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LETTERS TO THE EDITOR

COMMENTS AND ADDITIONS TO "TRANSVERSE VIBRATIONS OF CIRCULAR, ANNULAR PLATES WITH SEVERAL COMBINATIONS OF BOUNDARY CONDITIONS"

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The recent paper of Vera *et al.* [1] provides very accurate frequency parameters for annular plates with all the four possible combinations of simply supported and clamped edges. Only the lower frequency mode for each number n of nodal diameters is considered. These results are compared with the ones published in references [2, 3] with reasonably good agreement. Actually, some of the data given in reference [1] can be found in the recent papers of Amabili *et al.* [4] and Amabili and Dalpiaz [5] with a very good accuracy. The frequency parameters given in references [4, 5] are in excellent agreement with the ones given in reference [1] for annular plates clamped or simply supported at both edges; therefore they are more suitable for comparison than older ones. References [4, 5] provide data also for modes with different number m of nodal circles and for different boundary conditions. In particular, the following cases were considered in reference [4]: (i) both edges clamped; (ii) inner free edge and outer clamped edge; and (iii) both free edges. Reference [5] gives data for the following cases: (i) simply supported at both edges; (ii) free at the inner edge and simply supported at the outer edge.

The possible combinations of free, simply supported and clamped edges are nine for annular plates. All of them have been investigated by Vogel and Skinner [2]. More accurate results are given for seven of these cases in references [1, 4, 5]. Many other studies give frequency parameters for annular plates; a complete review of these, up to year 1987, has been given by Weisensel [6].

We thus has the opportunity to give accurate frequency parameters also for annular plates clamped at the inner edge and free at the outer edge for different Poisson's ratio ν; in fact, this case is interesting for engineering applications and it is not covered in references [1, 4, 5]. The frequency parameter, λ_{mn}, is related to the radian frequency, ω_{mn}, of the plate *in vacuo* by

$$\omega_{mn} = (\lambda_{mn}^2/a_2^2)\sqrt{D/\rho h}, \tag{1}$$

where a_2 is the outer radius of the plate, ρ is the mass density, h is the thickness and $D = Eh^3/[12(1-\nu^2)]$ is the flexural rigidity of the plate; ν and E are the

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TABLE 1

Frequency parameters λ_{mn} for annular plates clamped at the inner edge and free at the outer edge for $\nu = 0{\cdot}3$

m	n	$a = 0{\cdot}1$	$a = 0{\cdot}3$	$a = 0{\cdot}5$	$a = 0{\cdot}7$	$a = 0{\cdot}9$
0	0	2·0583	2·5806	3·6088	6·0787	18·557
0	1	1·8648	2·5596	3·6454	6·1234	18·578
0	2	2·3711	2·8206	3·8344	6·2670	18·640
0	3	3·5285	3·6434	4·3082	6·5309	18·743
0	4	4·6728	4·6981	5·0591	6·9332	18·887
0	5	5·7874	5·7927	5·9773	7·4752	19·074
0	6	6·8831	6·8842	6·9764	8·1405	19·302
1	0	5·0260	6·5278	9·2212	15·486	46·784
1	1	5·2603	6·6805	9·3114	15·533	46·798
1	2	6·0777	7·1376	9·5779	15·671	46·839
1	3	7·2940	7·8772	10·008	15·897	46·907
1	4	8·5765	8·8384	10·585	16·207	47·001
1	5	9·8364	9·9389	11·289	16·594	47·123
1	6	11·071	11·106	12·099	17·052	47·270
2	0	8·5965	11·111	15·610	26·089	78·457
2	1	8·8026	11·217	15·666	26·116	78·464
2	2	9·4963	11·536	15·832	26·196	78·489
2	3	10·631	12·070	16·108	26·329	78·524
2	4	11·939	12·809	16·490	26·514	78·577
2	5	13·256	13·726	16·973	26·750	78·645
2	6	14·552	14·781	17·552	27·037	78·727
3	0	12·111	15·626	21·919	36·584	109·89
3	1	12·299	15·709	21·960	36·603	109·89
3	2	12·905	15·958	22·084	36·661	109·92
3	3	13·935	16·375	22·289	36·757	109·92
3	4	15·218	16·957	22·575	36·891	109·97
3	5	16·562	17·698	22·939	37·062	110·02
3	6	17·897	18·585	23·379	37·271	110·08

TABLE 2

Frequency parameters λ_{mn} for annular plates clamped at the inner edge and free at the outer edge for $\nu = 0{\cdot}33$

m	n	$a = 0{\cdot}1$	$a = 0{\cdot}3$	$a = 0{\cdot}5$	$a = 0{\cdot}7$	$a = 0{\cdot}9$
0	0	2·0639	2·5877	3·6169	6·0873	18·566
0	1	1·8653	2·5610	3·6483	6·1285	18·585
0	2	2·3509	2·8044	3·8232	6·2620	18·644
0	3	3·5006	3·6155	4·2818	6·5121	18·741
0	4	4·6397	4·6647	5·0234	6·9003	18·877
0	5	5·7499	5·7551	5·9369	7·4305	19·053
0	6	6·8416	6·8427	6·9328	8·0876	19·270
1	0	5·0335	6·5352	9·2283	15·493	46·791
1	1	5·2648	6·6855	9·3170	15·539	46·804
1	2	6·0765	7·1372	9·5792	15·674	46·844

