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FORCED VIBRATIONS OF A CLAMPED, CIRCULAR PLATE OF RECTANGULAR ORTHOTROPY

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1. INTRODUCTION

In the case of free vibrations of a circular plate of rectangular orthotropy the vibration analyst encounters severe difficulties. Analytical, exact solutions appear possible in some instances [1]. Useful, approximate solutions are available [2–7]. Free vibrations of thin and elastic plates of complicated boundary shape of rectangular orthotropy have also been studied [8, 9]. On the other hand, apparently, forced vibrations have not been previously considered. This note deals with an approximate treatment of the title problem in the case where the plate is subjected to $a - p_0 \cos \omega t$-type excitation. The Galerkin method is employed.

2. APPROXIMATE ANALYTICAL SOLUTION

Forced vibration of the clamped orthotropic circular plate are described by the differential system (see Figure 1)

$$D_1 \frac{\partial^4 w}{\partial x^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 w}{\partial y^4} = p_0 \cos \omega t - \rho h \frac{\partial^2 w}{\partial t^2} \tag{1}$$

and, in polar co-ordinates the boundary conditions are

$$w(a, t) = \frac{\partial w}{\partial r}(a, t) = 0. \tag{2a,b}$$

Making

$$w(x, y, t) = W(x, y) \cos \omega t \tag{3}$$

and substituting in equation (1) one obtains

$$D_1 \frac{\partial^4 W}{\partial x^4} + 2D_3 \frac{\partial^4 W}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 W}{\partial y^4} - p_0 - \rho h \omega^2 W = 0. \tag{4}$$

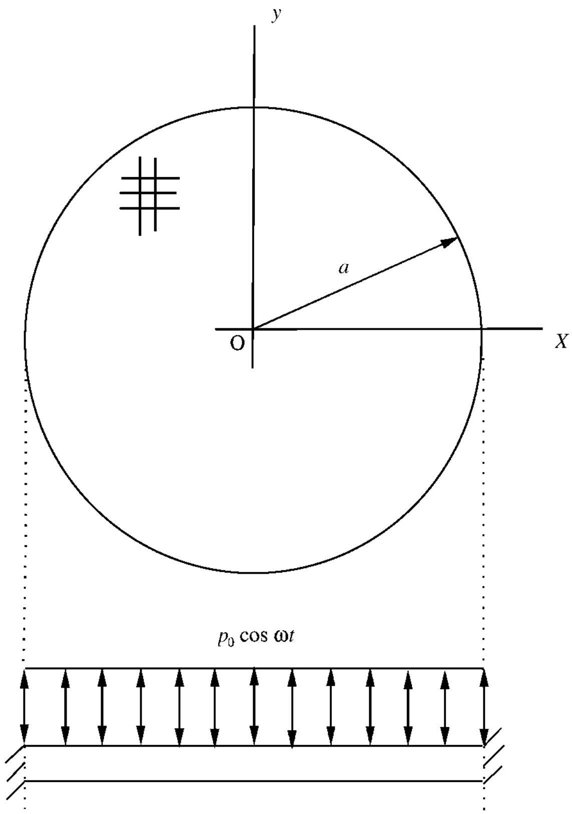

Figure 1. Forced vibrations of a clamped circular orthotropic plate.

A valid, approximate solution is given by [2]

$$W(x, y) \cong W_a(x, y) = A_1(a^2 - x^2 - y^2)^2 \tag{5}$$

and substituting in equation (4) yields the error or residual function

$$\varepsilon_1(x, y) = 8A_1(3D_1 + 2D_3 + 3D_2) - p_0 - \rho h \omega^2 A_1(a^2 - x^2 - y^2)^2 \tag{6}$$

which can be conveniently expressed as

$$\varepsilon(x, y) = D_1\left[8A_1\left(3 + 2\frac{D_3}{D_1} + 3\frac{D_2}{D_1}\right) - \frac{p_0}{D_1} - \frac{\Omega^2}{a^4}A_1(a^2 - x^2 - y^2)^2\right], \tag{7}$$

where $\Omega = \sqrt{\rho h/D_1}\,\omega a^2$.
Requiring now that the error function be orthogonal with respect to the co-ordinate function $W_a(x, y)$,

$$\iint \varepsilon(x, y) W_a \,\mathrm{d}x\,\mathrm{d}y = 0, \tag{8}$$

one obtains, substituting equations (5) and (7) in equation (8) and performing the required integration,

$$A_1 = \frac{5p_0}{(120D_1 - 3D_1\Omega^2 + 120D_2 + 80D_3)}. \tag{9}$$

The fundamental frequency coefficient of the system Ω_1 is obtained by requiring that the denominator of equation (9) approaches zero and one obtains

$$\sqrt{\rho h/D_1}\,\omega_1 a^2 = \Omega_1 = 6{\cdot}3245\sqrt{1 + 0{\cdot}6667\frac{D_3}{D_1} + \frac{D_2}{D_1}}. \tag{10}$$

