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THE BOUNDARY POINT METHOD FOR THE CALCULATION OF EXTERIOR ACOUSTIC RADIATION PROBLEM

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A new numerical method, the boundary point method, used for calculating the acoustic radiation problem caused by a vibrating body is presented. The gist of the new numerical method is to replace the coefficient matrices [**A**] and [**B**] in the system equation with the particular solution matrices which are formed of the particular solutions generated by fabricated sources. In the boundary point method, it is unnecessary to consider the interpolating operation and the singular integral which is indispensable for the BEM also does not exist. By avoiding the direct computation for the coefficient matrices, the boundary point method can improve the calculation speed substantially while maintaining the calculation precision. Another advantage of the method is that it can be used for calculating the acoustic parameters (such as the sound pressure, etc.) at any desired point in the sound field without calculation of the acoustic parameters on the surface. Finally, the boundary point method can overcome the non-uniqueness problem at the characteristic wavenumbers effectively.

The boundary point method put forward by the authors is applied to the calculation of the exterior acoustic radiation problem caused by a vibrating body. A detailed description of this method is presented. A test for the boundary point method is carried out on the aspects of its calculation precision and speed, adaptation to the geometric shape of vibrating body as well as effectiveness to overcome the non-uniqueness problem through various examples with different shapes and different boundary value distributions. An experiment on the exterior acoustic radiation of a vibrating rectangular box is performed in a semi-anechoic chamber.

1. INTRODUCTION

The boundary element method (BEM) has long been an effective numerical technique for the calculation of the exterior acoustic radiation problem. The major advantages of the BEM over domain methods are the reduction of the computational dimension of the problem by one and adaptation to the infinite domain problem. However, the BEM is not without shortcomings. One of its disadvantages is the formation of the coefficient matrices [**A**] and [**B**] in the system equation, which consumes a lot of CPU time. Another disadvantage is that it has non-unique solutions for exterior problems at certain characteristic frequencies associated with characteristic frequencies of corresponding interior problems [1].

A new numerical method, the boundary point method, used for calculating the acoustic radiation problem has been studied by the authors recently. In the boundary point method, a series of fabricated sources are constructed on the normal lines of the surface nodes of the vibrating body. The coefficient matrices [**A**] and [**B**] in the system equation can then be expressed by the particular solution matrices, which are formed of the particular solutions on the surface nodes generated by these fabricated sources. Comparing with the BEM, it is clear that the boundary point method can decrease the time consumed in the formation of the coefficient matrices greatly and avoid the treatment of the singular integral completely. Besides, the non-unique solution problem arising from the application of the boundary integral equation (BIE) or the BEM in exterior acoustic radiation problem no longer appears in the new numerical method [2, 3].

2. THE BOUNDARY POINT METHOD

Consider a vibrating finite body of enclosed arbitrary surface τ in an infinite homogeneous fluid whose density is ρ, and speed of sound c. The fluid fills the region D_+ exterior to τ. The region interior to τ is designated D_-. The steady state case in which the velocity potential is a harmonic function of time is adopted here. The system equation in matrix form for exterior problem, based on the classical Helmholtz integral equation, can then be written as

$$[\mathbf{A}]\{\Phi\} = [\mathbf{B}]\left\{\frac{\partial \Phi}{\partial \mathbf{n}}\right\}, \tag{1}$$

where $\{\Phi\}$ is an unknown m-vector composed of the velocity potential on the surface nodes, $\mathbf{n}$ is the unit normal on τ (directed away from $\mathbf{D}_-$), $\{\partial\Phi/\partial\mathbf{n}\}$ is a known m-vector determined by the normal velocity on the surface nodes. [**A**] and [**B**] are $m \times m$ coefficient matrices and m is the total number of the surface nodes.

