Journal of Sound and Vibration (1999) **228**(4), 837–844
Article No. jsvi.1999.2430, available online at http://www.idealibrary.com on IDEAL®

SOUND ATTENUATION CHARACTERISTICS OF RIGHT-ANGLE PIPE BENDS

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(*Received* 2 *October* 1996, *and in final form* 9 *June* 1999)

The numerical solution of the interior surface Helmholtz integral equation for toroidal pipes is presented. Acoustic surface pressures of various pipes in the shape of a quarter torus were calculated. Sound attenuation spectra of pipes possessing different bend sharpnesses are presented. A lower limit for the bend sharpness is determined for the attainment of a considerable level of sound attenuation.

1. INTRODUCTION

It is known that a one-dimensional sound wave generated at the entrance of a hard-walled straight pipe propagates along the pipe without attenuation in amplitude. The relation between the wavelength of the sound and the diameter of the pipe solely determines the propagation characteristics, these being either "one-dimensional" or "three-dimensional". However, bending a straight pipe or connecting two straight pipes with an elbow bend creates sound attenuation in some cases. It is the purpose of this analysis to investigate the dependence of the sound attenuation on the bend sharpness and the frequency. Pipes possessing right-angle circular bends are considered due to their extensive applications in industry.

Theoretical solutions of two- and three-dimensional acoustic wave propagation in hard-walled [1, 2] and acoustically lined [3, 4] ducts with rectangular cross-section have been presented. In these studies, cylindrical coordinate system has been used to describe the curved duct. The theoretical solution for a circular section curved duct has not been achieved yet due to the mathematical difficulties encountered in the solution of the wave equation. The wave equation is not separable in the toroidal coordinate system [5] which is required to describe the circular section curved bend. Cummings [1] has presented some measured and predicted results for the sound field in circular section curved ducts for which even numerical solutions are not available.

The present analysis is based on the numerical implementation of the interior surface Helmholtz integral equation by using the isoparametric boundary element technique. The pipes of this analysis are specifically in the shape of a quarter torus.

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Toroidal and polar coordinates are used to describe the walls and end sections respectively. Acoustic waves are generated by the piston-like vibration of the entrance section. The condition of no reflection at the exit is satisfied by an anechoic termination. There is no fluid flow in the rigid-walled pipes. Sound attenuation spectra for pipes possessing different bend sharpnesses are presented. The lower limit for the bend sharpness is determined for which a recordable sound attenuation level is attained.

2. FORMULATION OF THE PROBLEM

If the domain of interest is the interior acoustic field of a body with a boundary S and a unit outward normal $\mathbf{n}$ the Helmholtz integral equation may be written as

$$-C(\mathbf{x})p(\mathbf{x}) = \int_S \{p(\mathbf{y})\,[\partial G(R, k)/\partial n(\mathbf{y})] + \mathrm{i}\, z_0\, k\, u_n\,(\mathbf{y})\, G(R, k)\}\, \mathrm{d}s(\mathbf{y}), \qquad (1)$$

where p is the sound pressure amplitude, u_n is the normal velocity amplitude of the fluid particle at the boundary, $\mathbf{y}$ is any point on S, $\mathbf{x}$ is any point in space, $R = |\mathbf{x} - \mathbf{y}|$, $\mathrm{i} = \sqrt{(-1)}$. k denotes the wavenumber $2\pi f/c$ where f is the frequency, c is the speed of sound and $\exp(\mathrm{i}2\pi f t)$ time dependence is assumed. ds denotes a differential boundary element, $G(R, k)$ is the free-space Green function $G = \exp(-\mathrm{i}kR)/R$ for the Helmholtz operator in three dimensions. z_0 is the characteristic impedance of the medium $z_0 = \rho_0 c$ where ρ_0 is the density of the medium at rest. For $\mathbf{x}$ in the interior of the body, $C(\mathbf{x})$ is equal to 4π; for $\mathbf{x}$ in the exterior of the body, $C(\mathbf{x}) = 0$ and for $\mathbf{x}$ on S, $C(\mathbf{x})$ is given by [6]

$$C(\mathbf{x}) = -\int_S [\partial(1/R)/\partial n(\mathbf{y})]\, \mathrm{d}s(\mathbf{y}). \qquad (2)$$

