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LETTERS TO THE EDITOR

NATURAL FREQUENCIES OF ANNULAR PLATES HAVING CONTACT WITH FLUID

C.-C. LIANG

Department of Naval Architecture and Marine Engineering, Chung-Cheng Institute of Technology, Ta-Hsi, Taiwan 335, Republic of China

Y.-S. TAI

Institute of System Engineering, Chung-Cheng Institute of Technology, Ta-Hsi, Taiwan 335, Republic of China

AND

P.-L. LI

Navy Headquarters, Taipei, Taiwan 335, Republic of China

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1. INTRODUCTION

Annular and perforated plates constitute vital components in mechanical engineering, such as in nuclear reactors. Natural frequencies of components have attracted extensive interest when exploring the structural responses to various excitations; this preliminary involves high risk plants. Among perforated plates, annular plates have a particular importance, due to their axial symmetry. Using annular plates in design and engineering of a mechanical system is a conventional approach having become quite pervasive in certain industries. Specific applications of this type of mechanical component include turbines, saw blades, computer magnetic recording disks, gears, phonograph records and percussion musical instruments [1–3]. The prominent role of these components in engineering necessitates a comprehensive understanding of their mechanical behavior. In a pioneering work, Southwell [4] investigated natural frequency for annular plates vibration in a vacuum. These plates are important elements of machines and structures, with many investigators having explored their vibration problems in detail [5–19]. Narayana Raju [5], and Vogel and Skinner [6] analyzed this problem, demonstrating various combinations of boundary conditions to facilitate mechanical design purposes.

A critical aspect of such mechanical behavior is the vibration response of annular and perforated plates having contact with a fluid on one or both sides. As well known, a heavy fluid strongly influences the natural frequencies of a thin-walled structure. Therefore, annular plates having contact with a fluid is a topic of practical interest, particularly regarding the signaling problem of submarines and

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the vibration problems of local ship structures, ocean structures, dams and nuclear reactors. Limited investigators have examined annular and perforated plates having contact with a fluid on one or both sides. De Santo [20] experimentally investigated perforated plates used in nuclear reactors. Later, Kubota and Suzuki [21] theoretically and experimentally studied annular plates vibrating in an annular cylindrical cavity filled with a fluid. Amabili and Frosali [22] also theoretically investigated the free vibrations of annular plates placed on a free surface. Amabili [23], in using experimental coefficients, computed natural frequencies of annular plates placed on a free fluid surface or completely immersed in water. Amabili *et al.* [24] investigated the natural frequencies of annular plates on an aperture of an infinite rigid wall and in contact with a fluid on one side. Their work calculated only three boundary conditions: a clamped–clamped edge, a free–free edge and a clamped (outer)–free (inner) edge.

Based on Kwak's approach [24], this study thoroughly examines the annular plates placed on the fluid domain that is an annular aperture of an infinite rigid wall. The fluid is assumed to be an incompressible, inviscid and irrotational velocity potential. Nine boundary conditions, having non-dimensionally added virtual mass incremental (NAVMI) factors for annular plates having contact with fluid on one side, are calculated entirely by the Mathematica [25] software. Also, Ross's finite element method [26] is employed to calculate natural frequencies of circular and annular plates in air and having contact with water on one side or both sides. According to the calculation results, differences arise between natural frequencies in air and having contact with water on one or both sides. Both the proposed method and Ross's finite element method are compared with the experimental data, thereby yielding the mode's accurate applications. Moreover, the natural frequencies of a circular plate in fluid are also studied [27, 28]. Calculation results are appropriate for engineering design applications.

2. THEORETICAL BACKGROUND

Figure 1 depicts an annular plate having contact with fluid on one side, where a, b and h represent the outside radius, the inside radius and thickness of the annular plate, respectively, and L denotes a fluid domain. Not only is the proposed method based on the thin plate theory and Kwak's theory [24], but it also uses the Hankel transform to solve the fluid–plate coupled system; mixed boundary conditions are expressed by Titchmarch triple integral equations, [29]. The Rayleigh's quotient accounts for the squares of the natural frequencies of the annular plate in air are proportional to V_P/T_P^* [30], where V_P is the maximum potential energy and T_P^* is reference kinetic energy for annular plate [31]. On the other hand, the squares of the natural frequencies in fluids are proportional to the ratio between the maximum potential energy of the plate and the sum of the reference kinetic energy for both the plates, T_p^*, and the fluid, T_F^*. The non-dimensionalized added virtual mass incremental (NAVMI) factors are given by the ratio between the reference kinetic energy of the fluid and the kinetic energy of the plate. To easily comprehend the proposed method, i.e., a development of Kwak's theoretical approach, Tables 1 and 2 present the simplified theoretical procedures.

