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DAMAGE DETECTION IN SMART STRUCTURES THROUGH SENSITIVITY ENHANCING FEEDBACK CONTROL

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Non-destructive damage detection and isolation in structures using modal information is hindered by the sensitivity of modal frequencies to small changes in mass, stiffness, and damping parameters induced by damage. Here, a method of enhancing modal frequency sensitivity to damage using feedback control is introduced. Using state feedback, closed-loop modal frequencies are placed at locations in the complex plane that enhance sensitivity to particular types of damage. The method is intended for smart structures, which embody self-actuation and self-sensing capabilities. A simple example introduces the principle of sensitivity enhancing control for a single-degree-of-freedom structure. Then, the method is applied to finite-element models of a cantilevered beam to demonstrate the magnitude of sensitivity enhancement achievable for modest, local damage. Methods of implementing sensitivity enhancing full state or output feedback using point measurements of strain along the beam are described. Simulation results show that significant enhancement in sensitivity of modal frequencies of vibration to damage can be achieved using a single actuator and multiple strain sensors along the beam. The methodology enables a "dual use" smart structure—one that can be used for both vibration suppression and damage detection.

1. INTRODUCTION

Damage detection and localization is of interest in many structures, from aircraft wings, helicopter rotor blades, flexible structures, and rotating machinery, to civil structures such as buildings and bridges. A prevalent method of damage detection is based on identifying changes in modal frequencies resulting from local damage-induced changes in stiffness or mass. Modal methods assume that damage implies a detectable change in system parameters, and that the change in local parameters can be identified through examination of global modal frequencies. Numerous papers report experimental and theoretical results of damage detection through modal analysis. Representative examples, both past and recent, include Adams *et al.* [1], Cawley and Adams [2], Armon *et al.* [3], Papadopoulos and Garcia [4], Jian *et al.* [5], and Swamidas and Chen [6].

While damage normally does imply local changes in stiffness, mass, damping, or some combination of parameters, concerns in using modal frequencies to detect

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damage are as follows: (1) modal frequencies can be very insensitive to small changes in local stiffness or mass that would be indicative of damage; therefore, damage is extremely difficult to detect until it is substantial, and (2) sensitivity to changes in stiffness or mass can be inconsistent, with the sensitivity itself depending on modal properties and damage location. These drawbacks are easily demonstrated analytically and experimentally. For example, study of a single-degree-of-freedom system by Banks *et al.* [7] indicates both points. Here, the authors show that the sensitivity of natural frequency to changes in stiffness is inversely proportional to stiffness. Likewise, the sensitivity is inversely proportional to mass. Depending on the nominal frequency, stiffness, and mass values, small changes in these parameters system can produce changes in frequency undetectable by modern instrumentation and signal processing methods. An experimental illustration of these points is found in Adams *et al.* [1]. Adams *et al.* analyzed shifts in resonant frequencies of an aluminum bar under axial loading after initiating damage using two saw cuts on opposite sides of the bar. Thirty per cent of the cross-sectional area was removed. When damage was located at the center of the bar, frequency shifts in the first three modes were 0·8%, 0%, and 0·8% respectively. With damage near the end of the bar, frequency shifts of the first three modes were 0·7%, 1·3% and 1% respectively; hence, small frequency shifts for a structure with significant damage are indicated, and sensitivity dependence on damage location is indicated. In a finite element study of a cracked cantilevered plate conducted by Swamidas and Chen [6], a surface crack near the root of the plate with a width of 40% of plate width and depth of 70% of plate thickness produces frequency shifts in the first two bending modes of 0·68% and 0·27% respectively. Higher modes exhibit frequency shifts of less than 0·09%. Other authors confirm the modal sensitivity issue, and they also consider other important factors affecting modal sensitivity, including measurement noise (e.g., [8]) and model uncertainty (e.g., [9]).

Though numerous authors have demonstrated the use of modal frequency shifts for damage detection experimentally, removal of 10–40% of a cross-sectional area to illustrate damage detection is common. In uniform structures, damage levels introduced in these studies are often larger than the smallest crack that needs to be detected. Nevertheless, the argument for using modal properties to detect damage lies in the fact that the ability to detect and localize damage based on shifts in characteristic frequencies is desirable. Although sensitivity issues in using modal frequencies to detect damage remain a concern, there seems to be methods for localizing damage based on shifts in modal frequencies. For example, Adams *et al.* [1] develops and demonstrates a method to localize damage based on ratios of frequency shifts between the first two or three modes. They show that for a one-dimensional stress state and a symmetric structure it is possible to isolate damage to one of two locations using two modes of axial vibration. A more recent paper demonstrates adequate localization of damage by rank ordering of frequency shifts in the first four modes during transverse vibration [3]. Finally, a number of recent papers consider the use of other modal-based measures for damage localization, including mode shape changes [10], "strain" mode shapes [6], and measures based on mode shape curvature [11, 12].

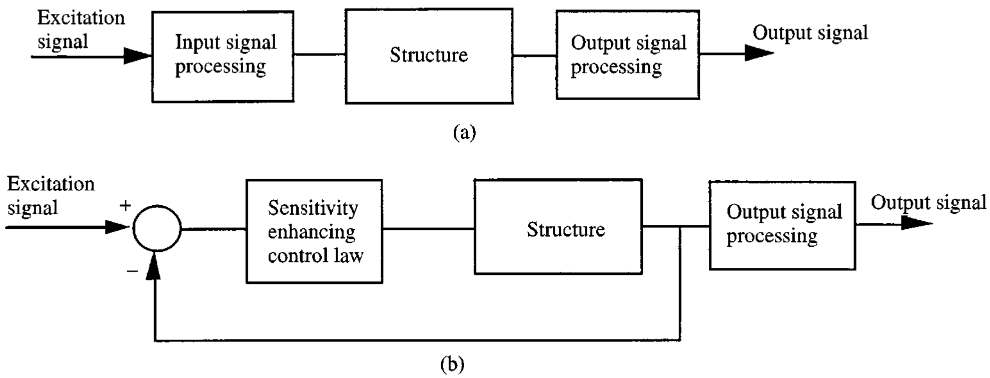

Figure 1. (a) Traditional versus (b) proposed approach for sensitivity enhancement of modal properties.