The points p and q are two arbitrary surface nodes of vibrating body as shown in Figure 1. A cube whose side is $2h$ may be constructed in the interior region $\mathbf{D}_-$. The cube is located on the normal line of node p and away from p for some distance. Suppose that uniform source is applied to the cube, the cube can then be used as a fabricated source (of course, other kinds of fabricated source can also be adopted, such as spherical surfaces, etc.). The solution on node q generated by the fabricated

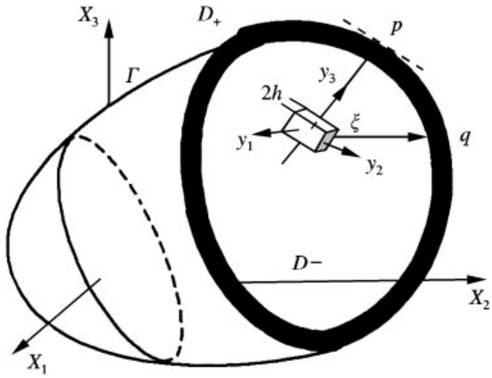

Figure 1. The diagram of the boundary point method.

source is

$$\Phi_T^*(p,q)=\int_{-h}^{+h}\int_{-h}^{+h}\int_{-h}^{+h}G(\xi,q)\,\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}y_3, \tag{2}$$

$$\frac{\partial \Phi_T^*}{\partial n_q}(p,q)=\int_{-h}^{+h}\int_{-h}^{+h}\int_{-h}^{+h}\frac{\partial G(\xi,q)}{\partial n_q}\,\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}y_3,$$

where

$$G(\xi,q)=\frac{1}{4\pi r}\mathrm{e}^{-\mathrm{j}kr},\qquad \frac{\partial G(\xi,q)}{\partial n_q}=-\left(\frac{1}{r}+\mathrm{j}k\right)\frac{\partial r}{\partial n_q}G(\xi,q),$$

$$r=(r_i r_i)^{1/2},\qquad r_i=q_i-\xi_i,\qquad r_{,i}=r_i/r,\qquad \frac{\partial \mathbf{r}}{\partial \mathbf{n}_q}=\mathbf{r}_{,i}\cdot\mathbf{n}_{q_i}$$

in which ξ_i and q_i (i = 1,2,3) are the co-ordinate components of point ξ throughout the cube and node q on τ, respectively, k is the wavenumber ω/c where ω is the circular frequency. $\Phi_T^*(p,q)$ and $(\partial\Phi_T^*/\partial n_q)\,(p,q)$ in formula (2) can be calculated by the 3-D standard Gaussian quadrature after transformation of the upper and lower limits.

The m-vectors $\{\Phi_T^*(p)\}$ and $\{\partial\Phi_T^*(p)/\partial\mathbf{n}\}$ are formed when node q replaces all the m nodes on the surface one by one and can be regarded as a particular solution for the system equation, i.e.,

$$[\mathbf{A}]\{\Phi_T^*(p)\}=[\mathbf{B}]\left\{\frac{\partial\Phi_T^*}{\partial\mathbf{n}}(p)\right\}. \tag{3}$$

Similarly, the $m\times m$ particular solution matrices $[\Phi_T^*]$ and $[\partial\Phi_T^*/\partial\mathbf{n}]$ consisting of m particular solutions can also be formed when node p replaces all the m nodes on the surface one by one and satisfy

$$[\mathbf{A}][\Phi_T^*] = [\mathbf{B}]\left[\frac{\partial \Phi_T^*}{\partial \mathbf{n}}\right]. \tag{4}$$

Formula (4) can be rewritten as

$$[\mathbf{A}]^{-1}[\mathbf{B}] = [\Phi_T^*]\left[\frac{\partial \Phi_T^*}{\partial \mathbf{n}}\right]^{-1}. \tag{5}$$