For $\mathbf{x}$ on S, equation (1) is called the interior surface Helmholtz integral equation and application of the boundary conditions is required before solution. In general, if the normal velocity amplitude of the boundary is denoted by u, on acoustically hard walls

$$u_n(\mathbf{y}) = u(\mathbf{y}) \qquad (3)$$

and on sound absorbing boundaries

$$u_n(\mathbf{y}) = u(\mathbf{y}) + \beta(\mathbf{y})\, p(\mathbf{y}), \qquad (4)$$

where β is the acoustic admittance of the absorbent material on the boundary [7].

Helmholtz integral equation (1) relates the sound pressure amplitude p and the normal velocity amplitude u subsequent to the application of the necessary boundary conditions. The numerical implementation of the interior surface Helmholtz integral equation by the boundary element method gives a set of complex algebraic equations which may be written as

$$[\mathbf{E} - \mathbf{K}\,(f)]\,\mathbf{p} = \mathbf{H}(f)\,\mathbf{u}, \qquad (5)$$

where **p** and **u** are the nodal sound pressure and prescribed nodal normal velocity vectors, respectively. Matrices **E** (a diagonal matrix), $\mathbf{K}(f)$ and $\mathbf{H}(f)$ are of size $N \times N$, where N is the total number of nodes. Nodal acoustic admittance values of the sound absorbing boundaries are included by matrix $\mathbf{K}(f)$.

The numerical solution is performed by a computer code which has the capability to generate equation (5) for arbitrarily shaped three-dimensional bodies possessing any type of boundary conditions. Quadrilateral quadratic isoparametric boundary elements are used in the discretization of the surfaces. The precision of the code for interior acoustic fields has been validated by using the analytical results for a spherical cavity [8].

3. SOUND ATTENUATION IN THE PIPES

In this analysis, all the curved pipes are in the shape of a quarter torus. The distinguishing characteristic of the pipes is the bend sharpness which is denoted by r/R. r is the pipe radius and R is the median arc radius, as shown in Figure 1. In the numerical solution, the pipes are modelled by using a total of 216 elements and 650 nodes. Each of the cross-sectional surfaces at the entrance and exit possesses 44 elements and 125 nodes. The boundary element discretizations of the walls and of a cross-sectional surface are shown in Figures 2 and 3 respectively. The entrance surface vibrates with a constant normal velocity amplitude while the exit surface behaves as an anechoic surface possessing an acoustic admittance of $\beta = 1/z_0$. These surfaces, due to their importance in the formation of a closed volume and in the application of the boundary conditions requiring precision, are represented by smaller elements than those for rigid walls. The application of the walls' boundary conditions to the nodes on the edges of the entrance and exit surfaces offers better

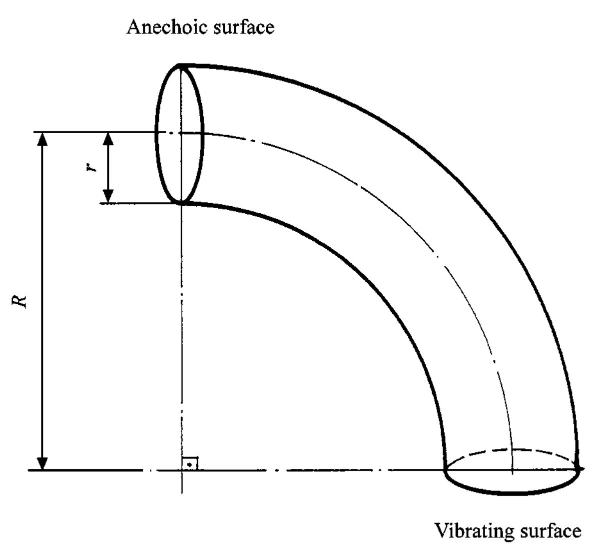

Figure 1. Pipe in the shape of a quarter torus.