This paper explores methods of enhancing sensitivity of modal frequencies to small changes in structure parameters and local geometry through use of feedback control. These methods are developed for smart structures, i.e., those capable of self-excitation, self-sensing, and closed-loop vibration control. Figure 1 compares other approaches to modal analysis with sensitivity-enhancing control approaches. In the absence of loop closure, the portions of the system that can be modified to enhance sensitivity of modal properties are the input to the structure (through careful selection of signals that excite characteristic frequencies) and the output signal (through improved sensors, signal processing, neural network identification, etc.). In smart structures, the open-loop scenario is implemented by using self-actuation of the structure to generate the input signal. Self-sensing provides the output signal. In contrast, closed-loop control modifies the system dynamics, such that modal frequencies are placed at values that optimize frequency sensitivity to damage-induced parameter variations. Here, self-actuation is used both to excite the structure and to affect closed-loop dynamics, with self-sensing providing the feedback signal as well as the output signal. In essence, closed-loop control provides a means to optimize input shaping for damage detection. Moreover, sensors and actuators required to implement sensitivity enhancing control are identical to those required to control vibrations; hence, a smart structure could be designed for the dual task of damage detection and vibration suppression. While other modal parameter sensitivities might also be modified through closed-loop control, here we focus primarily on modal frequency sensitivities.

2. A DEMONSTRATION OF SENSITIVITY ENHANCEMENT

Objectives of closed-loop control normally include enhancing stability, performance and robustness of the controlled system. Performance refers to characteristics such as damping and response speed, and in smart structures, closed-loop vibration control is normally applied to increase damping. One would also want the control law to be robust, which simply means that it should enhance performance independent of uncertainties in the structure model on which the control law is based, and that spillover, or excitation of unmodelled modes should be avoided. The method proposed here turns the robust control problem around. Instead of using a control law to make the system *insensitive* to changes in system parameters, it should enhance or *magnify sensitivity* to such changes in a predictable manner. In essence, we seek to increase observability of system parameter variations through feedback control. An example, based on the single-degree-of-freedom system used to discuss sensitivity issues in Banks *et al.* [7], is presented here to illustrate the concept. Consider a lightly damped second order system

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u, \tag{1}$$

where k, m, and b are stiffness, mass, and damping coefficients, respectively, u is an input force, and the state vector includes position and velocity. The sensitivity of the natural frequency $\omega_n = \sqrt{k/m}$ to small changes in k and m are [7]

$$\frac{\partial \omega_n}{\partial k} = \frac{\omega_n}{2k}, \qquad \frac{\partial \omega_n}{\partial m} = -\frac{\omega_n}{2m}. \tag{2}$$

Hence, the larger the stiffness and mass parameters, the smaller the sensitivity to changes in these parameters. If the system is controlled by state feedback with $u = -[K_1 \quad K_2][x \quad \dot{x}]^{\mathrm{T}}$, the closed-loop system characteristic equation is given by

$$\det\left(s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -(k+K_1)/m & -(b+K_2)/m \end{bmatrix} \right) = 0. \tag{3}$$

The closed-loop natural frequency depends on the control gain K_1:

$$\omega_{n_{cl}} = \sqrt{(k+K_1)/m}. \tag{4}$$

Accordingly, closed-loop natural frequency sensitivities are functions of the control gain K_1:

$$\frac{\partial \omega_{n_{cl}}}{\partial k} = \frac{\omega_{n_{cl}}}{2(k+K_1)}, \qquad \frac{\partial \omega_{n_{cl}}}{\partial m} = -\frac{\omega_{n_{cl}}}{2m}. \tag{5, 6}$$

Equations (5) and (6) show that for the single-degree-of-freedom system, one should reduce the natural frequency (make K_1 negative) to enhance sensitivity to changes in stiffness, and to enhance sensitivity to changes in mass, one should increase the natural frequency (make K_1 positive). For example, if the open-loop natural frequency and stiffness are 10 rad/s and 1000 N/m, respectively, the open-loop sensitivity to change in stiffness is 0·005 m/N s (equation (2)). Reducing the natural

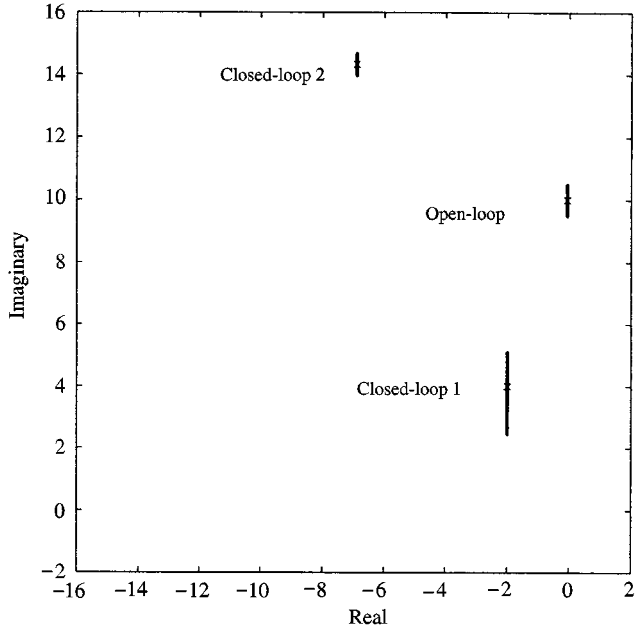

Figure 2. Sensitivity study for $\pm 10\%$ stiffness change in a single-degree-of-freedom structure. "$\times$" marks nominal root locations. The line shows how the roots change as stiffness varies within 10% of its nominal value.

