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TRANSVERSE VIBRATIONS OF A SIMPLY SUPPORTED RECTANGULAR PLATE OF GENERALIZED ANISOTROPY WITH AN INTERMEDIATE OBLIQUE SUPPORT

HILDA A. LARRONDO AND DANIEL R. AVALOS

Department of Physics, School of Engineering, Universidad Nacional de Mar del Plata, 7600 *Mar del Plata, Argentina*

AND

P. A. A. LAURA

Department of Engineering, Universidad Nacional del Sur and Institute of Applied Mechanics (*CONICET*), 8000 *Bahia Blanca, Argentina*

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1. INTRODUCTION

A recent publication dealt with the determination of the fundamental frequency of transverse vibration of rectangular continuous orthotropic plates in the case where the intermediate support is oblique with respect to the sides of the plate [1].

The present study makes use of a double Fourier series expansion which approximates the plate displacement amplitude and the classical Rayleigh–Ritz method to generate the natural frequency determinantal equation.

2. APPROXIMATE ANALYTICAL SOLUTION

For the rectangular plate under study, depicted in Figure 1, the Rayleigh–Ritz variational approach requires minimization of the functional

$$J[W'] = U_p[W'] - T_p[W'], \tag{1}$$

where $U_p[W']$ is the maximum strain energy and $T_p[W']$ is the maximum kinetic energy for the displacement amplitude of the plate.

In the case of rectangular plates of total general anisotropy its functional can be written (see, for example reference [2]).

$$J_p = \frac{1}{2}\int_{A_p}\left\{D_{11}\left(\frac{\partial^2 W'}{\partial x'^2}\right)^2 + 2D_{12}\frac{\partial^2 W'}{\partial x'^2}\frac{\partial^2 W'}{\partial y'^2} + D_{22}\left(\frac{\partial^2 W'}{\partial y'^2}\right)^2 + 4D_{66}\left(\frac{\partial^2 W'}{\partial x'\,\partial y'}\right)^2\right.$$

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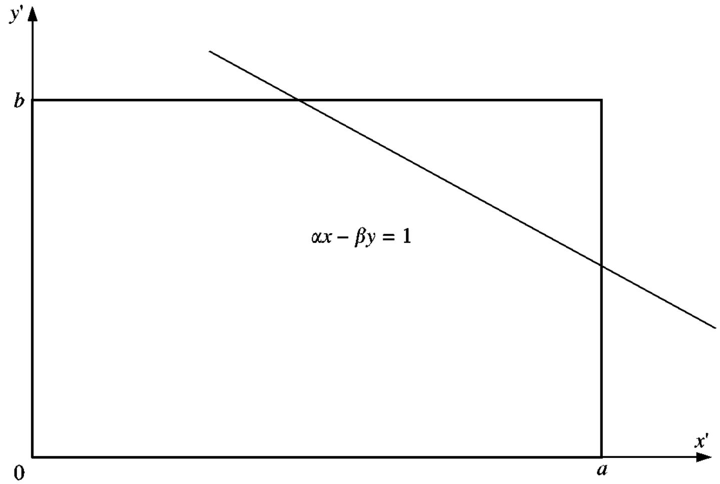

Figure 1. Vibrating system under consideration.

$$+ 4\left[D_{16}\left(\frac{\partial^2 W'}{\partial x'^2}\right) + D_{26}\left(\frac{\partial^2 W'}{\partial y'^2}\right)\right]\left(\frac{\partial^2 W'}{\partial x'\,\partial y'}\right)\Bigg\}\,\mathrm{d}x'\,\mathrm{d}y'$$

$$-\frac{\rho h \omega^2}{2}\int_{A_p} W'^2\,\mathrm{d}x'\,\mathrm{d}y', \qquad (2)$$

where W' is the true displacement amplitude of the plate; the first integral in equation (2) corresponds to $U_p[W']$ and the second measures the maximum kinetic energy of the plate $T_p[W']$. Both integrals are taken over the area A_p of the plate surface.

In equation (2), D_{ij} are the flexural rigidities of the (anisotropic) plate, which, in the case of an isotropic plate, take the simple form

$$D_{11} = D_{22} = \frac{E\mathrm{h}^3}{12(1-\nu^2)}, \quad D_{66} = \frac{(1-\nu)}{2}\frac{E\mathrm{h}^3}{12(1-\nu^2)}, \quad D_{12} = D_{16} = D_{26} = 0. \qquad (3)$$

Taking the lengths of the sides of the rectangular plate to be a and b in the x and y directions, respectively, and by introducing

$$W = W'/a, \quad x = x'/a', \quad y = y'/b, \quad r = b/a, \qquad (4)$$

equation (2) can be cast in a non-dimensional form.