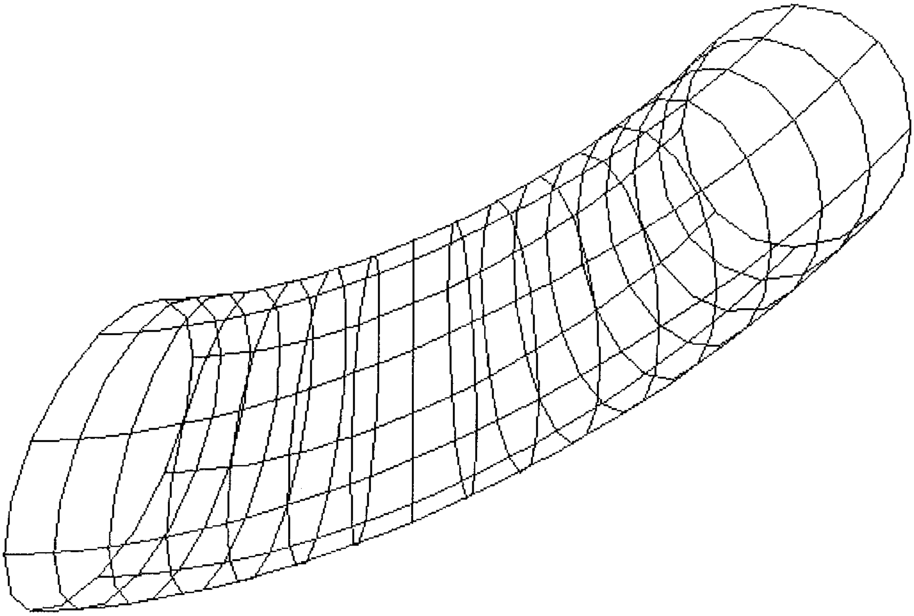

Figure 2. Boundary element discretization of the walls.

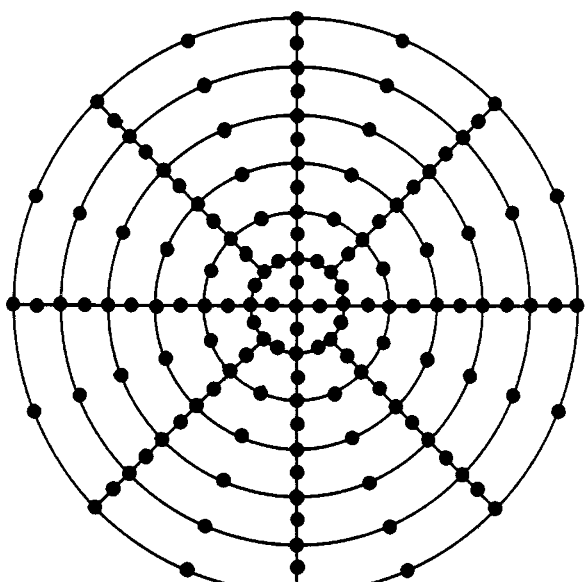

Figure 3. Boundary element discretization of a cross-sectional surface.

interpolation characteristics and reasonable results. Numerical integration is carried out by using 16-point Gaussian quadrature for every element.

Sound reduction characteristics of the pipes are calculated by using the average values of the nodal sound pressures at the entrance and exit surfaces. Since areas of these surfaces at a particular toroidal pipe are the same, the sound transmission loss of this pipe may be expressed by only average pressures. This change in expression is reflected to the label of the behavior of the pipe and the sound attenuation level is used, as was done by Ko [3, 4], to characterize sound reduction:

$$\text{sound attenuation level} = 20\ \log\left(\frac{p_{\text{inlet}}}{p_{\text{exit}}}\right)\ \mathrm{d}B. \tag{6}$$

Figure 4 presents the variation of the sound attenuation level for the pipe possessing a bend sharpness r/R of 0·8, with the non-dimensional wavenumber kr. The negative attenuation levels for the low values of kr are erroneous results associated with the numerical solution and may be accepted as zero. The maximum attenuation level of 7·2 dB is attained around $kr = 2{\cdot}3$. In order to make a check, a straight cylindrical pipe with the same radius, boundary conditions and boundary elements was solved. The attenuation for this pipe around $kr = 2{\cdot}3$ was calculated as $-0{\cdot}5$ dB, which may be accepted as zero. This result shows that the 7·2 dB attenuation is the characteristic of the curved pipe and does not originate from numerical inaccuracies.

