike-note of about E_6. Large amounts of metal, about two-thirds by mass, would have been subsequently turned off axi-symmetrically both inside and outside to bring the note down to C_5. The spectrum of a handbell is very much simpler than that of a church bell. Only two partials generally contribute significantly to the sound [2] and the strike note is determined entirely by the lowest one. Having two nodal diameters and no nodal circle, this is analogous to the church bell Hum.

The raw casting was struck near the rim with a standard Malmark E_6–G_7 mallet and its strike-note measured using the Korg II tone tester. The doublet components were (E_6 + 2 cents) and (E_6 − 5 cents), i.e., a mean frequency of 1317·85 Hz and a splitting of 7 cents (5·3 Hz at this frequency). The bell was then heat treated *in vacuo* by raising its temperature to 610°C for 2 h and subsequently allowing it to cool to room temperature over a 2 h period. The doublet components were then remeasured and found to have fallen to (E_6 − 15 cents) and (E_6 − 22 cents). Thus, the doublet splitting remained unchanged at 7 cents while the mean tone fell by an easily audible 17 cents. This is equivalent to a fall of 13·35 Hz, which is close to 1%.

3.3. DISCUSSION OF ANNEALING RESULTS

This fall in frequency on annealing appears to contradict the work of Arts who reported that heating a previously stressed bell until it glowed (i.e. to about 600°C) and then allowing it to cool slowly (i.e., relieving some of the internal stress) caused partial frequencies to rise. However, we held our bell at 610°C for 2 h before the slow cooling, which he evidently did not. Metallic phase changes are not instantaneous and can require many hours for a significant fraction of the metal to phase transform [18]. (In fact, "as cast" bell metal consists of a mixture of alpha and delta phases because the transformation to epsilon takes place extremely slowly, requiring about a 1000 h for a significant fraction of epsilon to be produced.) A possible explanation of the discrepancy is therefore that our treatment produced structural changes in the metal while his did not. This clearly required further investigation which the facilities available to us at the time did not permit.

While the fall of 17 cents was significant it should be remembered that the stress relief has taken place throughout the entire structure. Stress relief on debossing would occur primarily over that 2% of the bell's surface which is at the top of the crown and would fall away rapidly as one moves down the bell. It is therefore very likely that a change due to stress relief from debossing alone would be no more than about 1 cent. So, even if Arts were wrong, we would still expect the bending stiffness effect to dominate the debossing phenomenon.

4. A NEW SERIES OF DEBOSSING AND ANNEALING EXPERIMENTS

4.1. AIMS OF THE EXPERIMENTS

A special small carillon bell with a very long boss was cast for us by Messrs John Taylor & Co. Its profile, after having been debossed, is shown in Figure 2. The bell's rim diameter was 14·8 cm, vertical height 10·3 cm and mass after debossing 4·522 kg. The circular boss was of length 19 cm and diameter 4 cm. Such a boss should guarantee complete rigidity over the central crown region. For a carillon bell of this size only the Hum is musically important which, being at about 3 kHz gave the bell a strike note of about $F^{\#}_7$. Our intention was to use this bell to try to

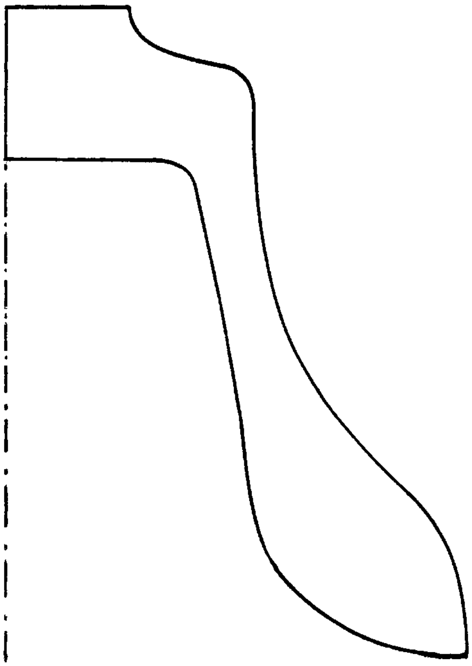

Figure 2. Profile of 4·5 kg small Taylor carillon bell after boss removal. Symmetry axis indicated by –·–·–·–·–·

unravel the contributions to partial frequency changes on debossing coming from (1) pure internal stress relief and (2) pure crown bending stiffness changes. It was also hoped to resolve the discrepancy in sign of frequency changes between our preliminary annealing experiment and the results of Arts.