Substituting equation (10) in equation (9) one determines the following convenient expression for the displacement amplitude at the plate center:

$$A_1 = \frac{5p_0}{3D_1\Omega_1^2[1 - (\Omega/\Omega_1)^2]} \tag{11}$$

and substituting equation (11) in equation (5) and expressing the result in terms of dimensionless variables one obtains

$$W_a\frac{D_1\Omega_1^2}{a^4 p_0} = \frac{5}{3}\frac{1}{[1 - (\Omega/\Omega_1)^2]}[1 - (x/a)^2 - (y/a)^2]^2, \tag{12}$$

where $0 \leqslant \Omega < \Omega_1$ and $\Omega_1 < \Omega < \Omega_2$ (Ω_2: second natural frequency corresponding to axisymmetric dynamic behavior).

When the plate is isotropic one has

$$D_1 = D_2 = D_3 = D$$

and then

$$\Omega_1 = \sqrt{\frac{320}{3}} = 10{\cdot}3279$$

which is approximately 1% higher than the exact value.

Accordingly equation (12) yields, for the static case ($\Omega = 0$),

$$W_a\frac{D(320/3)}{a^4 p_0} = \frac{5}{3}[1 - (x/a)^2 - (y/a)^2]^2$$

and rearranging one obtains

$$\frac{W_a}{(a^4 p_0/D)} = \frac{1}{64}[1 - (x/a)^2 - (y/a)^2]^2 \tag{13}$$

which coincides with the exact solution.

3. NUMERICAL RESULTS

Figure 2 depicts the variation of plate amplitudes for $y = 0$ as a function of the dimensionless ratio Ω/Ω_1.

Clearly, expression (5) constitutes a first order approximation since it does not take into account the azimuthal variation of the plate response. This variation will

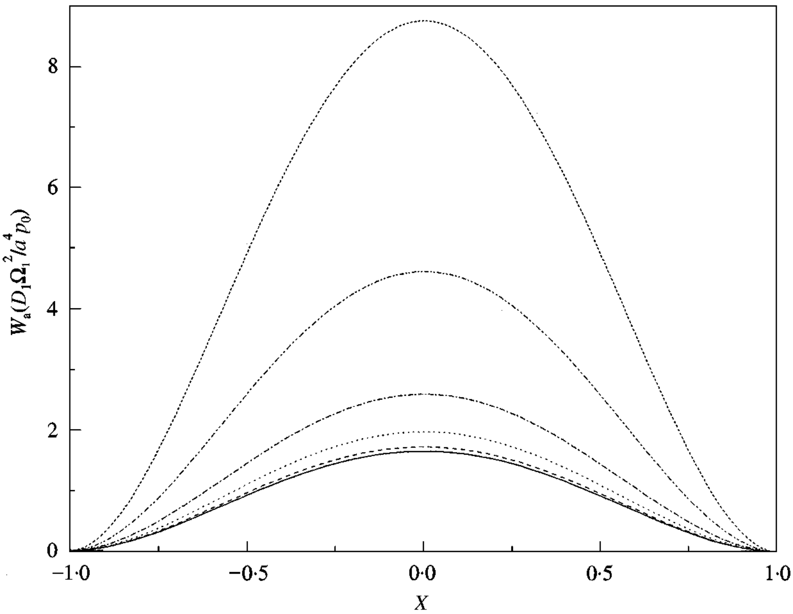

Figure 2. Dimensionless dynamic amplitudes at $y = 0$ and as a function of Ω/Ω_1: ——, $\Omega/\Omega_1 = 0$; – – –, 0·2; ------, 0·7; – · – · –, 0·6; – · · – · ·, 0·8; – – –, 0·9; $a = 1$.

be accounted for, if one makes

$$W_a = \sum_{n=0}^{N} \sum_{m=0}^{M} A_{nm} (a^2 - x^2 - y^2)^2 x^{2n} y^{2m}. \tag{14}$$

A one-term approximation will suffice if an "average" response, from a global viewpoint, is sought.

On the other hand, using equation (14) one will be able, in principle, to obtain a detailed description of the dynamic response. Accordingly, dynamic stress resultants may be obtained. The approach is similar in the case of a simply supported edge although, for practical reasons, one prefers to satisfy only the essential boundary condition.

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