Therefore, $\{\Phi\}$ can be evaluated when $\{\partial\Phi/\partial\mathbf{n}\}$ is specified as

$$\{\Phi\} = [\mathbf{A}]^{-1}[\mathbf{B}]\left\{\frac{\partial \Phi}{\partial \mathbf{n}}\right\} = [\Phi_T^*]\left[\frac{\partial \Phi_T^*}{\partial \mathbf{n}}\right]^{-1}\left\{\frac{\partial \Phi}{\partial \mathbf{n}}\right\}. \tag{6}$$

Besides, the boundary point method can also be used for calculating the velocity potential at any desired point x in the sound field, denoted by $\Phi(x)$, without calculation of the velocity potentials on the surface nodes. $\Phi(x)$ can be expressed, in the form of the boundary values $\{\Phi\}$ and $\{\partial\mathbf{\Phi}/\partial\mathbf{n}\}$, as [3]

$$\Phi(x) = \{\mathbf{C}\}^{\mathrm{t}}\{\mathbf{\Phi}\} + \{\mathbf{D}\}^{\mathrm{t}}\left\{\frac{\partial \mathbf{\Phi}}{\partial \mathbf{n}}\right\}, \tag{7}$$

where $\{\mathbf{C}\}$ and $\{\mathbf{D}\}$ are coefficient m-vectors, and the superscript "t" denotes the transposition operation.

Combining equations (1) and (7) yields

$$\Phi(x) = (\{\mathbf{C}\}^{\mathrm{t}}[\mathbf{A}]^{-1}[\mathbf{B}] + \{\mathbf{D}\}^{\mathrm{t}})\left\{\frac{\partial \mathbf{\Phi}}{\partial \mathbf{n}}\right\} = \{\mathbf{E}\}\left\{\frac{\partial \mathbf{\Phi}}{\partial \mathbf{n}}\right\}, \tag{8}$$

where $\{\mathbf{E}\} = \{\mathbf{C}\}^{\mathrm{t}}[\mathbf{A}]^{-1}[\mathbf{B}] + \{\mathbf{D}\}^{\mathrm{t}}$.

Suppose that $\{\mathbf{\Phi}_T^*(x)\}$ is a m-vector composed of the velocity potentials at point x generated by the fabricated sources, and $[\partial\mathbf{\Phi}_T^*/\partial\mathbf{n}]$ an $m\times m$ matrix composed of the normal derivatives of the velocity potentials on the surface nodes generated by the fabricated sources, we have

$$\{\mathbf{\Phi}_T^*(x)\}^{\mathrm{t}} = \{\mathbf{E}\}\left[\frac{\partial \mathbf{\Phi}_T^*}{\partial \mathbf{n}}\right]. \tag{9}$$

Formula (9) can be rewritten as

$$\{\mathbf{E}\} = \{\mathbf{\Phi}_T^*(x)\}^{\mathrm{t}}\left[\frac{\partial \mathbf{\Phi}_T^*}{\partial \mathbf{n}}\right]^{-1}. \tag{10}$$

Therefore, $\Phi(x)$ can be evaluated when $\{\partial\mathbf{\Phi}/\partial\mathbf{n}\}$ is specified:

$$\Phi(x) = \{\mathbf{\Phi}_T^*(\mathbf{x})\}^{t}\left[\frac{\partial\mathbf{\Phi}_T^*}{\partial\mathbf{n}}\right]^{-1}\left\{\frac{\partial\mathbf{\Phi}}{\partial\mathbf{n}}\right\}. \qquad (11)$$

Once the velocity potential and its normal derivative are known, the sound pressure P, sound intensity I and sound power W can be obtained subsequently.

3. NUMERICAL EXAMPLES

3.1. THE PULSATING AND OSCILLATING SPHERE

For the problem of acoustic radiation from a pulsating sphere or an oscillating sphere, the analytical solution of the sound pressure on the surface for a pulsating sphere of radius a pulsating with uniform radial velocity v is

$$P(a) = \mathrm{j}v\rho cka/(1 + \mathrm{j}ka) \qquad (12)$$

and the analytical solution of the sound pressure on the surface for an oscillating sphere of radius a oscillating with radial velocity $v\cos\theta$ ($\theta = 0$ is the direction of oscillation) is

$$P(a) = (v\cos\theta)\,\frac{\mathrm{j}\rho cka(1 + \mathrm{j}ka)}{2(1 + \mathrm{j}ka) - (ka)^2}, \qquad (13)$$

where $\rho = 1{\cdot}21\ \mathrm{kg/m^3}$ is the gas density and $c = 344$ m/s the speed of sound in the gas.

