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A NOTE ON THE STABILITY AND CHAOTIC MOTIONS OF A RESTRAINED PIPE CONVEYING FLUID

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Prediction of the behavior of a pipe carrying flowing fluid is of considerable importance in engineering. Jin [1] has studied the stability and chaotic motions of a restrained pipe conveying fluid (Figure 1). He determined the fixed points of the equation of motion of the pipe, which represent the configuration of static deformation of the pipe (equilibria), and then analyzed their stability in a parameter plane. This information was used by Jin for the determination of flow behavior of the system in phase space.

However, it should be noted that the determination of vibration amplitudes of the pipe carrying flowing fluid is not of less importance than that of the flow behavior of the system in phase space. As it is shown below, Jin's analytical model gives a very simple method for the determination of the vibration amplitude for the case of some periodic motions.

In this note we use the following four-dimensional first order ordinary differential equation of motion of the pipe which was obtained by Jin [1] for the system of two degree of freedom ($N = 2$):

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{F}(\mathbf{X}), \tag{1}$$

where the dot denotes differentiation with respect to τ, $\tau = (EI/[M + m])^{1/2} t/L^2$, EI is the flexural rigidity of the pipe, L its length and m its mass per unit length, M is the mass of the fluid per unit length, t is the time variable:

$$\mathbf{X} = (x_1, x_2, x_3, x_4)^{\mathrm{T}}, \qquad x_1 = q_1(\tau), \qquad x_2 = q_2(\tau), \qquad x_3 = \dot{q}_1(\tau), \qquad x_4 = \dot{q}_2(\tau),$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \end{bmatrix}, \qquad \mathbf{F}(\mathbf{X}) = (0, 0, F_3, F_4)^{\mathrm{T}}.$$

For the sake of simplicity, consideration is restricted to the case of a system with the one degree of freedom ($N = 1$) which is obviously contained in Jin's analytical

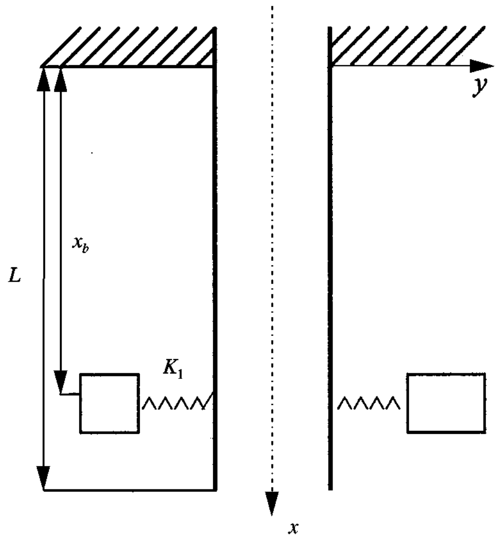

Figure 1. Schematic of the system treated in this paper.

model [1] as a special case. Then, writing equation (1) in the scalar form for $N = 1$ leads to

$$\ddot{q}_1 = a_1 q_1 + a_3 \dot{q}_1 + F_3, \tag{2}$$

$$0 = b_1 q_1 + b_3 \dot{q}_1, \tag{3}$$

where

$$a_1 = -(\lambda_1^4 + u^2 c_{11} + \gamma e_{11} + k_1 g_{11}), \qquad a_3 = -(\alpha\lambda_1^4 + 2\sqrt{\beta} u b_{11}),$$

$$b_1 = -(u^2 c_{21} + \gamma e_{21} + k_1 g_{21}), \qquad b_3 = -2\sqrt{\beta} u b_{21}, \qquad F_3 = -k_2 q_1^3 \varphi_1^4(\xi_b),$$

$$b_{11} = 2,$$

$$c_{11} = 2\lambda_1\sigma_1 - \lambda_1^2\sigma_1^2, \qquad d_{11} = c_{11}/2 = \lambda_1\sigma_1 - \lambda_1^2\sigma_1^2/2,$$

$$e_{11} = b_{11} + d_{11} - c_{11} = 2 - \lambda_1\sigma_1 + \lambda_1^2\sigma_1^2/2, \qquad g_{11} = \varphi_1^2(\xi_B), \qquad b_{21} = -4,$$

$$c_{21} = -4\lambda_1\sigma_1,$$

$$d_{21} = -4(\lambda_1\sigma_1 + 2) + 12 = -4\lambda_1\sigma_1 + 4,$$

$$e_{21} = b_{21} + d_{21} - c_{21} = -4 - 4\lambda_1\sigma_1 + 4 + 4\lambda_1\sigma_1 = 0, \qquad g_{21} = 0,\ \xi = x/L,$$