Figures 5–9 show the sound attenuation spectra for the pipes possessing bend sharpness r/R of 0·7, 0·6, 0·5, 0·4 and 0·45 respectively. Comparison of Figures 4–9 reveals three characteristics of the curved pipes as a function of bend sharpness. As the bend sharpness decreases (1) the maximum value of the sound attenuation level decreases, (2) the non-dimensional wavenumber kr, corresponding to the maximum attenuation decreases, and (3) the range of attenuation becomes narrower while the value of kr corresponding to the beginning of attenuation remains almost constant.

These features show that if the change in r/R is established by a change in radius r, with the mean arc radius R held constant, narrowing the curved pipe results in

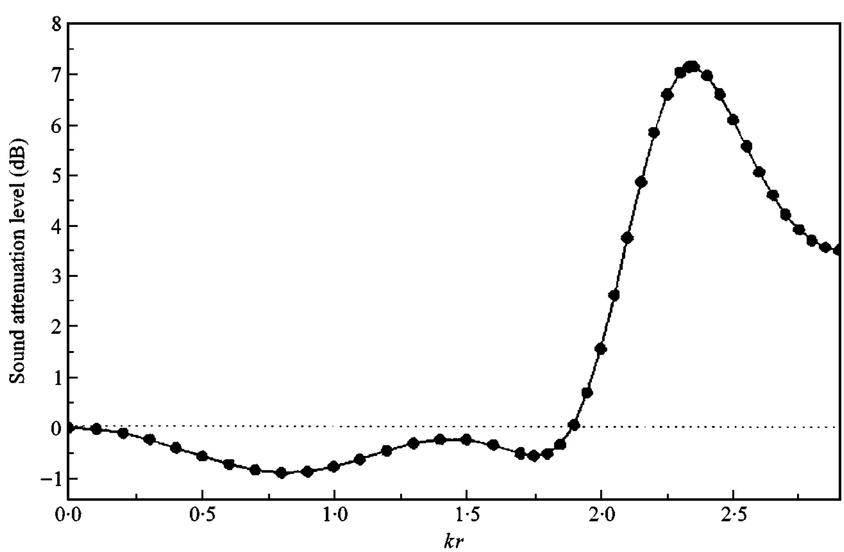

Figure 4. Sound attenuation spectrum for the pipe with a bend sharpness r/R of 0·8.

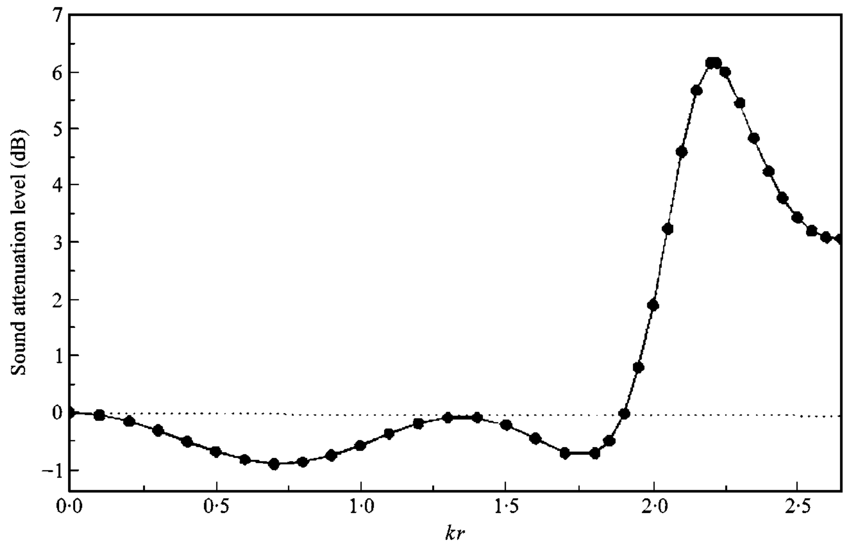

Figure 5. Sound attenuation spectrum for the pipe with a bend sharpness r/R of 0·7.