Figure 2 shows the discretization of the spherical surface with $a = 0{\cdot}1$ m, $v = 0{\cdot}1$ m/s. The total number of the surface nodes is 20. The numerical and

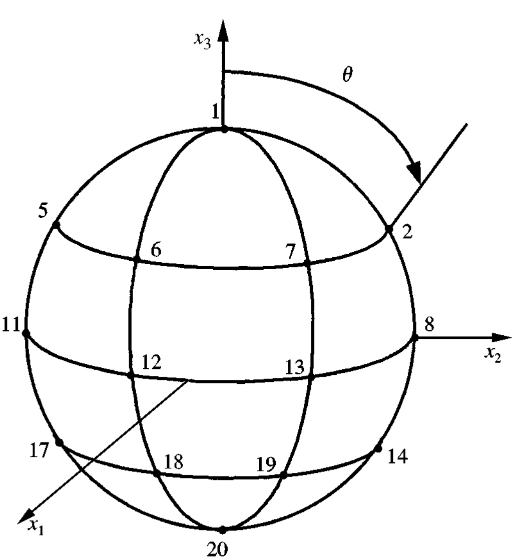

Figure 2. The sphere.

TABLE 1

Results for the pulsating sphere

ka	Numerical solution	Analytical solution
1	20·4706 + j20·4600	20·4706 + j20.4600
2	32·7308 + j16·3722	32·7307 + j16·3720
3	36·8277 + j12·2771	36·8273 + j12·2770
*3·14	37·1551 + j11·8267	37·1554 + j11·8269
4	38·5130 + j9·6278	38·5132 + j9·6277
5	39·3472 + j7·8685	39·3466 + j7·8681
6	39·8140 + j6·6362	39·8139 + j6·6363
*6·28	39·9096 + j6·3515	39·9091 + j6·3517
7	40·1007 + j5·7287	40·1016 + j5·7288
8	40·2906 + j5·0357	40·2905 + j5·0360

TABLE 2

Results for the oscillating sphere

ka	Numerical solution	Analytical solution
1	5·7947 + j17·3649	5·7966 + j17.3687
2	23·1368 + j17·3632	23·1404 + j17·3669
3	27·5699 + j11·2320	27·5727 + j11·2348
4	28·4873 + j8·0089	28·4898 + j8·0122
*4·49	28·6512 + j7·0070	28·6534 + j7·0106
5	28·7492 + j6·2053	28·7509 + j6·2091
6	28·8450 + j5·0706	28·8458 + j5·0752
7	28·8870 + j4·2895	28·8867 + j4·2951
*7.73	28·9037 + j3·8600	28·9025 + j3·8624
8	28·9082 + j3·7193	28·9066 + j3·7260

analytical solutions of the sound pressure on the surface for the pulsating and oscillating sphere at different wavenumbers are shown in Tables 1 and 2 (without loss of generality, the calculating results for node 2 are listed for the oscillating sphere). The errors between the analytical solutions and the numerical ones are less than 0·5%.

The authors have tried using the BEM with cubic spline interpolating function to evaluate the sound pressure on the surface for two examples [4]. From the results it is found that the numerical solutions will be severely in error at the characteristic wavenumbers $ka = \pi$, 2π for the pulsating sphere and $ka = 4{\cdot}4934$, $7{\cdot}7252$ for the oscillating sphere. However, satisfied numerical solutions for the characteristic wavenumbers can be obtained by the boundary point method.