4.2. USE OF X-RAY DIFFRACTION

After preliminary modal measurements had been made a slice of the boss about 1 cm in length was cut off from its remote end. This slice was retained for use in X-ray diffraction measurements. Its initial X-ray spectrum was obtained and it was subsequently subjected to exactly the same sequence of heat treatments as the residual bell. A fresh spectrum was obtained by using the slice after each of these treatments. By looking at small changes in the *positions* of the important spectral peaks it was possible to make some estimate of the change in internal strain [19, 20]. However, since the slice was no longer attached to the bell, the quantitative changes it gave, may not have been representative of the bell itself. Likewise any changes in phase structure of the bell metal could be checked by comparing the *heights and positions* of the peaks.

4.3. EXPERIMENTAL PROCEDURE AND RESULTS

The frequencies and nodal patterns of the first few partials were measured, before removing the 1 cm slice, but this time in a fully equipped laboratory rather than

Table 3

Hum and Tierce for a 4·5 kg small Taylor carillon bell before and after various heat treatments and stages of boss removal; the mean of each split pair is quoted; errors are estimated overall at about ± 0·3 Hz on the means and ± 0·3 cents on the changes

Bell condition (see text)	Mean frequency (Hz)		Changes (cents)		Splittings (Hz)	
	Hum	Tierce	Hum	Tierce	Hum	Tierce
(1) As cast (19 cm boss)	2996·9	7029·3	—	—	25·8	8·0
(2) Boss cut to 5 cm	2995·9	7026·1	– 0·6	– 0·8	27·7	7·7
(3) Annealed at 340°C	2999·1	7032·5	+ 1·8	+ 1·6	27·8	8·7
(4) Complete boss removal	2987·8	7028·0	– 6·5	– 1·1	27·5	8·1
(5) Re-annealed at 340°C	2990·1	7034·2	+ 1·3	+ 1·5	27·8	8·4
(6) Quench annealed from 340°C	2966·8	6975·5	– 13·5	– 14·5	28·5	7·6
(7) Re-annealed at 340°C	2985·5	7022·0	+ 9·6	+ 9·8	27·0	8·0
(8) Annealed at 610°C	2980·5	7009·0	– 2·9	– 3·2	27·0	6·0

with the portable equipment used in the preliminary experiments in Australia. More accurate methods, described previously in detail by some of the present authors [1, 3], were now able to be employed.

After having established that the removal of the 1 cm slice had no significant effect on the partial frequencies, the boss was cut down to 5 cm. Even this gave changes of less than 1 cent. The doublet averages and splittings for the 19 and 5 cm lengths are incorporated into Table 3 for the Hum, the only musically important partial for this bell, and for the Tierce, as a representative of the higher partials.

The bell, with the remaining 5 cm boss, and the separate 1 cm slice were now annealed by being heated at 340°C in an oven for 2 h *in vacuo* (~6 μbar) and then allowed to cool slowly under a flow of inert gas. The frequencies *increased*, as shown in Table 3, in line with expectations from Arts. However, these increases were of only a couple of cents and due to stress relief throughout the *entire* bell structure. Relief from conventional debossing would be largely restricted to the central crown region, about 2% of the total bell surface area, and so would result in much smaller frequency falls than this. The temperature of 340°C was selected by reference to the Sn–Cu alloy phase diagram [18, 21], reproduced in Figure 3, as being low enough to guarantee that no phase changes would occur yet high enough to produce some internal stress relief. The X-ray spectra confirmed that no significant phase changes had occurred and that residual stress had been reduced.

When the remaining 5 cm of boss was cut off there were further falls in partial frequencies with 6·5 cents for the Hum and 1·1 cents for the Tierce. This makes it clear that the primary mechanism responsible for the church bell result was almost certainly crown stiffness with any stress relief being only a minor factor.

In order to check that we had not seriously underestimated the influence of stress relief due to 2 h being an insufficient time the whole annealing process was repeated.