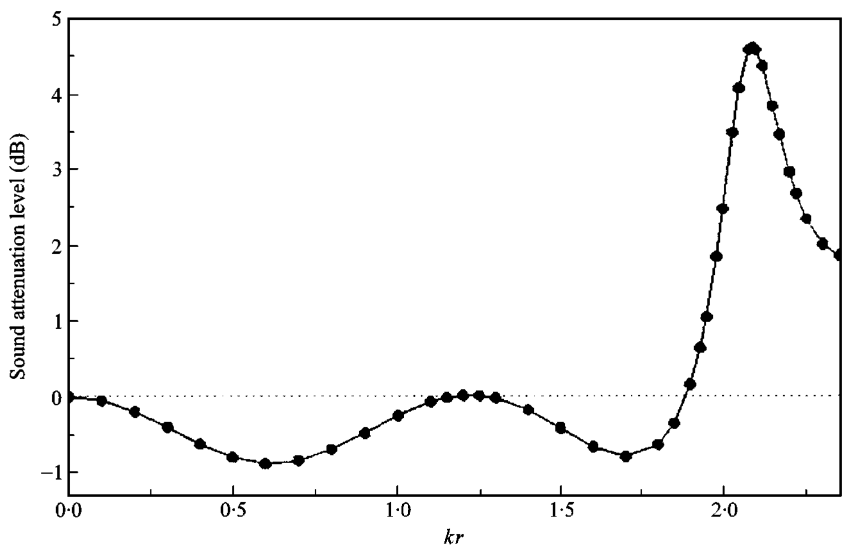

Figure 6. Sound attenuation spectrum for the pipe with a bend sharpness r/R of 0·6.

a decrease in the peak level of the sound attenuation which is attained at comparatively higher frequencies. As the pipe is narrowed, the attenuation begins at higher frequencies and extends over a narrower frequency range.

The spectrum in Figure 9 for the pipe with $r/R = 0{\cdot}4$ shows minor values for sound attenuation. Since the results are obtained by a numerical solution, peak levels which are lower than 1 dB may not be reliable; in fact such a small attenuation is also unimportant in practice. On the other hand, Figure 8 shows

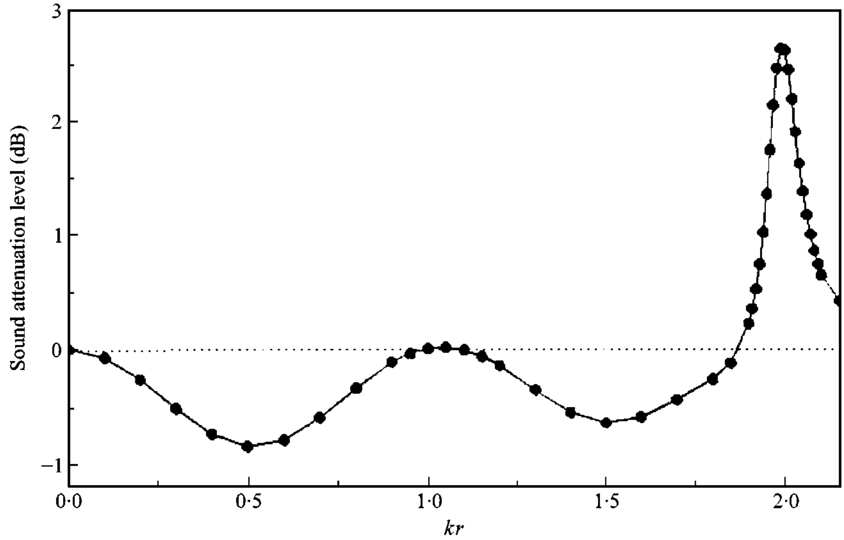

Figure 7. Sound attenuation spectrum for the pipe with a bend sharpness r/R of 0·5.