x is the longitudinal co-ordinate, $\xi_b = x_b/L$, $u = (M/EI)^{1/2}UL$, U is the fluid velocity, $\alpha = (EI/[M+m])^{1/2}\,a/L^2$, $\beta = M/(M+m)$, $\gamma = (M+m)gL^3/EI$, g is the acceleration due to gravity, $k_1 = K_1 L^3/EI$, $k_2 = K_2 L^5/EI$, K_1 is the stiffness of the spring of the elastic support, K_2 is the stiffness of the cubic spring, which represents the effect of the motion constraints, $\varphi_1(\xi) = \cosh\lambda_1\xi - \cos\lambda_1\xi - \sigma_1 \sinh\lambda_1\xi - \sin\lambda_1\xi)$ is the eigenfunction of the cantilever beam and λ_1 represents the eigenvalue of the cantilever beam,

$$\sigma_1 = [\sinh\lambda_1 - \sin\lambda_1]/[\cosh\lambda_1 + \cos\lambda_1].$$

Further, we will use the assumption of harmonic time dependence for the function $q_1(\tau)$:

$$q_1(\tau) = B\sin\Omega\tau + D\cos\Omega\tau, \tag{4}$$

where $\Omega = L^2\omega/(EI/[M+m])^{1/2}$ and ω is the circular frequency.

In the work presented here, the assumption of harmonic time dependence for the function $q_1(\tau)$ is used for purposes of illustration of possible applications of Jin's analytical model.

Substituting equation (4) into equation (2), taking the inner product with $\sin\Omega\tau$, and noting that

$$\int_0^{2\pi/\Omega} \sin k\Omega\tau \sin n\Omega\tau \,\mathrm{d}\tau \begin{cases} = 0, & k \neq n, \quad k, n = 1, 2, 3, \ldots, \\ \neq 0, & k = n, \end{cases}$$

$$\int_0^{2\pi/\Omega} \cos k\Omega\tau \sin n\Omega\tau \,\mathrm{d}\tau = 0, \quad \text{for all } k, n = 1, 2, 3, \ldots$$

one obtains

$$B\left[\frac{3k_2\varphi_1^4(\xi_b)}{4\Omega}\mathrm{P}^2 - \left(\Omega + \frac{a_1}{\Omega}\right)\right] + Da_3 = 0, \tag{5}$$

where $P = \sqrt{B^2 + D^2}$ is the vibration amplitude in the transverse direction.

Substituting equation (4) into equation (3), taking the inner product with $\cos\Omega\tau$, and noting that

$$\int_0^{2\pi/\Omega} \cos k\Omega\tau \cos n\Omega\tau \,\mathrm{d}\tau \begin{cases} = 0, & k \neq n, \quad k, n = 1, 2, 3, \ldots, \\ \neq 0, & k = n, \end{cases}$$

$$\int_0^{2\pi/\Omega} \cos k\Omega\tau \sin n\Omega\tau \,\mathrm{d}\tau = 0, \quad \text{for all } k, n = 1, 2, 3, \ldots$$

one obtains

$$Bb_3 + D\frac{b_1}{\Omega} = 0. \tag{6}$$

Inspection of homogeneous equations (5) and (6) in B and D shows that for a non-trivial pair of parameters B and D, the determinant of the corresponding coefficient matrix must be zero:

$$\begin{vmatrix} \dfrac{3k_2\varphi_1^4(\xi_b)}{4\Omega}P^2 - \left(\Omega + \dfrac{a_1}{\Omega}\right) & a_3 \\ b_3 & \dfrac{b_1}{\Omega} \end{vmatrix} = 0. \tag{7}$$

The determinantal equation (7) yields the following expression for the vibration amplitude in the transverse direction:

$$P = \frac{2\Omega}{\varphi_1^2(\xi_b)} \sqrt{\frac{1 + a_3 b_3/b_1 + a_1/\Omega^2}{3k_2}}. \tag{8}$$

This last equation can be also deduced as follows.

Equation (6) leads to

$$D = -\frac{b_3\Omega}{b_1} B.$$

Substituting this into equation (5) yields

$$B\left[\frac{3k_2\varphi_1^4(\xi_b)}{4\Omega} P^2 - \left(\Omega + \frac{a_1}{\Omega}\right) - \frac{a_3 b_3 \Omega}{b_1}\right] = 0.$$

Since the solution to equations (5) and (6), (B, D), is non-trivial, the expression inside the square brackets must be equal to zero:

$$\frac{3k_2\varphi_1^4(\xi_b)}{4\Omega} P^2 - \left(\Omega + \frac{a_1}{\Omega}\right) - \frac{a_3 b_3 \Omega}{b_1} = 0.$$

It is easy to verify that this equation written in terms of P is the same as equation (8).

REFERENCES

1. J.-D. Jin 1997 *Journal of Sound and Vibration* **208**, 427–439. Stability and chaotic motions of a restrained pipe conveying fluid.