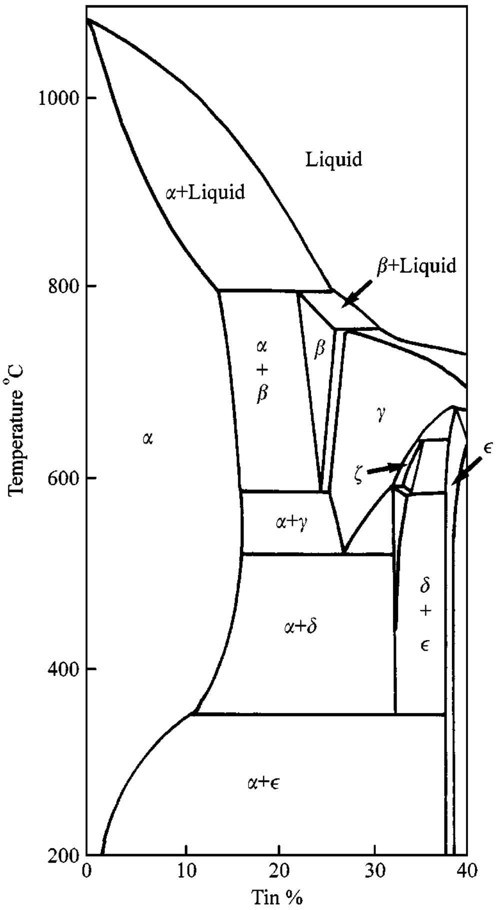

Figure 3. Phase diagram for the Cu–Sn system, after Hedges.

There were further increases in frequencies, but only of the size similar to those from the first application of the process.

The bell was again heated up to 340°C, this time for $1\frac{1}{2}$ h and not under reduced pressure, and then dropped into a bucket of cold water. This quench annealing led to relatively large frequency drops of about 14 cents. Annealing again at 340°C *in vacuo* for 2 h brought the frequencies back up by about 10 cents: i.e., almost back to the lowest stress state. These stress changes were again consistent with the X-ray diffraction results, which still indicated no changes in phase.

While these results were entirely consistent with those of Arts, they were not so with those of our preliminary experiment with the handbell raw casting. To try and understand this we took the small carillon bell, now in a state of low internal stress, and annealed it again for 2 h under *vacuo* but this time using 610°C as in the preliminary experiment. This led not to frequency increases but rather to drops of about 3 cents, 610°C is still well below the alloy's melting point but, as can be seen from Figure 3, is high enough to allow the possibility of changes from the delta into the beta and gamma phases. These transformations proceed slowly [18]. It was expected that a period of 2 h would be sufficient to allow these changes to occur but that heating to about 600°C for a short period, as Arts appears to have done, would not be. The X-ray diffraction patterns were consistent with this explanation.

4.4. DOUBLET SPLITTING

From Table 3 we see that there were hardly any changes in doublet splitting throughout any of the processes involved. The symmetry breaking producing the split must therefore have had its origin elsewhere than in residual stress, as was also the case with the raw casting. Thus, all our experiments confirm that annealing produces frequency changes without necessarily improving warble, confirming the bellfounders' bias against using this expensive process.

4.5. SOME CONCLUSIONS

Debossing causes frequency drops entirely due a reduction in mechanical crown stiffness and any internal stress relief it produces will be small and tend to increase the frequency. Since annealing at 340°C produces only stress relief and gives frequency rises, we are in agreement with Jan Arts. However annealing at 610°C, while it no doubt also gives stress relief, causes structural changes in the alloy resulting in an overall rise in the frequencies. This is in agreement with our preliminary experiment, described in section 3, and explains the apparent disagreement with Arts.

5. MOUNTING OF BELLS

5.1. HOW BELLS ARE MOUNTED

After being tuned bells have to be mounted for use. A swinging church bell is attached by bolts (usually four) through its crown to a massive headstock [22, 23]. Historically bolts were not used but rather metal straps passing through "canons" cast into the top of the crown. However, this approach is now obsolescent in Northern Europe. Headstock and bell swing together as one unit during ringing. Clock bells and carillon bells are "huge dead" by one or more bolts through their crowns attaching them rigidly to a massive framework. Those modern handbell manufacturers who use a boss cut if off after tuning and attach a handle or strap by a single bolt tightly screwed through a single hole in the centre of the crown. The same bolt can be used to attach the clapper assembly to the inside of the bell, which also allows a choice of impact point. Other makers do not use a hole passing right through the crown but rather cast a tang and attach some kind of handle assembly onto that.