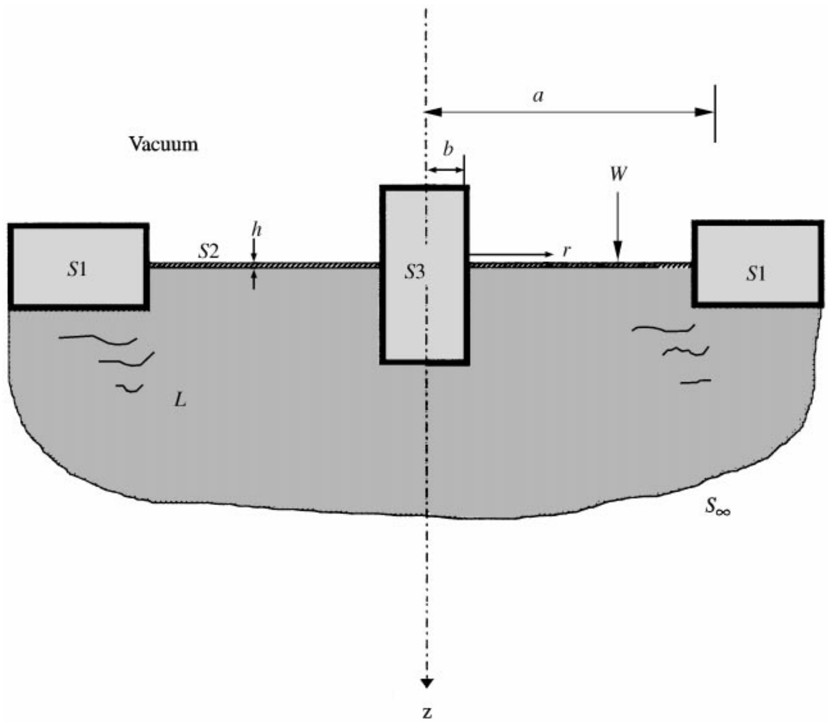

Figure 1. Geometry of the annular plate having contact with water: a outer radius; b inner radius; h thickness of plate; L unbounded fluid domain; r radial direction; w displacement; $S1$ and $S3$ infinite rigid wall; $S2$ annular plate region; S_∞ infinite surface.

The frequency parameters (in air) and NAVMI factors (in fluid) are two prominent factors in the theoretical procedure of the natural frequencies plates in fluid. The frequency parameters (in air) are derived on the basis of the thin plate theory. In 1965, Vogel and Skinner [6] calculated the parameters using machine computations. The Bessel functions were replaced by their approximate polynomial equivalents [32]. In this study, we use Mathematica software of Numerical Root Finding and Numerical Integration [25] to obtain numerical computations of natural frequency parameters (in air) and the NAVMI factors (in fluid) based on the thin plate theory and Kwak's theory which are the first and second kinds of Bessel and modified Bessel functions.

3. VERIFICATION AND NUMERICAL EXAMPLES

In the air or water domain, five different sizes plates are adopted to verify the proposed method and Ross's finite element method. Plate #1 is circular and the material is carbon steel; plate #2 is annular but its material is an aluminum alloy. The other plates' (#3, #4, #5) material is low carbon steel. For circular and annular plates, Table 3 lists the detailed properties. The boundary conditions of clamped and simply supported edge are applied for circular plates, whereas a free

TABLE 1

The theoretical procedure for annular plate vibration in a vacuum [7, 8]

1. Assumption

(1) Thin plate theory
(2) Absence of loss
(3) The continuous system of the annular plate
(4) Linearly elastic homogeneous and isotropic material
(5) Effects of shear deformation and rotary inertia are neglected
(6) The maximum displacement is small
(7) Deformation due to gravity can be neglected