To illustrate the computational efficiency of the boundary point method, the computing time of the BEM with cubic spline interpolating function for one wavenumber is compared with that of the boundary point method. The former is 18·8 s and the latter is 2·2 s.

3.2. THE FINITE CYLINDER

Let us consider the acoustic radiation from a finite cylinder of radius a and length $2b$ where $b/a = 2$. A uniform radial velocity is prescribed on the periphery of the cylinder. The ends of the cylinder are motionless. This problem is a modelling challenge for the numerical method because the finite cylinder processes most of the features of an arbitrary body (e.g. a combination of curved and flat surfaces connected at edges). Many investigators [5–7] took this problem as a test example and calculated its farfield sound pressure patterns on a circle of $r = 5a$ centered at the origin of co-ordinates. Herein the same calculations for this problem are carried out by the boundary point method. Figure 3 shows the discretization of the surface of the finite cylinder. The total number of the surface nodes is 32. The farfield ($r = 5a$) sound pressure patterns are calculated and plotted against the polar angle for $ka = 1$, 2, 2·53, respectively ($ka = 2{\cdot}53$ is corresponding to a characteristic wavenumber of the finite cylinder), as shown in Figure 4. The results of the boundary point method agree with those obtained in references [5–7] for the same problem. The computing time of the boundary point method is 12·5 s and much less than that of the BEM.

3.3. SOUND POWER FOR A CUBE

To show how well the method proposed in this paper handles bodies with edges and corners, a cube with side of length 0·2 m is considered. The test problem is set up by constructing a substitute problem [8]: the surface normal velocity is given by the values obtained from the point source at the center of the cube. A point source of unit strength produces sound power of $k^2\rho c/8\pi$ where k is the wavenumber. Hence, a point source of strength $8\pi/k^2\rho c$ has unit power and it is used as the

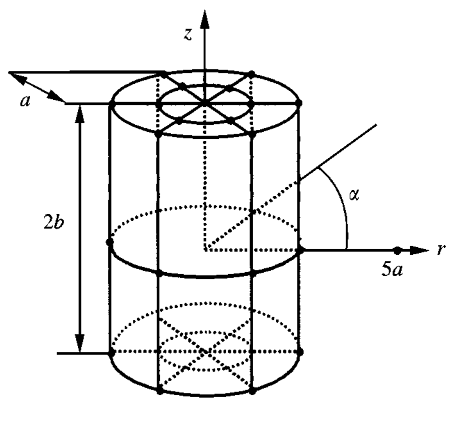

Figure 3. The cylinder.

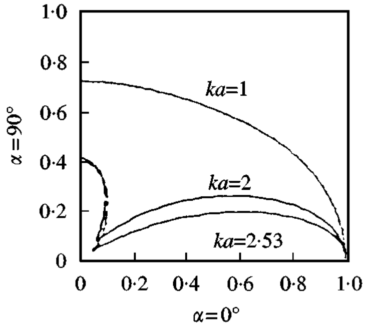

Figure 4. The farfield sound pressure patterns.

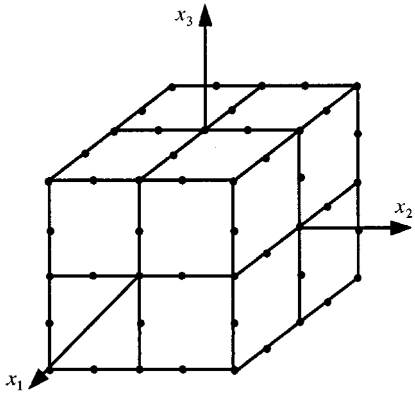

Figure 5. The cube.

TABLE 3

Sound power for the cube

ka	Numerical solution	Theoretical solution	*ka*	Numerical solution	Theoretical solution
1	1·0010	1	5	1·0016	1
2	0·9974	1	6	1·0007	1
3	0·9959	1	7	0·9981	1
4	0·9967	1			

substitute source in the test. The surface of the cube is divided as shown in Figure 5. The total number of the surface nodes is 74. The numerical solutions of the boundary point method at different wavenumbers are given in Table 3. It can be found that these numerical solutions coincide well with the theoretical solution. The errors between the numerical solutions and the theoretical ones are less than 0·5%. The computing time for one wavenumber is less than 2 min.