Church bells cast with canons are often too old (pre 1890s) to have had the benefit of modern tuning [6]. They are regularly to be seen in foundries waiting to be returned and/or refurbished. The founders' first act is often to cut off their cannons so that they can be remounted in the modern way. It has been reported to us that this often results in falls in the pitches of musical partials [8], which is not surprising since a set of cannons is just a boss of complex shape whose removal will cause a reduction in crown stiffness.

5.2. MOUNTING AND CROWN STIFFNESS

Clearly attaching a bell tightly with bolts through its crown, whether to headstock, mounting frame or handle assembly, will increase stiffness in the crown region. This will cause the frequencies of all the partials to increase or, at best, remain unchanged. Such a phenomenon does not seem to have been reported in the literature. Given the modal forms of the musical partials one would expect the frequency changes to be small. However this was equally true of debossing and annealing, both of which we have seen can produce measurable and even musically detectable changes in frequency. We now report the results of two simple experiments which demonstrate the existence of this phenomenon and indicate the level of its significance.

5.3. PRELIMINARY EXPERIMENTS WITH MALMARK HANDBELLS

This experiment was performed in Australia with two finished Malmark handbells which has already had their bosses removed after tuning. One was a $C_5^\#$ weighing 460 g when disassembled and the other an A_6 weighing 140 g. As mentioned in section 3.2, only two normal modes are significant for the handbell sound. The lower of these determines the strike note, the higher being tuned to a perfect 5th in the upper octave. The pitches of both doublet members of the strike notes were measured using the Korg II tone tester. First, measurements were made on the disassembled bells, then they were repeated when the bells had been tightly attached at their crowns to a large external mass (550 g) by a nut and bolt. The nut was inside the bell, the bold passed through the central hole in the crown and there were two flat washers, one on the inside and the other on the outside of the bell. The combined mass of the nut, bolt and washers was 45 g.

For each bell, the pitch rose with increase in crown stiffness and, as one would expect, the effect was greater for the smaller bell. When disassembled again the bells' pitches returned to their initial values. The maximum changes from the disassembled state were 7 cents for the $C_5^\#$ bell and 25 cents for the A_6. A change of 7 cents is probably undetectable by the ear but 25 cents (a quarter of a semitone) is easily noticeable. The extra stiffness produced when mounting handle assemblies is unlikely to be as great as in this experiment. Nevertheless, the effects is real and could be useful in counteracting the falls in pitch reported for tang removal after tuning [13].

5.4. EXPERIMENT WITH A SMALL CARILLON BELL

This experiment used a small carillon bell of mass 15 kg cast by John Taylor & Co in the 1960s. It was mounted on a horizontal lathe and attached by a single

TABLE 4

Musical partials of a 15 kg small Taylor carillon bell on (nut tight) and off the lathe; the upper member of each split pair is quoted

Partial	On lathe (Hz)	Off lathe (Hz)	Change Δf (Hz)	$\Delta f/f$	Change (cents)
Hum	2000	1993	− 7	− 0·0035	− 6·1
Fundamental	4080	4050	− 30	− 0·0074	− 12·8
Tierce	4635	4635	0	0·0000	0·0
Quint	7789	7788	− 1	− 0·0001	− 0·2
Nominal	7172	7170	− 2	− 0·0003	− 0·5

bolt through a hole in the centre of its crown. The frequencies of the first five partials were measured using the accurate methods referred to in section 4.1. The bell was then removed from the lathe and the frequencies remeasured. The results are listed in Table 4 showing only the higher component of each split pair. As with any small carillon bell, only the lowest one or two partials were acoustically significant [2, see section 5.2]. The frequencies of the higher partials had therefore not been tuned to the musical ratios used for larger bells. Conventional partial names are nevertheless used for convenience.