frequency by a factor of 2 through closed-loop control increases the closed-loop sensitivity by a factor of 2. These concepts are illustrated in Figures 2 and 3 using a stochastic root locus.† In Figure 2, the locus of roots as k varies between $\pm 10\%$ of its nominal value are shown for the system under open-loop control, closed-loop control that decreases the natural frequency (K_1 negative), and closed-loop control that increases the natural frequency (K_1 positive). For both closed-loop systems, damping is simultaneously increased using gain K_2, since this is a desirable closed-loop characteristic. "$\times$" marks the nominal root locations, and the light, dotted lines show the extent of root location variation, indicating the sensitivity of the system to changes in parameters (the longer the line, the more the root varies for the specified uncertainty). By comparing the open-loop root variation to closed-loop system 1, we see that reducing the natural frequency through control increases the sensitivity, since the roots migrate farther from their nominal values. Comparison to closed-loop system 2 shows that increasing the natural frequency decreases sensitivity. The directional nature of root migration is also of importance, as it shows that stiffness changes at constant mass (and damping parameter b) directly affect the damped natural frequency.

Figure 3 illustrates the effect of each control law on sensitivity to $\pm 10\%$ changes in mass. Here, nominal root locations are marked by "$\circ$". An increase in

† This stochastic root locus [16], a tool for visualizing sensitivity to parameter uncertainty, shows the migration of system eigenvalues (open-loop or closed-loop) as one or more parameters change. Parameters may follow specific distributions, such as uniform, Gaussian, binary, or some other distribution, and the distribution of root locations is determined by Monte Carlo simulation.

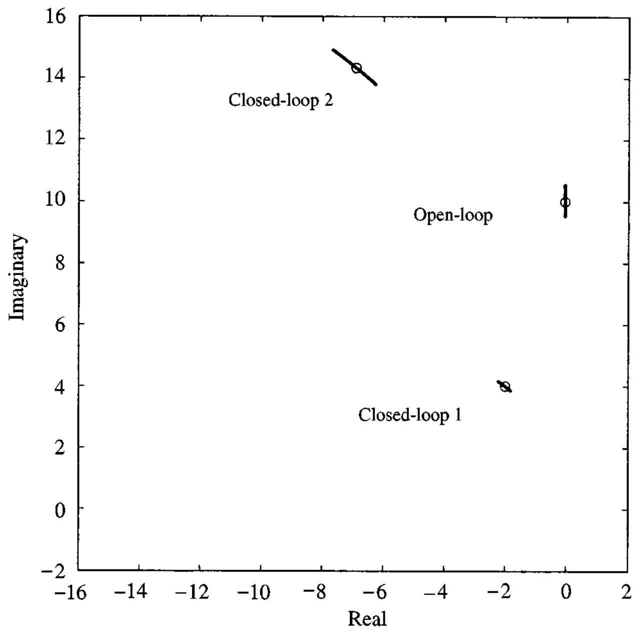

Figure 3. Sensitivity study for $\pm 10\%$ mass change in a single-degree-of-freedom structure "○" marks nominal root locations. The line shows how the roots change as mass varies within 10% of its nominal value.

closed-loop natural frequency increases sensitivity to mass changes, as reflected in closed-loop system 2, and the direction of root migration differs from Figure 2. The fact that effects of the two control laws on sensitivity to specific parameter variations (mass or stiffness) are opposite suggests that multiple control laws can be designed to enhance sensitivities of different "damage mechanisms", providing the ability to distinguish between types of damage by successive application of different control laws during maintenance of the structure. Moreover, direction of root migration may also aid in distinguishing one damage mechanism from another.

3. CONTROL SYSTEM DESIGN FOR SENSITIVITY ENHANCEMENT

3.1. SYSTEM AND CONTROL MODEL

State feedback control laws for sensitivity enhancement are designed for homogeneous, one-dimensional beams under bending, whose transverse displacement $y(x, t)$ is described by a finite-element model of an Euler–Bernoulli beam with Kelvin–Voigt damping and appropriate boundary conditions. The control model of the beam, or model of the beam from which state feedback control gains are designed, consists of an n node finite-element model. A higher order finite-element model with nt nodes constitutes the model of the actual system to which the control law is applied. For either model, the dynamics describing temporal variation of transverse displacement for the ith element are

$$\mathbf{M}_i\ddot{\mathbf{x}}_i + \mathbf{C}_i\dot{\mathbf{x}}_i + \mathbf{K}_i\mathbf{x}_i = F_i, \tag{7}$$

where $\mathbf{x}_i = [y(x_{i-1}, t)\ \theta(x_{i-1}, t)\ y(x_i, t)\ \theta(x_i, t)]^{\mathrm{T}}$. $\mathbf{x}_i$ is the position of the ith node along the beam, y_i is the transverse displacement at the ith node, θ_i is the slope $\mathrm{d}y_i/\mathrm{d}x$, and F_i is the distributed force acting on element i. Elemental mass, and stiffness matrices for the ith element of a beam subject to transverse vibration are given by [13]

$$\mathbf{M}_i = \frac{\rho_i A_i L_i}{420}\begin{bmatrix} 156 & 22L_i & 54 & -13L_i \\ 22L_i & 4L_i^2 & 13L_i & -3L_i^2 \\ 54 & 13L_i & 156 & -22L_i \\ -13L_i & -3L_i^2 & -22L_i & 4L_i^2 \end{bmatrix},$$

$$\mathbf{K}_i = \frac{E_i I_i}{L_i^3}\begin{bmatrix} 12 & 6L_i & -12 & 6L_i \\ 6L_i & 4L_i^2 & -6L_i & 2L_i^2 \\ -12 & -6L_i & 12 & -6L_i \\ 6L_i & 2L_i^2 & -6L_i & 4L_i^2 \end{bmatrix}.$$