One gets, for the functional for the whole system of Figure 1,

$$J_{nd} = \frac{2J}{rD_{11}} = \int_{A_p}\Bigg[\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \frac{2d_{12}}{r^2}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + \frac{d_{22}}{r^4}\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + \frac{4d_{66}}{r^2}\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)^2$$

$$+ 4\left(\frac{d_{16}}{r}\frac{\partial^2 W}{\partial x^2} + \frac{d_{26}}{r^3}\frac{\partial^2 W}{\partial y^2}\right)\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)\Bigg]\mathrm{d}x\,\mathrm{d}y$$

$$-\Omega^2\int_{A_p} W^2\,\mathrm{d}x\,\mathrm{d}y, \qquad (5)$$

where as usual,

$$\Omega = \sqrt{(\rho h/D_{11})}\omega a^2 \tag{6}$$

is the non-dimensional frequency coefficient and

$$d_{ij} = \frac{D_{ij}}{D_{11}}, \quad (i, j) = (1, 2, 6). \tag{7}$$

The displacement amplitude $W(x, y)$ of the plate is expressed in an approximated way by means of the expression

$$W_a(x, y) = (\alpha x - \beta y - 1)\left[\sum_{n=1}^{N}\sum_{m=1}^{M} b_{mn}\sin(m\pi x)\sin(n\pi y)\right], \tag{8}$$

where the first factor takes into account the existence of the oblique support and the second factor is the double Fourier series used in the previous works [1].

Needless to say, the presence of the first factor in equation (8) makes the resulting analytical expressions rather lengthy, the resulting expression for the functional has 88 terms in total. The calculus although tedious is, however, straightforward.

In order to minimize the functional (5), one has to take its partial derivatives with respect to the coefficients b_{ij} of expression (8) and equal these derivatives to zero.

That is to say,

$$\frac{\partial J_{nd}}{\partial b_{ij}} = 0, \quad (i, j) = 1, 2, \ldots . \tag{9}$$

System (9) yields an $M \times N$ homogeneous linear system of equations in the b_{ij}'s. A secular determinant in the natural frequency coefficients of the system results from the non-triviality condition.

The present study is concerned with the determination of the first four frequency coefficients, Ω_1 to Ω_4, in the case of anisotropic rectangular plates for different locations of the oblique support.

3. NUMERICAL RESULTS

All calculations were performed for an anisotropic simply supported rectangular plate of uniform thickness taking $D_{12}/D_{11} = 0{\cdot}3$, $D_{22}/D_{11} = 0{\cdot}5 = D_{66}/D_{11}$; and $D_{16}/D_{11} = 1/3 = D_{26}/D_{11}$. In each of the five tables presented, a different value of the aspect ratio b/a has been taken, from $b/a = 0{\cdot}5$ in Table 1 to $b/a = 2$ in Table 5.

For the double Fourier series in equation (8), $M = N = 20$ have been used, that is to say a secular determinant of the order 400 was generated for all the situations. For these values of M and N satisfactory convergence is achieved for all situations as has been checked by incrementing M and N to 30 (i.e., a determinant of order 900). As usual, special care has been taken to manipulate such large determinants and 80-bit floating point variables (IEEE-standard temporary reals) have been used in order to obtain accurate results.

Table 1 depicts values for a rectangular plate of aspect ratio $r = \frac{1}{2}$. The table is divided into three sections each showing values for the frequency coefficients for

TABLE 1

Values of the first four frequency coefficients Ω_1–Ω_4 in the case of an anisotropic rectangular plate of aspect ratio $b/a = \frac{1}{2}$

Inner support	Support position	Ω_1	Ω_2	Ω_3	Ω_4
	$y_0 = 1/4$	76·21	103·13	147·15	206·81
Horizontal	$y_0 = 1/2$	124·48	155·09	186·25	189·80
(Figure 2)	$y_0 = 3/4$	76·21	103·13	147·15	206·81
	$y_0 = 1$	52·47	78·58	121·25	150·99
	$x_0 = 1/4$	51·80	102·36	140·03	166·69
Vertical	$x_0 = 1/2$	75·44	87·55	151·01	157·81
(Figure 3)	$x_0 = 3/4$	51·80	102·36	140·03	166·69
	$x_0 = 1$	41·86	75·40	122·97	125·30
	(a)	110·82	128·89	185·90	205·15
Diagonal	(b)	60·88	93·19	134·46	167·55
(Figure 4)	(c)	40·99	69·93	112·98	126·06