From the table we see that, as expected, all frequencies fell on removal from the lathe, except for an unchanged Tierce. On the basis of the modal forms in Figure 1, the *relative* sizes of the falls should be similar to those found on debossing the 80 kg church bell. The change in the Fundamental was somewhat larger than expected. At 12·8 cents its fall is the only one which should be noticeable to the ear. In reality, with such a bell, the sound heard is due almost entirely to the Hum. So, although the effect is real, it is unlikely to matter in practice for such small bells. With larger bells, where the fundamental is important, the effect could be significant.

As mentioned in section 2.1, the profile of the 80 kg debossed church bell was very similar to that of the 214 kg Taylor D_5 church bell in Figure 1. With the 15 kg carillon bell the differences were more substantial, it being much thicker relative to its height and its inner profile completely different. In the hope of explaining the larger than expected drop in the Fundamental we therefore carried out a finite element calculation of the normal modes of this bell. The methods used were as previously reported [1]. The results for the musical partials, with no constraints applied at the central hole, are shown in Figure 4. It should be noted that the structure in the figure ends at the edge of the central bolt hole and not at the symmetry axis. There is thus no symmetry-based compulsion for the amplitudes to vanish at this edge, although one would still expect them to be small there. While the forms of the modal functions are similar to those of the church bell there are some interesting differences. The Tierce and Nominal now have no nodal circle, which clearly shows that they belong to the same generic family as the Hum. However, the regions where they have significant motion are even more remote

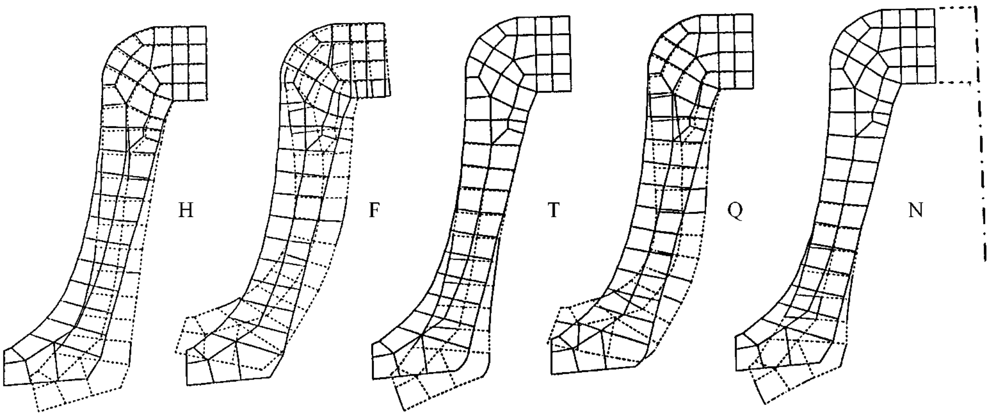

Figure 4. Modal shapes for the musical partials of a 15 kg small Taylor carillon bell. Full lines represent the equilibrium configurations and dotted lines the modal shapes for the Hum (H), Fundamental (F), Tierce (T), Quint (Q) and Nominal (N). The structure shown for the crown ends at the outer edge of a central circular bolt hole. The symmetry axis is indicated by –· –· –· –· –·

from the crown that in the case of the church bell. Hum and Quint are very similar to their church bell counterparts but the Fundamental has much more motion in the crown, especially along the bolt hole. Clamping the bell here will therefore produce a greater increase in modal stiffness and hence in frequency for this particular partial.

6. CONCLUSIONS

The frequencies of all musical partials fall when a church, carillon or handbell is debossed. This is due to falls in the bending stiffness in the central region, not to the internal stress relief. The relative sizes of the falls for a given bell can be understood qualitatively from the forms of the modal functions. Although the effect is small, it is greatest for the Hum and can be acoustically significant.

Similar behaviour occurs in small carillon bells when their central mounting bolt is released. Differences in modal function details from those of the church bell result in the Fundamental now being the most affected. Again the drop is small but acoustically significant. Handbells behave similarly and there is little doubt that church bells also do, although in these cases it will again be the Hum which matters most.

Warble in a bell is not necessarily reduced by annealing but the process can change the partial frequencies. Quench annealing, which must increase internal stress, makes the frequencies fall. Annealing a quenched belt at 340°C causes the frequencies to rise back towards their unstressed values while leaving the alloy's phase structure unchanged. However, annealing an unquenched bell at 610°C can result in falls in frequency because it produces changes in the phase structure.