L_i is the element length, and ρ_i, A_i, I_i, E_i are the elemental density, area, area moment of inertia, and Young's modulus respectively. The damping matrix $\mathbf{C}_i$ is assumed to be proportional to strain rate, as in Kelvin–Voigt damping. Combining elemental matrices, mass (**M**), stiffness (**K**), and damping (**C**) matrices are formed, and the relevant state-space representation of the control model is

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BF}, \tag{8}$$

where

$$\mathbf{A} = \begin{bmatrix} -\mathbf{M}^{-1}\mathbf{C} & -\mathbf{M}^{-1}\mathbf{K} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} \mathbf{M}^{-1} \\ \mathbf{0} \end{bmatrix},$$

$$\mathbf{X} = [\dot{y}_1 \quad \dot{\theta}_1 \quad \dot{y}_2 \quad \dot{\theta}_2 \ldots\ldots \dot{y}_n \quad \dot{\theta}_n \quad y_1 \quad \theta_1 \quad y_2 \quad \theta_2 \ldots\ldots y_n \quad \theta_n]^{\mathrm{T}},$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{0}_{2n\times m} & & & \\ F_{11} & F_{21} & \cdots & F_{m1} \\ 0 & 0 & \cdots & 0 \\ F_{12} & F_{22} & \cdots & F_{m2} \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \\ F_{1n} & F_{2n} & \cdots & F_{mn} \\ 0 & 0 & \cdots & 0 \end{bmatrix}.$$

F_{ij} is the total force applied at node j due to actuator i, and m is the number of actuators. After appropriate boundary conditions for the beam are applied, some state variables can be eliminated; for example, state variables y_1 and θ_1 are eliminated for a cantilevered beam based on boundary conditions.

The approach to constructing a higher order finite-element model to represent the actual system is identical, with nt replacing n. The finite-element model of the actual system is used to study damage detection. By changing one or more parameters in a single element or several contiguous elements of the actual system, and applying feedback control laws designed based on the control model to the actual system, local damage can be simulated. For example, a reduction in the thickness of a single element at zero reduction in mass simulates a crack as best as possible with this linear model, while reduction in thickness and mass of one or more contiguous elements may represent damage in the form of a hole or slot. Spillover, or excitation of higher order modes is also examined through the application of a control law developed using the low order structure model to the higher order actual model.

3.2. IMPLEMENTATION OF STATE FEEDBACK CONTROL LAWS

In a smart structure, strain is usually the measured variable, and it is necessary to estimate the state vector of equation (8) from strain measurements. Gopinathan *et al.* [14] proposes a method for estimating transverse and angular displacement along a beam based on strain measurements at each node. Strain (ε) is related to deflection (y) through the curvature equation

$$\varepsilon(x, t) = -\frac{t_b}{2}\frac{\partial^2 y(x, t)}{\partial x^2}, \tag{9}$$

where t_b is the beam thickness. Approximating transverse displacement distribution by a cubic spline, and strain distribution by a linear spline, a recursive relationship between strain measured at point locations and transverse displacement is derived [14]:

$$y_i = -\frac{L_i^2}{3t_b}(\varepsilon_i + 2\varepsilon_{i-1}) + y_{i-1} + \theta_{i-1}, \qquad \theta_i = -\frac{L_i^2}{t_b}(\varepsilon_i + \varepsilon_{i-1}) + \theta_{i-1}, \tag{10, 11}$$

L_i is the element length. Velocities at each node, $\dot{y}_i$ and $\dot{\theta}_i$, can be estimated by differentiation or by implementing a state estimator using equations (10) and (11) as observations. As an alternative to this recursive observer, a model-based observer can be designed using standard linear-quadratic or pole-placement methods. To design the model-based observer, the relationship between measured strain and state elements must be provided through an output equation. The output equation depends on the electromechanical coupling relationship between measured strain and mechanical displacements. The analytic derivation of such an output relationship is given in [15]. Alternatively, a beam model can be developed experimentally using frequency response methods, providing the transfer function between the input voltage to an actuator and the strain output directly.

With one of these state estimation methods available, full-state feedback control laws can be applied for vibration control or sensitivity enhancement by using estimated transverse displacements and velocities at each node of the control model as state feedback variables. Based on the single-degree-of-freedom system analyzed above, placing closed-loop modal frequencies at values lower than open-loop frequencies should enhance sensitivity to stiffness changes; hence, we aim to reduce modal frequencies through pole placement for detecting local reduction in stiffness. Analysis of controllability of the system shows that the beam is controllable for a single input; however, multiple inputs may be desirable to maximize sensitivity enhancement. A state feedback control law for a system with m actuators takes the form $\mathbf{F} = -\mathbf{K_c X}$, where $\mathbf{K_c}$ is a $4n \times m$ control gain matrix, and $\mathbf{X}$ and $\mathbf{F}$ are as given above. Denoting the control model of the system by $(\mathbf{A}, \mathbf{B})$, and the actual system by $(\mathbf{A_a}, \mathbf{B_a})$, the roots of the closed-loop system formed by applying control law $\mathbf{F} = -\mathbf{K_c X}$ to the actual system are given by the solutions to

$$\lambda \mathbf{I} - (\mathbf{A_a} - \mathbf{B_a K_{ca}}) = 0, \tag{12}$$

where, for a clamped-free beam,

$$\mathbf{K_{ca}} = [\mathbf{K}_{\mathbf{b}1} \quad \mathbf{K}_{\mathbf{b}2} \quad \ldots \quad \mathbf{K}_{\mathbf{b}n-1}], \qquad \mathbf{K}_{\mathbf{b}i} = [0_{m \times 2(n-1)-2} \quad \mathbf{K}_{\mathbf{c}i}].$$