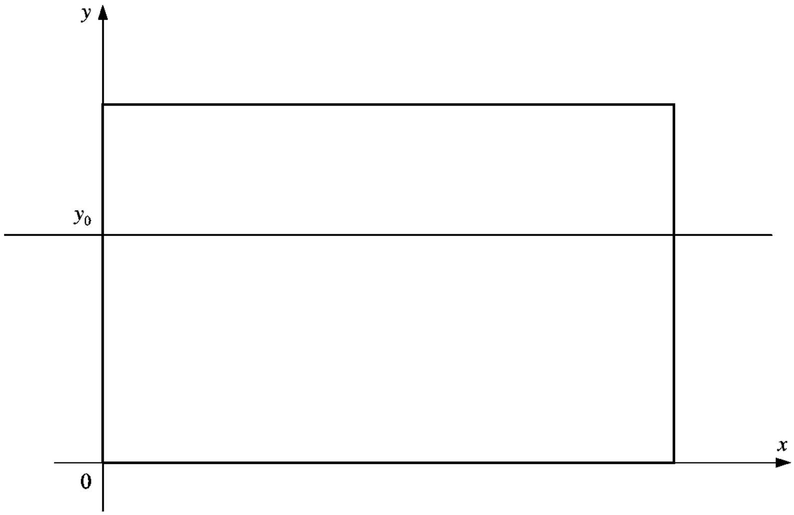

Figure 2. Inner support parallel to the x-axis.

different locations of the intermediate support. The first section shows values corresponding to the intermediate support parallel to the x-axis (Figure 2). Four of them have been chosen, intersecting the (non-dimensional) co-ordinate y at $y_0 = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$, and 1. As expected values for $y_0 = \frac{1}{2}$ are identical to those for $y_0 = \frac{3}{4}$, but both have been quoted in order to show the reliability of the method employed for the calculation.

The second section of Table 1 depicts values for a set of four intermediate supports parallel to the y-axis, as shown in Figure 3. Their intersections to the x-axis are at $x_0 = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ and 1. Again, values for $x_0 = \frac{1}{2}$ agree with those for $x_0 = \frac{3}{4}$.

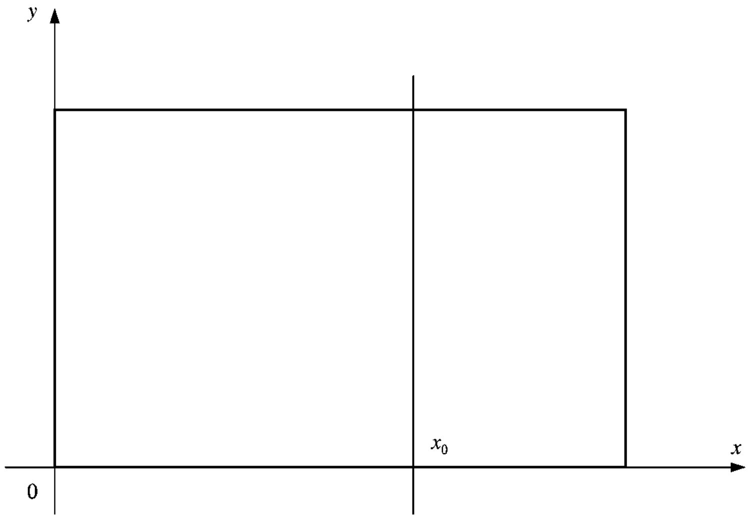

Figure 3. Inner support parallel to the y-axis.

Table 2

Values of the first four frequency coefficients Ω_1–Ω_4 in the case of an anisotropic rectangular plate of aspect ratio $b/a = \frac{2}{3}$

Inner support	Support position	Ω_1	Ω_2	Ω_3	Ω_4
Horizontal (Figure 2)	$y_0 = 1/4$	47·34	74·31	116·98	136·02
	$y_0 = 1/2$	75·91	99·18	117·03	123·87
	$y_0 = 3/4$	47·34	74·31	116·98	136·02
	$y_0 = 1$	33·80	59·75	90·91	98·67
Vertical (Figure 3)	$x_0 = 1/4$	39·97	78·51	103·44	127·95
	$x_0 = 1/2$	61·11	75·83	112·55	114·64
	$x_0 = 3/4$	39·97	78·51	103·44	127·95
	$x_0 = 1$	30·43	61·08	80·21	99·31
Diagonal (Figure 4)	(a)	74·85	98·66	131·93	151·73
	(b)	42·45	69·01	100·68	117·07
	(c)	28·30	54·89	78·92	91·75

Finally, section three of Table 1 shows values of the frequency coefficients for three positions of the inner support parallel to the diagonal as shown in Figure 4.

The same pattern is repeated in Tables 2–5, one of each value of the plate aspect ratio $r = b/a$. In Table 2, $r = \frac{2}{3}$ have been taken, whereas Tables 3–5 depict values for $r = 1$, $\frac{3}{2}$ and 2 respectively.