To summarize we can say that bending stiffness in the crown, internal stress and alloy phase structure enable us to explain a range of phenomena in various types of bells. All these involve changes in the frequencies of musical partials which, while small, can be perceptible. They give a plausible explanation of why traditional bell founders always use vertical lathes for tuning church and carillon bells, even when they are relatively light, and do not try to control warble by annealing.

ACKNOWLEDGMENTS

The authors are indebted to Mr. Hervey Bagot of Bagot Bellfoundries in Adelaide, South Australia, for supplying the 80 kg bell used in the debossing study and the raw casting of the Malmark handbell, for assistance in taking some measurements and for information about other bells produced at his foundry. They also wish to thank Messrs John Taylor and Company, Bellfounders of Loughborough, for supplying the 4·5 and 15 kg carillon bells.

REFERENCES

1. R. Perrin, T. Charnley and J. De Pont, 1983 *Journal of Sound and Vibration* **90**, 29–49. Normal modes of the modern English church bell.
2. T. D. Rossing and R. Perrin 1987 *Applied Acoustics* **20**, 41–70. Vibrations of bells.
3. R. Perrin and T. Charnley 1987 *Journal of Sound and Vibration* **117**, 411–420. A comparative study of the normal modes of various modern bells.
4. R. Perrin and T. Charnley 1978 *Musical Instrument Technology* **3**, 109–117. The suppression of warble in bells.
5. H. Bagot 1986 *Acoustics Australia* **14**, 35–41. Bells–their design and tuning.
6. R. Perrin, T. Charnley and G. M. Swallowe, 1995 *Applied Acoustics* **46**, 83–101. On the tuning of church and carillon bells.
7. E. W. Van Heuven 1949 *Acoustical Measurements of Church Bells and Carillons* de Gebroeders van Cleef, 's-Gravenhage, chapter 4.
8. M. Milsom 1985 private communication.
9. J. R. Pierce 1992 *The Science of Musical Sound*, New York: Freeman, chapter 10, revised edition.
10. L. E. Kinsler, A. R. Frey, A. B. Coppens and J. V. Sanders 1982 *Fundamental of Acoustics*, p. 264. New York: Wiley, third edition.
11. C. A. Culver 1956 *Musical Acoustics*. New York: McGraw-Hill, chapter 7, fourth edition.
12. I. Johnston 1989 *Measured Tones*, p. 78. Bristol: IOP Publishing.
13. T. D. Rossing and H. J. Sathoff 1980 Journal of the Acoustic Society of America **68**, 1600–1607. Modes of vibration and sound radiation from tuned handbells.
14. A. H. Wilson 1965 *The Theory of Metals*. Cambridge: Cambridge University Press. Section 3.4, second edition.
15. J. Arts 1946 *Journal of the Acoustic Society of America* **18**, 503. The effect of heating and cooling on the pitch of bells.
16. J. Arts 1950 *Journal of the Acoustic Society of America* **22**, 511–512. Changes in the pitch of bells.
17. G. M. Swallowe, R. Perrin, T. Charnley, D. J. Parry, P. J. Webster, G. Mills and K. Low 1990 *Applied Acoustics* **30**, 267–278. Warble in carillon bells: an investigation using neutron diffraction.
18. E. S. Hedges (editor) 1960 *Tin and Its Alloys*. London: Edward Arnold, chapter XI.
19. N. F. Henry, H. Lipson and W. Wooster 1961 *The Interpretation of X-ray Diffraction Photographs*. New York: McMillan.

20. I. C. Noylan and J. B. Cohen 1987 *Residual Stress: Measurement by Diffraction and Interpretation*. New York: Springer, chapter 5.
21. M. Hansen 1958 *Constitution of Binary Alloys*, p. 370. New York: McGraw-Hill.
22. T. S. Jennings 1988 *Bellfounding*. Princes Risborough: Shire Publications.
23. T. S. Jennings 1991 *The Development of British Bell Fittings*. Published privately by Trevor S. Jennings. Loughborough, 1991. (ISBN 0950007641)