$\mathbf{K}_{\mathbf{c}i}$ denotes the elements of the matrix $\mathbf{K_c}$ corresponding to common nodes of the actual beam model and control model. Pole placement provides an established methodology for finding state feedback control gains to demonstrate sensitivity enhancement. For systems where it is undesirable, due to computation requirements, to estimate the state from strain measurements, output feedback may be useful for sensitivity enhancement. In output feedback, strain measurements are used directly to affect closed-loop dynamics. With output feedback, closed-loop roots can no longer be placed arbitrarily, as with full state feedback; however, root locus analysis can be used to design a sensitivity enhancing output feedback system, as shown by example, below.

Sensitivity enhancement is demonstrated by simulation for a cantilevered beam whose properties are given in Table 1. The beam is modelled by a low-order (nine node) finite-element model for control law design, and a 65 node finite-element model is chosen for the actual system. Table 1 shows insignificant differences in the two models for the first four modes. It is assumed that a single actuator is available, and two damage conditions are considered: (1) change in cross-sectional area of a single element by 10%, with corresponding change in mass, and (2) a 5% change in thickness of a single element with no change in mass. Damage to a single element such as reduction in thickness corresponds to damage across the entire width of the beam and element thickness (0·5/64 or 0·78 cm). A change in thickness (with or

TABLE 1

Beam and properties and open-loop modal frequencies of the control and actual models

Property	Value
Length	0·5 m
Young's Modulus	7·17E10 Pa (Aluminum)
Thickness	1·6 mm
Width	2 cm
Density	2800 kg/m^3
Control model nodes (n)	9
Actual model nodes (nt)	65
Modal frequencies of actual model	$-0{\cdot}54 \pm 32{\cdot}87$j, $-21{\cdot}22 \pm 204{\cdot}91$j, $-166{\cdot}36 \pm 552{\cdot}30$j, $-638{\cdot}82 \pm 932{\cdot}49$j
Modal frequencies of control model	$-0{\cdot}54 \pm 32{\cdot}87$j, $-21{\cdot}24 \pm 204{\cdot}92$j, $-166{\cdot}56 \pm 552{\cdot}61$j, $-641{\cdot}68 \pm 933{\cdot}60$j

TABLE 2

Desired and actual closed-loop root locations for full-state feedback

Property	Value
Desired closed-loop root locations	$-0{\cdot}54 \pm 24$j, -21 ± 150j, -166 ± 320j
Undamaged closed-loop roots of the actual model	$-0{\cdot}54 \pm 24$j, -21 ± 150j, $-165{\cdot}89 \pm 320$j, $-638{\cdot}82 \pm 932{\cdot}54$j
Undamaged closed-loop roots of the control model	$-0{\cdot}54 \pm 24$j, -21 ± 150j, -166 ± 320j, $-641{\cdot}68 \pm 933{\cdot}60$j

without mass reduction) changes the stiffness at the damage location. The change in stiffness without mass reduction may reflect change in cross-sectional properties due, for example, to crack initiation; however, the model remains linear, and local plastic deformation and yielding are not modelled.

A full-state feedback closed-loop control law is designed to shift the first three modal frequencies down (at approximately constant $\zeta\omega$) as indicated in Table 2. The feedback control law is then applied to the actual system, with damage located at a single element, and sensitivity enhancement is measured as a percent decrease in closed-loop natural frequency of the damaged beam over the nominal closed-loop natural frequency. Table 2 shows the closed-loop roots selected for pole placement, the nominal (undamaged) closed-loop roots of the actual model, and the nominal root locations for the control model. Again, no significant difference in closed-loop root locations arises due to the reduced-order control model illustrating minimal spillover effects.

Figures 4 and 5 present results of the full-state feedback control law applied to the cantilevered beam, as compared to open-loop vibration. Figure 4 shows open-loop sensitivity of the first four modal frequencies to damage as a function of