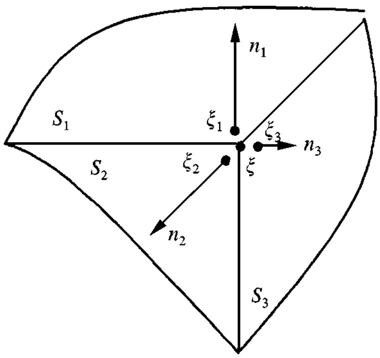

Figure 6. The treatment for the corner.

In the boundary point method, the nodes which are on the corners and edges are treated in a manner different from those in the BEM. For example, the node ξ in Figure 6 is a corner of three surfaces s_1, s_2 and s_3. In the calculation of the boundary point method, the node is replaced by nodes ξ_1, ξ_2 and ξ_3 which are slightly away from ξ and on three surfaces respectively. The corners and edges are smoothed after the treatment and correspond to the circular beads of the mechanical parts.

4. EXPERIMENT

Many mechanical parts such as the axial box of the lathe tool, etc. have a shape similar to that of a rectangular box in engineering. Therefore, it is valid to study the acoustic radiation problem caused by a vibrating rectangular box which can be considered to be the representative of such a problem. For this purpose, a steel rectangular box with side of length $0{\cdot}28\,\mathrm{m(L)} \times 0{\cdot}27\,\mathrm{m(W)} \times 0{\cdot}26\,\mathrm{m(H)}$ is manufactured. Each surface of the box, except for the bottom which will be fixed firmly on the ground in the experiment, has been arranged with 25 surface nodes regularly. Experimental measurements are taken for the vibrating velocity (including both the amplitude and the phase) on the total 125 surface nodes of the rectangular box and the sound pressure level at 10 points on a half-spherical surface around the rectangular box shown in Figure 7 corresponding to homogeneous excitation of 500 Hz in a semi-anechoic chamber. Figure 8 shows the diagram of the whole measuring equipment. Taking the vibrating velocities as input, the sound pressure levels at the above 10 points can be calculated by the boundary point method. From the comparison of the calculated results and measured results shown in Table 4, the effectiveness of the boundary point method for the calculation of acoustic radiation problem is further verified.

The measurement error is one of the important factors that influences the computational accuracy. Due to the limitation of the experimental facilities, the normal velocities on the 125 surface nodes cannot be measured simultaneously. Thus any slight fluctuation of the measurement devices, such as the temperature drift, will affect the measurement accuracy as well as the computational accuracy subsequently. Instead of being measured, however, the normal velocities on the