2. Procedure

Step	Descriptions	Formulations
1	The governing equation	$D\nabla^4 w + \rho_p h \dfrac{\partial^2 w}{\partial t^2} = 0$
2	The Laplacian operator in the polar co-ordinates r, θ:	$\nabla^2 = \dfrac{\partial^2}{\partial r^2} + \left(\dfrac{1}{r}\right)\dfrac{\partial}{\partial r} + \left(\dfrac{1}{r^2}\right)\dfrac{\partial^2}{\partial \theta^2}$
3	The exact solutions	$w = w(r,\theta,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} W_{mn}(r)\cos(m\theta)\mathrm{g}_{mn}(\mathrm{t})$ where $W_{mn}(r) = A_{mn}J_m(\lambda_{mn}r/a) + B_{mn}Y_m(\lambda_{mn}r/a) + C_{mn}I_m(\lambda_{mn}r/a) + D_{mn}K_{mn}(\lambda_{mn}r/a)$
4	Frequency	$f_{mn} = \left(\dfrac{\lambda_{mn}^2}{2\pi a^2}\right)\sqrt{\dfrac{D}{\rho_P h}}$

outside edge and either a clamped or free inner edge are applied for annular plates. For Ross's finite element method [26], Figure 2 displays the plate element and is described by internal and external nodal circles; it has two degrees of freedom per node. The fluid element is also of annular form in Figure 3: it has two internal and two external nodal circles in line with the nodal circles of the plate. Figure 4 displays the typical mesh of the circular plate in fluid, and Figure 5 displays the mesh of the annular plate in fluid. Both the proposed method and Ross's FEM employ the added virtual mass approach to solve the frequencies of the plates vibration coupled with fluid. The verification of the proposed method and Ross's FEM in air or in water correspond more closely to each other than the experimental data for the fundamental mode and frequency. However, the computations are not as accurate for higher modes.

TABLE 2

The theoretical procedure for annular plate vibration having contact with fluid [24]

1. Assumption

(1) Wavelength in a fluid is smaller than diameter of plate, i.e., fluid motion is small
(2) Dynamic fluid motion is incompressible, inviscid and irrotational
(3) The mode shapes in fluids and in a vacuum are to be the same
(4) Thin plate theory
(5) Linearly elastic homogenous and isotropic material
(6) Free surface boundary condition, $\omega = 0$, $\partial\phi/\partial z = 0$

2. Procedure

Step	Descriptions	Fomulations
1.	Velocity potential function	$U(r,\theta,z,t) = \phi(r,z)\cos(m\theta)\,\dot{g}_{mn}(t)$ where $g_{mn}(t) = e^{i\omega_{mn}t}$
2.	$\phi(r,z)$ satisfies Laplace equation	$\dfrac{\partial^2\phi}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial\phi}{\partial r} + \dfrac{\partial^2\phi}{\partial z^2} - \dfrac{m^2}{r^2}\phi = 0$ in L
3.	$\phi(r,z)$ boundary conditions	$\left.\dfrac{\partial\phi(r,z)}{\partial z}\right\|_{z=0} = 0$ on S_1 and S_3
		$\left.\dfrac{\partial\phi(r,z)}{\partial z}\right\|_{z=0} = W_{mn}(r)$ on S_2
		$\phi(r,z), \dfrac{\partial\phi(r,z)}{\partial r}, \dfrac{\partial\phi(r,z)}{\partial z} \to 0$ for $r, z \to \infty$ on S_∞
4.	Hankel Transform	$\bar{\phi}(\xi,z) = \displaystyle\int_0^\infty r\phi(r,z)J_m(\xi r)\,\mathrm{d}r$
	let $\dfrac{\partial^2\phi}{\partial r^2} \to (1), \dfrac{1}{r}\dfrac{\partial\phi}{\partial r} \to (2)$,	$\displaystyle\int_0^\infty r((1)+(2)+(4))J_m(\xi r)\,\mathrm{d}r = -\xi^2\bar{\phi}(\xi,z)$
	$\dfrac{\partial^2\phi}{\partial z^2} \to (3), -\dfrac{m^2}{r^2}\phi \to (4)$,	$\displaystyle\int_0^\infty r(3)J_m(\xi r)\,\mathrm{d}r = \dfrac{\mathrm{d}^2}{\mathrm{d}z^2}\bar{\phi}(\xi,z))$
	integrating	
5.	Step 2 is simplified the o. d. e.	$\dfrac{\mathrm{d}^2}{\mathrm{d}z^2}\bar{\phi} - \xi^2\bar{\phi} = 0$
6.	The solution of the o. d. e	$\bar{\phi}(\xi,z) = B(a\xi)\mathrm{e}^{-\xi z}, z \geqslant 0$
7.	Defined the inverse Hankel Transform	$\phi(r,z) = \displaystyle\int_0^\infty \xi\bar{\phi}(\xi,z)J_m(\xi r)\,\mathrm{d}\xi$
8.	Instead of step 6	$\phi(r,z) = \displaystyle\int_0^\infty \xi B(a\xi)\mathrm{e}^{-\xi z}J_m(\xi r)\,\mathrm{d}\xi$

