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LETTERS TO THE EDITOR

NUMERICAL EXPERIMENTS ON A VIBRATING CIRCULAR PLATE OF NON-UNIFORM THICKNESS AND A CONCENTRIC, INNER, CIRCULAR SUPPORT

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1. INTRODUCTION

The problem of transverse vibrations of a circular plate of uniform thickness, free edge and an internal, concentric circular support was studied by Bodine in a well-known paper [1, 2].[1]

The present note may be considered as a generalization of Bodine's problem in the sense that a circular plate of thickness varying in a discontinuous fashion is considered; see Figure 1. The fundamental frequency coefficient of the structural system is determined by two approximate techniques: (1) the optimized Rayleigh–Ritz method [4], (2) a very accurate and well-known finite element code [5].

2. SOLUTION BY MEANS OF THE OPTIMIZED RAYLEIGH–RITZ METHOD

The governing functional is (see Figure 1)

$$J(W)=\iint_p D(\bar{r})\left[\left(W''+\frac{W'}{\bar{r}}\right)^2-2(1-\upsilon)\frac{W'W''}{\bar{r}}\right]\bar{r}\,\mathrm{d}\bar{r}\,\mathrm{d}\theta-\rho\omega^2\iint_p h(\bar{r})\,W^2\,\bar{r}\,\mathrm{d}\bar{r}\,\mathrm{d}\theta, \tag{1}$$

subject to the boundary conditions

$$W(\bar{r}_0)=0, \qquad W''(a)+\frac{\upsilon}{a}W'(a)=0, \tag{2a, b}$$

$$W'''(a)+\frac{W'(a)}{a}-\frac{W'(a)}{a^2}=0. \tag{2c}$$

[1] A recent study [3] concluded that Bodine's results did not possess sufficient accuracy.

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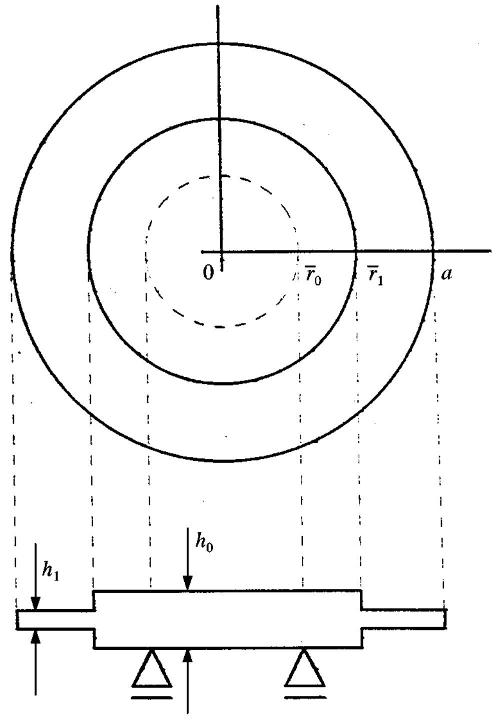

Figure 1. Circular plate of non-uniform thickness considered in the present study.

Defining $\eta_h = h_1/h_0$, $r = \bar{r}/a$ and

$$\eta = \begin{cases} 1, & 0 \leqslant r \leqslant r_1, \\ \eta_h, & r_1 < r \leqslant 1. \end{cases}$$

expression (1) becomes

$$\frac{a^2}{2\pi D_o} J(W) = \int_0^1 \eta^3 \left[\left(W'' + \frac{W'}{r} \right)^2 - 2(1-\upsilon)\frac{W'W''}{r} \right] r\,\mathrm{d}r - \Omega^2 \int_0^1 \eta W^2 r\,\mathrm{d}r, \quad (3)$$

where $\Omega^2 = (\rho h_0 a^4 \omega^2)/D_0$.

The boundary conditions, expressed in terms of the dimensionless variable r, become

$$W(r_0) = 0, \quad W''(1) + \upsilon W'(1) = 0, \quad W'''(1) + W''(1) - W'(1) = 0. \quad \text{(4a–c)}$$

Following reference [3] one approximates the displacement amplitude by means of

$$W_a = \sum_{j=1}^{N} C_j \varphi_j(r), \quad (5)$$

where

$$\varphi_j(r) = \alpha_j r^{p+j-1} + \beta_j r^{j+1} + 1.$$

The α_j's and β_j's are determined substituting φ_j in equations (4a) and (4b) [3]. The fundamental frequency coefficients were determined making $N = 9$ in (5). The frequency coefficient was, finally, minimized with respect to p [3].

3. FINITE ELEMENT DETERMINATIONS

A well-known finite element code was used [5]. One-quarter of the plate domain was subdivided into 5895 elements resulting in 6043 nodes. The resulting number of equations varied for the different structural configurations considered; see Table 1.