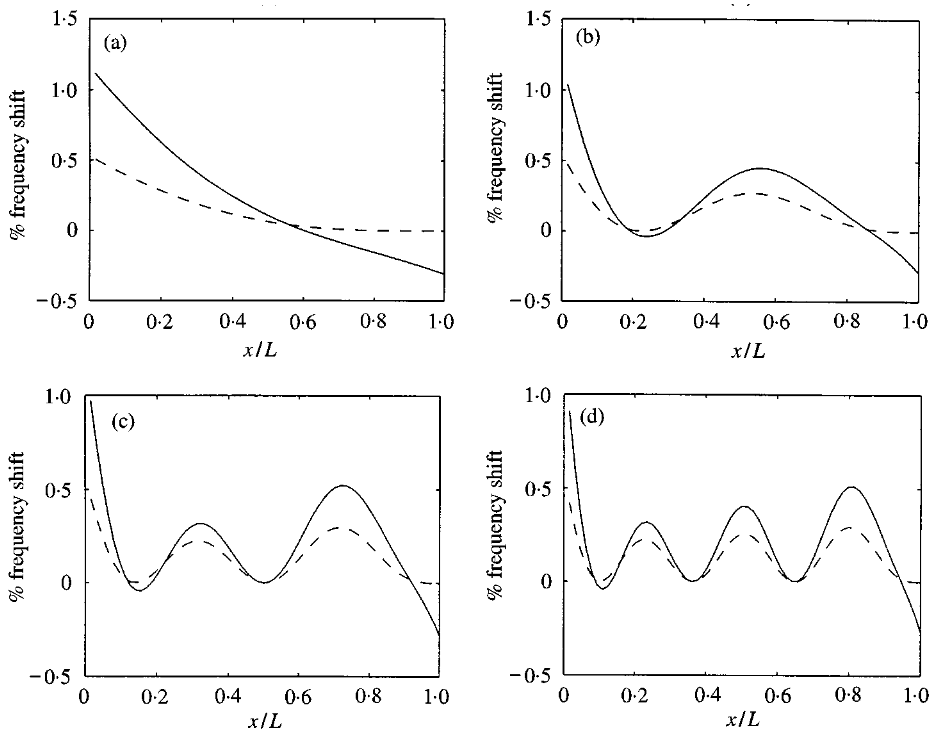

Figure 4. Frequency shifts in first four modes of cantilevered beam under *open-loop vibration* for damage in a single element along the beam at location x/L: ——, 10% thickness reduction, with corresponding change in cross-sectional area, element mass, and area moment of inertia; - - - -, 5% thickness reduction with no change in element mass. (a) Mode 1, (b) mode 2, (c) mode 3, (d) mode 4.

damage location, for each damage condition. Maximum sensitivities under open-loop vibration occur at the root of the beam, with the maximum sensitivity barely exceeding a 1% frequency shift for severe damage, and never exceeding 0·5% for modest damage. Figure 5 shows the same results for closed-loop vibration. Here, sensitivity to damage near the root of the beam has increased by a factor of approximately 60 (first mode) to a factor of 5 (third mode). Sensitivity of mode 4 is unchanged, since this mode was not controlled. Additional modal sensitivities (not shown in figure) also remain largely unchanged. The fact that sensitivity of uncontrolled modes remains unchanged indicates minimal spillover due to design of the control law based on a low order model.

Figures 4 and 5 demonstrate enhanced sensitivity of the controlled system over open-loop vibration for a single actuator at node 2. Control laws also were designed for identical closed-loop root locations as given in Table 2, and a single actuator at nodes 4 and 6 respectively. Figure 6 shows the resulting frequency shifts in the first four modes for these actuator locations. Here, we note that the direction of frequency shift and the magnitude of sensitivity enhancement as a function of damage location depends significantly on actuator location. For the actuator located at node 2, near the root of the beam, sensitivity is enhanced significantly near the root, and less as the damage location progresses toward the tip. For an

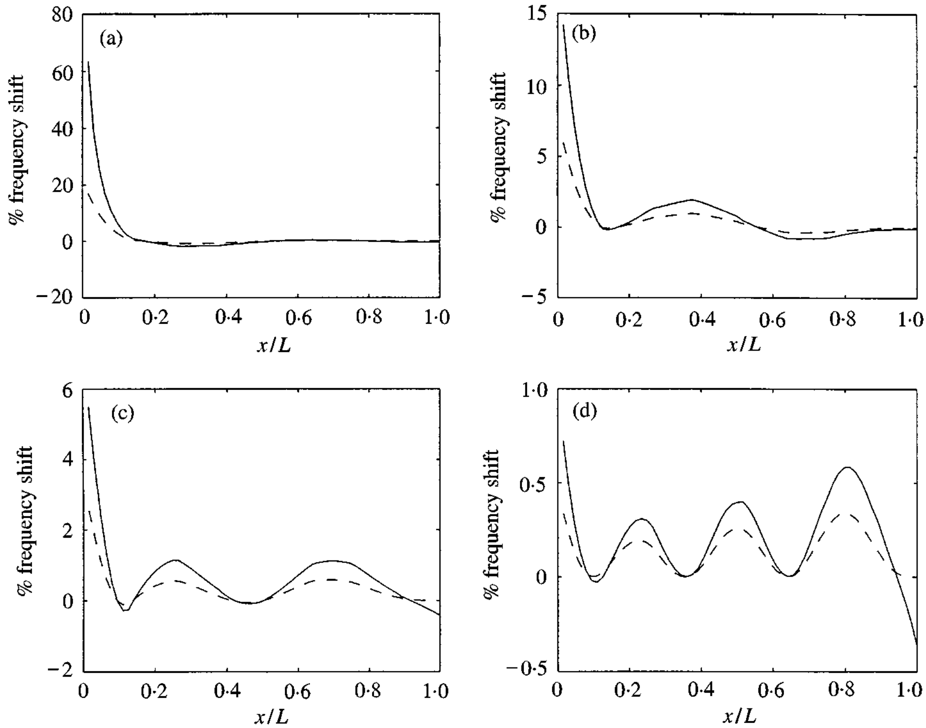

Figure 5. Frequency shifts in first four modes of cantilevered beam under *closed-loop vibration* for damage in a single element along the beam at location x/L: ——, 10% thickness reduction, with corresponding change in cross-sectional area, element mass, and area moment of inertia; – – –, 5% thickness reduction with no change in element mass. (a) Mode 1, (b) mode 2, (c) mode 3, (d) mode 4.

actuator located at node 4 or node 6, sensitivity is enhanced significantly in the first two modes along most of the beam.