TABLE 2 Continued

9.	Mixed boundary conditions (Titchmarch triple integral equations)	$\int_0^\infty \xi^2 B(a\xi) J_m(\xi r)\,\mathrm{d}\xi = 0,\ 0 \leqslant r \leqslant b$ $\int_0^\infty \xi^2 B(a\xi) J_m(\xi r)\,\mathrm{d}\xi = -W_{mn}(r),$ $b \leqslant r \leqslant \mathrm{a}$ $\int_0^\infty \xi^2 B(a\xi) J_m(\xi r)\,\mathrm{d}\xi = 0,\ a \leqslant r$
10.	Non-dimensionalized variables	$\rho = r/a,\ \eta = a\xi,\ A(\eta) = \eta B(\eta),\ c = b/a$
11.	Non-dimensionalized mixed boundary conditions triple integral equations	$\int_0^\infty \eta A(\eta) J_m(\eta\rho)\,\mathrm{d}\eta = 0,\ 0 \leqslant \rho \leqslant c$ $\int_0^\infty \eta A(\eta) J_m(\eta\rho)\,\mathrm{d}\eta = -a^3 W_{mn}(a\rho),$ $c \leqslant \rho \leqslant 1$ $\int_0^\infty \eta A(\eta) J_m(\eta\rho)\,\mathrm{d}\eta = 0,\ \rho > 1$
12.	Hankel inversion theorem	$\int_0^\infty \eta A(\eta) J_m(\eta\rho)\,\mathrm{d}\eta$ $= \begin{cases} -a^3 W_{mn}(r),\ \mathrm{c} < r < 1 \\ 0, 0 < r < c,\ 1 < r < \infty \end{cases}$ $A(\eta) = \int_c^1 -a^3 W_{mn}(a\rho) J_m(\eta\rho)\rho\,\mathrm{d}\rho,$ $0 \leqslant c \leqslant 1 \leqslant \infty.$
13.	The solution $A(\eta)$	$A(\eta) = -a^3 \int_c^1 \rho W_{mn}(a\rho)\, J_m(\eta\rho)\,\mathrm{d}\rho$ $A(\eta) = -a^3 H_{mn}(\eta)$

where

$$H_{mn}(\eta) = A_{mn} H_{Amn}(\eta) + B_{mn} H_{Bmn}(\eta) + C_{mn} H_{Cmn}(\eta) + D_{mn} H_{Dmn}(\eta)$$

$$H_{Amn}(\eta) = \frac{1}{\lambda^2 - \eta^2}[\lambda J_m(\eta) J_{m+1}(\lambda) - \eta J_{m+1}(\eta) J_m(\lambda)] - \frac{c}{\lambda^2 - \eta^2}[\lambda J_m(\eta c) J_{m+1}(\lambda c) - \eta J_{m+1}(\eta c) J_m(\lambda c)]$$

$$H_{Bmn}(\eta) = \frac{1}{\lambda^2 - \eta^2}[\lambda J_m(\eta) Y_{m+1}(\lambda) - \eta J_{m+1}(\eta) Y_m(\lambda)] - \frac{c}{\lambda^2 - \eta^2}[\lambda J_m(\eta c) Y_{m+1}(\lambda c) - \eta J_{m+1}(\eta c) Y_m(\lambda c)]$$