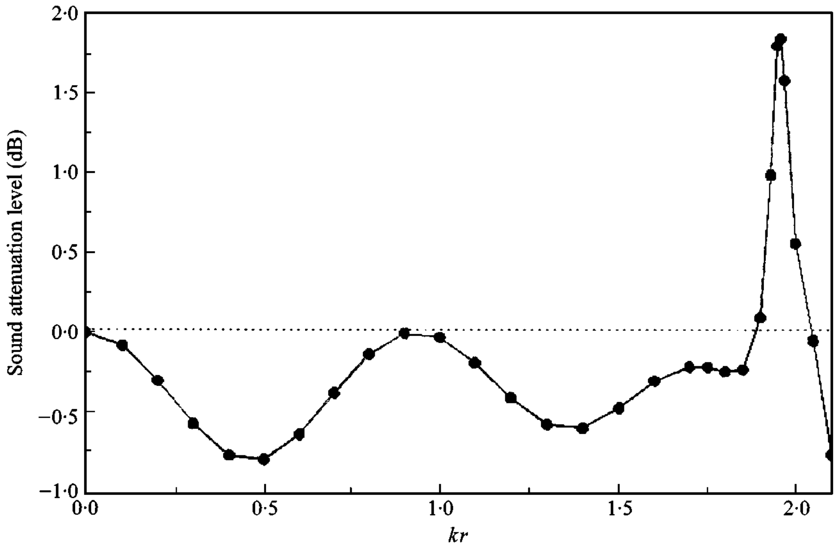

Figure 8. Sound attenuation spectrum for the pipe with a bend sharpness r/R of 0·45.

attenuation values higher than 1 dB for the pipe of $r/R = 0{\cdot}45$; however, the range of attenuation is rather limited. Therefore, it may be stated that bending a pipe at a right angle with a sharpness r/R less than 0·5 would not provide a considerable sound attenuation.

4. CONCLUSION

A boundary element method solution for the acoustic pressures of curved pipes described in toroidal coordinates was presented. Interior surface pressure

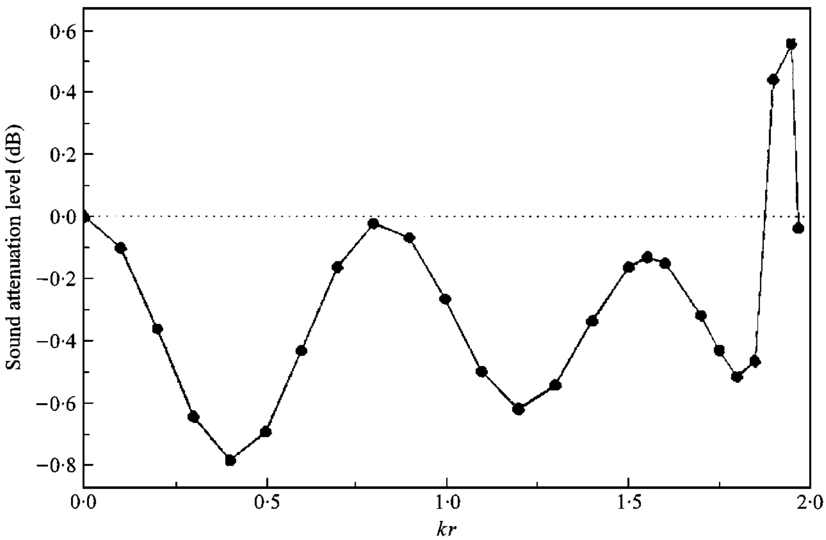

Figure 9. Sound attenuation spectrum for the pipe with a bend sharpness r/R of 0·4.

amplitudes of hard-walled pipes bent at a right angle were calculated under the conditions of a uniformly vibrating cross-sectional surface at the entrance and an anechoic surface at the exit. Sound attenuation characteristics of several pipes possessing different bend sharpnesses were examined. The analysis showed that sharper bends provide more sound attenuation corresponding to higher non-dimensional wavenumbers in a wider range. It would be a useless effort to bend a straight pipe in the shape of a quarter torus with a sharpness r/R less than 0·5, in order to obtain a recordable sound attenuation.

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