3.3. OUTPUT FEEDBACK

Results shown in Figures 4–6 demonstrated that full-state feedback closed-loop control can significantly enhance sensitivity to stiffness reduction using a single actuator. Next, we seek to enhance sensitivity using output feedback, in which state estimation would not be required, and hence significantly less real-time computation would be required. With output feedback, closed-loop roots cannot be arbitrarily placed, and root locus design methods are used to determine control gains. Figure 7 shows a root locus for (positive) output feedback of a single sensor collocated at node 2 of the beam with the actuator. The sensor location determines the zero locations in the complex plane, and for the chosen sensor location, closed-loop roots of all four modes migrate to lower frequencies as the gain increases, indicating that sensitivity enhancement should occur. Closed-loop roots of the first three modes as the feedback gain increases are shown in Figure 7. In Figure 8, the sensitivity to damage for a feedback gain $K = -5000$ and the two

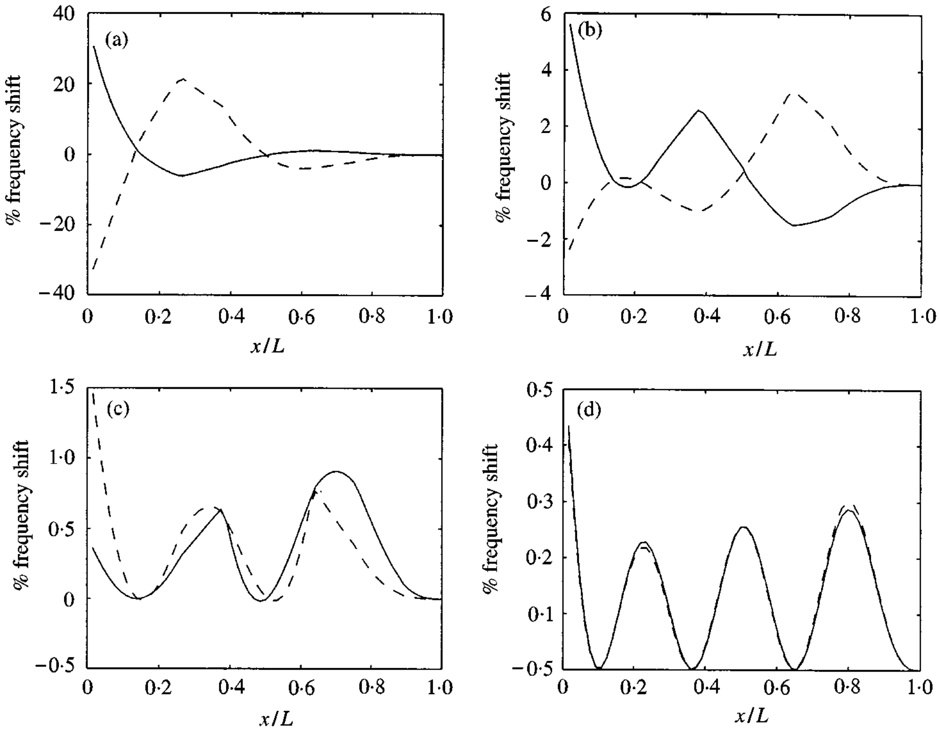

Figure 6. Frequency shifts in first four modes of cantilevered beam under *closed-loop vibration* for damage in a single element along the beam at location x/L. Damage mode is 5% thickness reduction with no change in element mass: ——, actuator at node 4; – – –, actuator at node 6. (a) Mode 1, (b) mode 2, (c) mode 3, (d) mode 4.

damage cases is examined. Results show that increases in sensitivity comparable to the full-state feedback case are achievable with output feedback.

3.4. ENHANCEMENT OF MODE SHAPE CHANGES

Cornwell *et al.* [11] and Stubbs and Kim [12] present damage detection measures that are based on the change in mode shape curvature due to damage. While we focus on shaping modal frequency sensitivity here, we also demonstrate modest sensitivity enhancement of mode shape changes through feedback control. Figure 9 shows the difference in the undamaged and damaged normalized mode shapes under open-loop control and closed-loop control when the beam is damaged at element 10. The damage mode is 5% thickness reduction with no mass reduction. Here, two control laws are implemented. Control law 1 is that designed to target frequency sensitivity, as presented in Table 2, while Control law 2 shifts the first three modal frequencies to higher values in order to enhance amplitude sensitivity. The results show that a significant spike in the mode shape difference occurs at the damage location for both open-loop and closed-loop control. However, sensitivity is an issue, as the percent change in mode shape at the damage