TABLE 2 Continued

$$H_{Cmn}(\eta) = \frac{1}{\lambda^2+\eta^2}[\lambda J_m(\eta) I_{m+1}(\lambda) + \eta J_{m+1}(\eta) I_m(\lambda)] - \frac{c}{\lambda^2+\eta^2}[\lambda J_m(\eta c) I_{m+1}(\lambda c) + \eta J_{m+1}(\eta c) I_m(\lambda c)]$$

$$H_{Dmn}(\eta) = \frac{1}{\lambda^2+\eta^2}[-\lambda J_m(\eta) K_{m+1}(\lambda) + \eta J_{m+1}(\eta) K_m(\lambda)] - \frac{c}{\lambda^2+\eta^2}[-\lambda J_m(\eta c) K_{m+1}(\lambda c) + \eta J_{m+1}(\eta c) K_m(\lambda c)]$$

14.	Velocity potential function ϕ at the fluid–plate interface	$\phi(\rho, 0) = -a\int_0^\infty H_{mn}(\eta) J_m(\eta\rho)\,\mathrm{d}\eta$
15.	Reference kinetic energy of fluid	$T_F^* = \frac{1}{2}\rho_F \int_0^{2\pi}\int_0^\infty \left(\frac{\partial\phi(r,z)}{\partial z}\right)_{z=0} \phi(r,0) \times \cos^2(m\theta)\, r\,\mathrm{d}r\,\mathrm{d}\theta$ $T_F^* = -\frac{1}{2}\rho_F a^2 \psi\theta \int_c^1 \phi(\rho,0) W_{mn}(a\rho)\rho\,\mathrm{d}\rho,$ where $\left.\frac{\partial\phi(r,z)}{\partial z}\right\|_{z=0} = W_{mn}(r)$ on S_2 $\left.\frac{\partial\phi(r,z)}{\partial z}\right\|_{z=0} = 0$ on S_1 and S_3 $\begin{cases}\psi_\theta = 2\pi, & m=0\\ \psi_\theta = \pi, & m>0\end{cases}$
	Instead of step 14	$T_F^* = -\frac{1}{2}\rho_F a^2 \psi_\theta \int_c^1 (-a\int_0^\infty H_{mn}(\eta) \times J_m(\eta\rho)\,\mathrm{d}\eta) W_{mn}(a\rho)\rho\,\mathrm{d}\rho$
	Reference kinetic energy of fluid	$= \frac{1}{2}\rho_F a^3 \psi_\theta \int_C^1 \int_0^\infty H_{mn}(\eta) J_m(\eta\rho) W_{mn}(a\rho) \times \rho\,\mathrm{d}\rho\,\mathrm{d}\eta$ $= \frac{1}{2}\rho_F a^3 \psi_\theta \int_0^\infty H_{mn}^2(\eta)\,\mathrm{d}\eta$ where $H_{mn}(\eta) = \int_c^1 \rho W_{mn}(a\rho) J_m(\eta\rho)\,\mathrm{d}\rho$
16.	Reference kinetic energy of plate	$T_P^* = \frac{1}{2}\rho_P h \int_h^a \int_0^{2\pi} W_{mn}^2(r)\cos^2(m\theta) r\,\mathrm{d}r\,\mathrm{d}\theta$ $= \frac{1}{2}\rho_P a^2 h \psi_\theta \int_c^1 W_{mn}^2(a\rho)\rho\,\mathrm{d}\rho$ $= \frac{1}{2}\rho_P a^2 h\psi_\theta$ where $\int_c^1 \mathrm{W}_{mn}^2(\mathrm{a}\rho)\rho\,\mathrm{d}\rho = 1$ (*normalized*)

TABLE 2 Continued

17	Application of Rayleigh's quotient	$f_{mn}^2 \propto \left(\frac{V_p}{T_p^*}\right)_{vacuum}$ $f_F^2 \propto \left(\frac{V_p}{T_P^* + T_F^*}\right)_{fluid}$.
18.	Defined the added virtual mass incremental (AVMI) β_{mn}	$= \frac{T_F^*}{T_P^*}$ $= \Gamma_{mn}(\rho_F/\rho_P)(a/h)$ where $\Gamma_{mn} = \int_0^\infty H_{mn}^2(\eta)\,\mathrm{d}\eta$ Γ_{mn} = non-dimensionalized added virtual mass incremental (NAVMI) factor having contact with fluid on one side
19.	Natural frequencies having contact with fluid on one side f_{Fmn}	$= \frac{f_{mn}}{\sqrt{1+\beta_{mn}}} = \frac{f_{mn}}{\sqrt{1+\Gamma_{mn}\frac{\rho_F}{\rho_P}\frac{a}{h}}}$
20.	Natural frequencies having contact with fluid on both sides f_{Fmn}^*	$= \frac{f_{mn}}{\sqrt{1+2\beta_{mn}}} = \frac{f_{mn}}{\sqrt{1+(2\Gamma_{mn})\frac{\rho_F}{\rho_P}\frac{a}{h}}}$