TABLE 2 Continued

m	n	$a = 0{\cdot}1$	$a = 0{\cdot}3$	$a = 0{\cdot}5$	$a = 0{\cdot}7$	$a = 0{\cdot}9$
1	3	7·2880	7·8713	10·003	15·896	46·911
1	4	8·5669	8·8283	10·575	16·201	47·004
1	5	9·8238	9·9258	11·274	16·582	47·122
1	6	11·055	11·091	12·081	17·034	47·267
2	0	8·6006	11·115	15·614	26·093	78·461
2	1	8·8057	11·220	15·669	26·119	78·467
2	2	9·4971	11·537	15·834	26·198	78·493
2	3	10·629	12·068	16·108	26·330	78·527
2	4	11·935	12·804	16·487	26·513	78·580
2	5	13·250	13·719	16·967	26·747	78·646
2	6	14·544	14·772	17·543	27·031	78·728
3	0	12·114	15·629	21·922	36·587	109·89
3	1	12·301	15·711	21·963	36·606	109·89
3	2	12·906	15·960	22·086	36·663	109·93
3	3	13·935	16·375	22·290	36·758	109·92
3	4	15·216	16·955	22·574	36·891	109·97
3	5	16·558	17·694	22·936	37·061	110·02
3	6	17·891	18·579	23·374	37·268	110·08

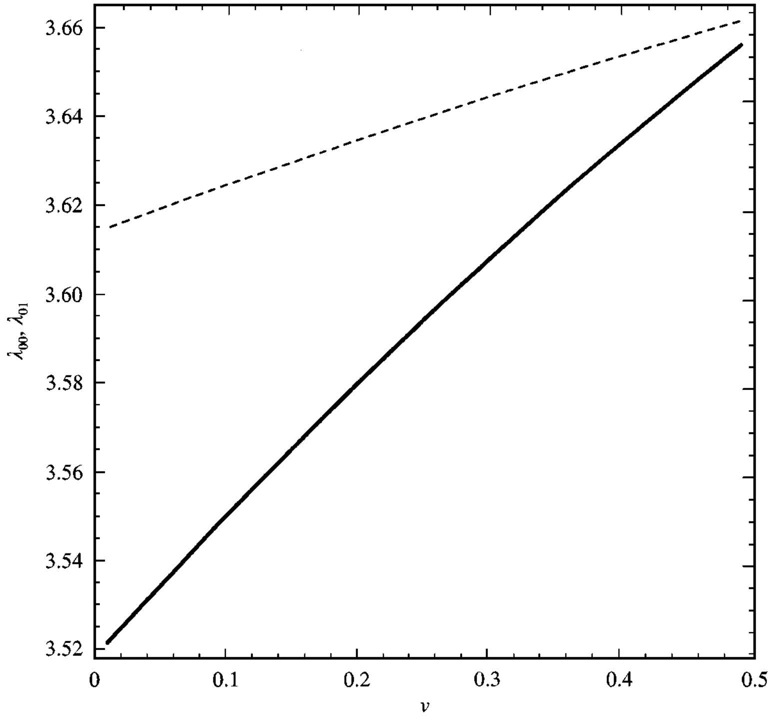

Figure 1. Effect of Poisson's ratio on the frequency parameters λ_{mn} of annular plates clamped at the inner edge and free at the outer edge; $a = 0{\cdot}5$. ——, $n = 0$, $m = 0$; – – – , $n = 1$, $m = 0$.

Poisson's ratio and the Young's modulus, respectively. It is useful to define the parameter $a = a_1/a_2$, where a_1 is the inner radius of the plate. The values of λ_{mn} are given in Tables 1 and 2 with five significant digits for $\nu = 0{\cdot}3$ and $\nu = 0{\cdot}33$, respectively, and different values of $a = 0{\cdot}1$, $0{\cdot}3$, $0{\cdot}5$, $0{\cdot}7$, $0{\cdot}9$. They have been obtained by using the classical Kirchhoff theory of thin plates, with neglect of rotary inertia and shear deformation. Computations have been made on a *Silicon Graphics Onyx 2* parallel computer by using a self-made *C* code.

It is interesting to observe that, for $\nu = 0{\cdot}3$, the fundamental mode is ($n = 1$, $m = 0$) for $a = 0{\cdot}1$, $0{\cdot}3$; for $a = 0{\cdot}5$, $0{\cdot}7$, $0{\cdot}9$ the fundamental mode is the first axisymmetric mode ($n = 0$, $m = 0$). The frequency parameters increase with a; in particular, for $a = 0{\cdot}9$ the effect of the number of nodal diameters n is almost negligible and the frequency parameters depend mainly on the number of nodal circles m. The values given in Table 1 are in reasonably good agreement with those given in reference [2] except for mode ($n = 1$, $m = 0$) and $a = 0{\cdot}1$ and for modes ($n = 0$, $m = 0$, 1) and $a = 0{\cdot}9$; for these the values given in reference [2] must be considered inaccurate or wrong. The effect of the Poisson's ratio on natural frequencies of modes ($n = 0$, $m = 0$) and ($n = 1$, $m = 0$) in shown in Figure 1 for $a = 0{\cdot}5$; it can be considered significant, especially for the first axisymmetric mode ($n = 0$, $m = 0$), in very accurate computations. It is interesting to note that the frequency parameters considered in Figure 1 have almost a linear behavior versus the Poisson's ratio.

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