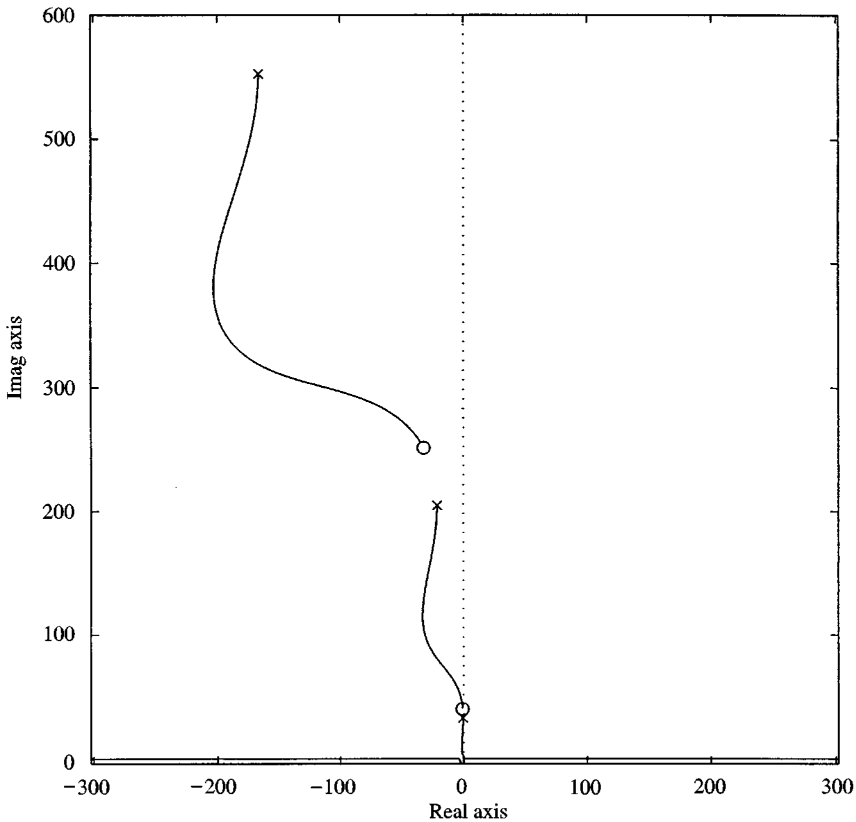

Figure 7. Output feedback root locus for system with a single sensor and actuator collocated at node 2. "×" marks open-loop root locations, and "○" marks zero locations.

location for the open-loop system is at best 0·6% (mode 1) and as low as 0·18% (mode 3). Under Control law 1, the magnitude of the mode shape change is enhanced significantly only in mode 3, while for Control law 2, all of the controlled modes exhibit modest mode shape change sensitivity enhancement. As a percent change from the undamaged mode shape, mode 3 sensitivity changes from 0·18% in the open-loop system to 0·89% in the closed-loop system.

4. CONCLUSIONS

A method of enhancing modal sensitivity to local damage using feedback control is presented and demonstrated. Applying appropriate state or output feedback control laws developed for a low order control model of a cantilevered beam to higher order model of the beam results in significant enhancement of the sensitivity of controlled modes to damage. State feedback can be implemented through estimation of state variables from strain measurements, or output feedback can be implemented using the measured strains directly. The concepts presented here form a basis for initial experiments and development of a control system design methodology for enhancing modal sensitivity to damage in smart structures.

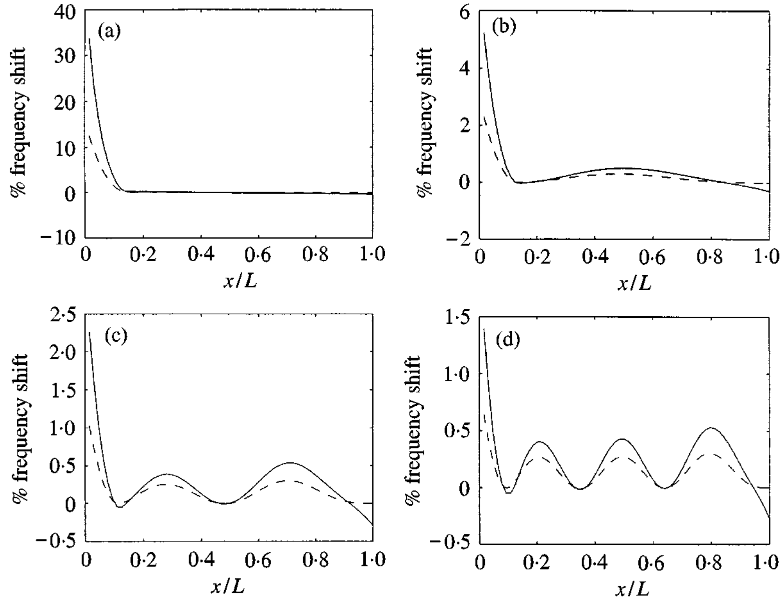

Figure 8. Frequency shifts in first four modes of cantilevered beam under *closed-loop vibration* using output feedback for damage in a single element along the beam at location x/L: ——, 10% thickness reduction, with corresponding change in cross-sectional area, element mass, and area moment of inertia; – – –, 5% thickness reduction with no change in element mass. (a) Mode 1, (b) mode 2, (c) mode 3, (d) mode 4.

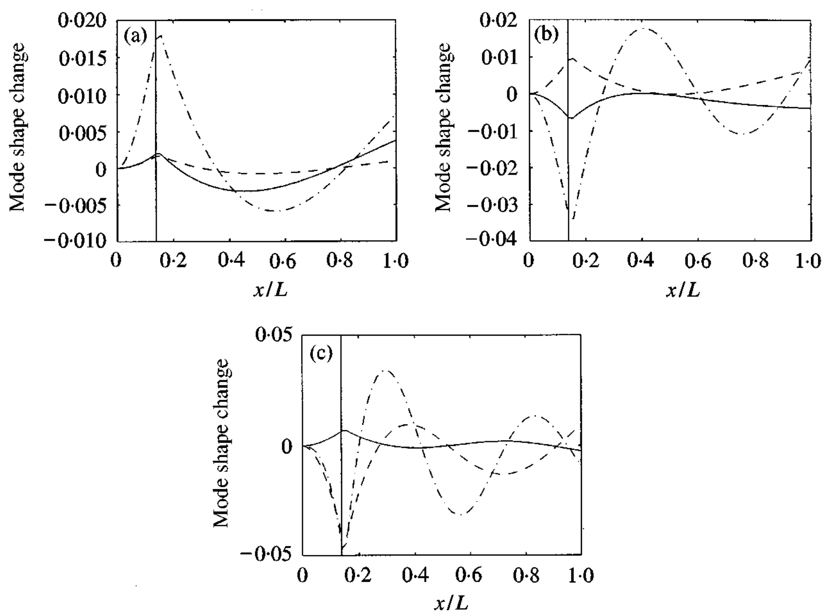

Figure 9. Difference between undamaged and damaged mode shape for open-loop vibration and for two closed-loop systems: ——, open-loop; – – –, control law 1; · – · – · –, control law 2. (a) Mode 1, (b) mode 2, (c) mode 3.

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