The present approach can be extended in a straightforward fashion to the case of plates of non-uniform thickness, presence of orifices, etc. In the case of other

TABLE 3

Values of the first four frequency coefficients Ω_1–Ω_4 in the case of an anisotropic square plate

Inner support	Support position	Ω_1	Ω_2	Ω_3	Ω_4
Horizontal (Figure 2)	$y_0 = 1/4$	26·87	51·94	70·12	85·37
	$y_0 = 1/2$	41·11	50·67	113·53	118·52
	$y_0 = 3/4$	26·87	51·94	70·12	85·37
	$y_0 = 1$	20·57	41·48	53·14	66·80
Vertical (Figure 3)	$x_0 = 1/4$	31·22	50·64	79·32	87·74
	$x_0 = 1/2$	49·75	66·47	77·19	85·32
	$x_0 = 3/4$	31·22	50·64	79·32	87·74
	$x_0 = 1$	22·29	40·29	59·02	65·71
Diagonal (Figure 4)	(a)	49·31	66·84	85·90	103·92
	(b)	28·03	46·49	67·43	77·08
	(c)	18·89	37·27	51·67	61·39

TABLE 4

Values of the first four frequency coefficients Ω_1–Ω_4 in the case of an anisotropic rectangular plate of aspect ratio $b/a = \frac{3}{2}$

Inner support	Support position	Ω_1	Ω_2	Ω_3	Ω_4
Horizontal (Figure 2)	$y_0 = 1/4$	17·67	34·04	48·63	54·45
	$y_0 = 1/2$	24·96	28·23	49·82	53·87
	$y_0 = 3/4$	17·67	34·04	48·63	54·45
	$y_0 = 1$	14·64	25·52	40·34	44·29
Vertical (Figure 3)	$x_0 = 1/4$	27·13	36·42	50·42	68·58
	$x_0 = 1/2$	44·27	55·77	65·68	69·39
	$x_0 = 3/4$	27·13	36·42	50·42	68·58
	$x_0 = 1$	18·58	27·11	40·16	53·26
Diagonal (Figure 4)	(a)	37·88	43·85	63·37	70·55
	(b)	20·65	32·11	45·97	56·79
	(c)	14·35	23·96	37·33	44·39

combinations of boundary conditions, one would use the corresponding combination of “beam functions” popularly used when dealing with isotropic and orthotropic structural elements.

TABLE 5

Values of the first four frequency coefficients Ω_1–Ω_4 in the case of an anisotropic rectangular plate of aspect ratio $b/a = 2$

Inner support	Support position	Ω_1	Ω_2	Ω_3	Ω_4
Horizontal (Figure 2)	$y_0 = 1/4$	14·34	24·53	37·89	43·89
	$y_0 = 1/2$	18·86	20·33	37·11	41·45
	$y_0 = 3/4$	14·34	24·53	37·89	43·89
	$y_0 = 1$	12·54	19·07	28·16	39·20
Vertical (Figure 3)	$x_0 = 1/4$	25·65	31·09	39·42	50·29
	$x_0 = 1/2$	42·30	49·23	59·18	63·77
	$x_0 = 3/4$	25·65	31·09	39·42	50·29
	$x_0 = 1$	17·23	22·21	29·90	40·05
Diagonal (Figure 4)	(a)	33·15	35·15	48·46	56·29
	(b)	17·42	25·92	35·26	45·90
	(c)	12·62	18·59	26·83	37·13

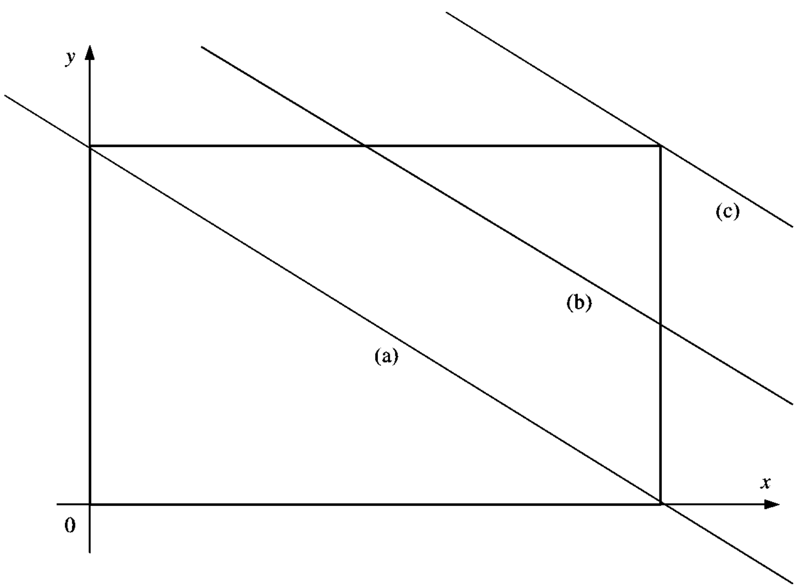

Figure 4. Intermediate support parallel to the diagonal.

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