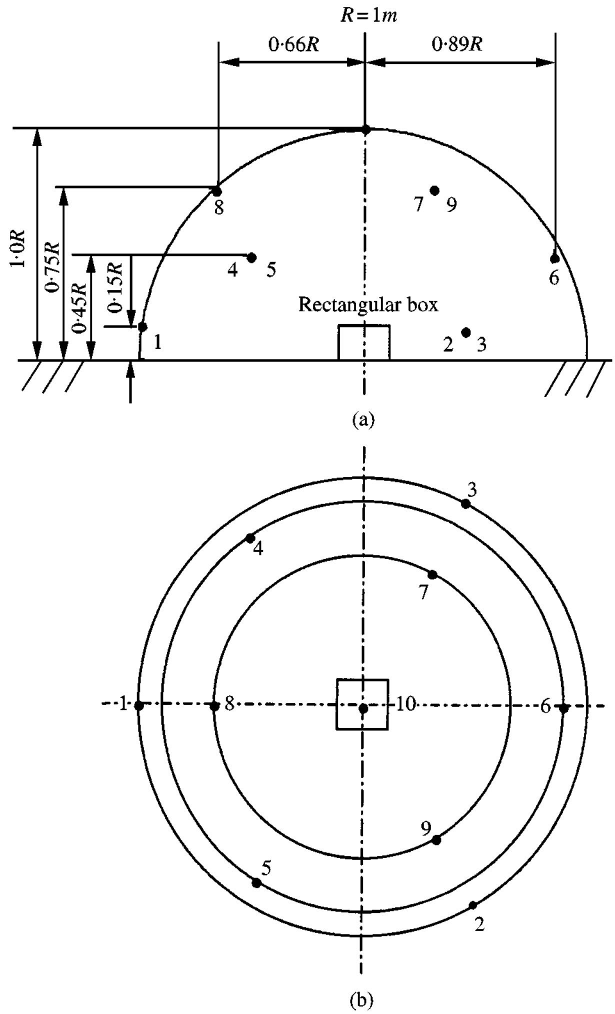

Figure 7. The measuring points of the sound pressure level.

TABLE 4

The Comparison between the calculated results and the measured results (dB)

No.	Calculated result	Measured result	Error
1	53·8973	56·5	2·6
2	53·9822	56·6	2·6
3	51·8076	54·0	2·2
4	59·7566	58·8	1·0
5	52·2220	53·5	1·3
6	55·7863	55·6	0·2
7	67·0140	64·1	2·9
8	57·8129	57·7	0·1
9	48·1200	48·6	0·5
10	62·4169	62·0	0·4

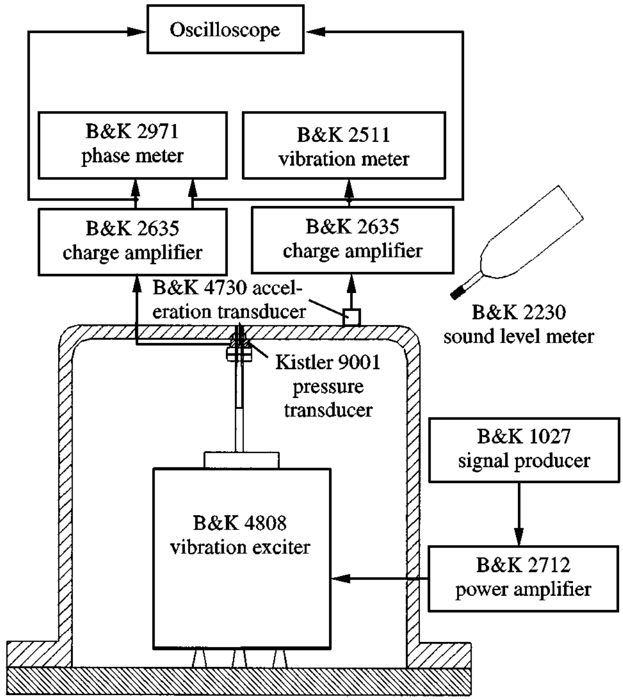

Figure 8. The diagram of the measuring equipment.

surface nodes can be calculated by means of the structural analysis. The computing errors introduced by the non-simultaneous measurement can then be cancelled.

5. CONCLUSIONS

The gist of the new numerical method is to replace the coefficient matrices [**A**] and [**B**] in the system equation with the particular solution matrices which are formed of the particular solutions generated by fabricated sources. In the boundary point method, it is unnecessary to consider the interpolating operation, and the singular integral which is indispensable for the BEM also does not exist. By avoiding direct computation for the coefficient matrices, the boundary point method can improve the calculation speed substantially while maintaining the calculation precision. Another advantage of the method is that it can be used for calculating the acoustic parameters (such as the sound pressure, etc.) at any desired point in the sound field without calculation of the acoustic parameters on the surface. Finally, the boundary point method can overcome the non-uniqueness problem at the characteristic wavenumbers effectively.

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