4. NUMERICAL RESULTS

Fundamental eigenvalues $\Omega_1 = \sqrt{(\rho h_0/D_0)}\,\omega_1 a^2$ were computed for the configuration shown in Figure 1 for several combinations of the geometric parameters $\eta_h = h_1/h_0$. $R_1 = \bar{r}_1/a$ and $R_0 = \bar{r}_0/a$ for the Poissons ratio (υ) equal to 0·3.

Table 1 depicts comparison of values of Ω_1 determined by means of the optimized Rayleigh–Ritz method and the finite element algorithmic procedure in the case of a plate of uniform thickness. The agreement is excellent for all the configurations considered, specially if one takes into account the fact that only nine polynomial co-ordinate functions were used when carrying out the analytical procedure.

Tables 2 and 3 show values of Ω_1 for the non-uniform thickness case, determined by means of the optimized Rayleigh–Ritz method and the finite element technique respectively. The agreement is very good from an engineering viewpoint for all the configurations under study. Certainly, for $\eta_h = 0{\cdot}8$ and $R_1 = 0{\cdot}6$ the agreement is remarkably good.

In general, the finite element results, presumably of extremely high accuracy, are lower than the upper bounds determined using the analytical approach exception

TABLE 1

Comparison of fundamental frequency coefficients determined by means of the optimized Rayleigh–Ritz method and the finite element technique: case of uniform thickness

	Values of $\Omega_1 = \sqrt{\rho h_0/D_0}\,\omega_1 a^2$									
Method	$R_c = 0{\cdot}1$	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
Variational	3·910	4·275	4·852	5·708	6·930	8·397	8·960	7·810	6·236	4·935
Finite element	3·909	4·275	4·851	5·706	6·929	8·396	8·959	7·809	6·235	4·935
Number of equations	17 954	17 940	17 926	17 912	17 900	17 886	17 872	17 858	17 846	17 832

Note: The number of equations employed when using the finite element method is indicated for the sake of completeness.

TABLE 2

Fundamental frequency coefficients of the system shown in Figure 1 determined by means of the optimized Rayleigh–Ritz method ($\eta_h - h_1/h_0$, $R_1 = \bar{r}_1/a$, $R_0 = \bar{r}_0/a$)

η_h	R_1	$R_0 = 0{\cdot}1$	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
	0·4	3·864	4·172	4·643	5·301	6·254	7·489	7·965	6·848	5·394	4·222
0·8	0·5	3·978	4·311	4·831	5·567	6·564	7·787	8·233	7·046	5·538	4·325
	0·6	4·089	4·456	5·030	5·869	7·010	8·275	8·607	7·330	5·754	4·490
	0·7	4·127	4·513	5·122	6·018	7·270	8·637	8·880	7·537	5·936	4·644
	0·4	3·797	4·024	4·352	4·759	5·397	6·363	6·790	5·714	4·398	3·381
0·6	0·5	4·098	4·381	4·807	5·364	6·049	6·980	7·387	6·152	4·688	3·570
	0·6	4·406	4·782	5·356	6·164	7·175	8·181	8·327	6·878	5·203	3·932
	0·7	4·483	4·912	5·585	6·559	7·867	9·096	8·938	7·363	5·666	4·309

TABLE 3

Fundamental frequency coefficients of the system shown in Figure 1 determined by means of the finite element method

η_h	R_1	$R_0 = 0{\cdot}1$	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
	0·4	3·844	4·144	4·605	5·246	6·195	7·436	7·907	6·801	5·359	4·198
0·8	0·5	3·995	4·333	4·857	5·605	6·606	7·832	8·269	7·072	5·553	4·335
	0·6	4·090	4·457	5·032	5·868	7·012	8·273	8·611	7·328	5·752	4·486
	0·7	4·126	4·511	5·119	6·013	7·259	8·624	8·873	7·531	5·927	4·634
	0·4	3·689	3·892	4·182	4·548	5·186	6·169	6·549	5·517	4·259	3·289
0·6	0·5	4·107	4·392	4·812	5·364	6·028	6·968	7·362	6·117	4·650	3·541
	0·6	4·388	4·758	5·321	6·098	7·071	8·046	8·250	6·787	5·127	3·866
	0·7	4·474	4·898	5·562	6·522	7·792	8·999	8·901	7·322	5·596	4·232

made of the configurations corresponding to $\eta_h = 0{\cdot}8$ and $R_1 = 0{\cdot}5$ and also for $\eta_h = 0{\cdot}6$, $R_1 = 0{\cdot}5$ and $R_0 \leqslant 0{\cdot}4$.

This is also the case for some eigenvalues corresponding to $\eta_h = 0{\cdot}8$ and $R_1 = 0{\cdot}6$. It is felt that in some instances these differences may be caused by round-off errors, specially when carrying out the analytical procedure.

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