3.1. CIRCULAR PLATE #1 (CLAMPED, SIMPLY SUPPORTED)

Natural frequencies of plate #1 in air and in water are calculated by the proposed method and Ross's FEM. Table 4 indicates that by using the proposed method to obtain the natural frequency, the fundamental mode of a clamped edge circular plate is 66 Hz in water on one side and 165 Hz in air. The same case by the Ross's FEM it is 60 Hz in water and 165 Hz in air. Two different kinds of behavior are observed for the mode; the first mode in air correlates well with the theoretical data, while the other has only slightly higher frequencies. On the other hand, the natural frequencies in water approximately have a value of 40% in air for the lowest mode. The phenomena for a simply supported circular plate are the same. Table 5 reveals that by using the proposed method to obtain the natural frequency, the fundamental mode is 31 Hz in water on one side and 80 Hz in air. For the same case by Ross's FEM, it is 27 Hz in water and 80 Hz in air.

3.2. ANNULAR PLATE #2 (OUTSIDE/INSIDE, FREE/CLAMPED)

The proposed method and FEM data obtained for the annular plate having contact with water on both sides are compared with the experimental data.

TABLE 3

The plates' geometrical and material properties

Plate no.	Plate 1	Plate 2	Plates 3–5
Geometry	Circular	Annular	Annular
Material	Carbon steel	Aluminum alloy	Low carbon steel
Inside radius (mm)	—	15	15,30,50
Outside radius (mm)	175	200	100
Thickness (mm)	2	3·251	1·5
Mass density (kg/m^3)	7870	2560	7800
Young's modulus (Gpa)	207	70	206
The Poisson ratio	0·3	0·3	0·3
Water density (kg/m^3)	1000	1000	1000

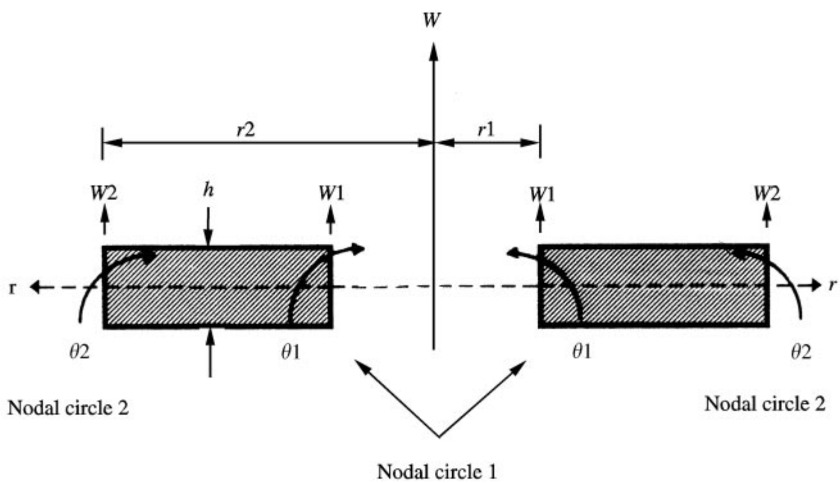

Figure 2. Ross's annular plate element.

Table 6 summarizes the results for plate #2; the same modes are listed in Tables 4 and 5. According to these results, experimental natural frequencies in water on both sides are slightly higher than the proposed method's data. The natural frequency of the fundamental mode obtained by the proposed method is 15 Hz in water on both sides and 83 Hz in air. This subsequently obtained by Ross's FEM is 14 Hz in water, 83 Hz in air and by experimental data is 17 Hz in water and 78 Hz in air. For the lowest mode, the natural frequencies in water have roughly a value of 18% in air.

3.3. ANNULAR PLATE #3, #4 AND #5 (BOTH FREE EDGES)

The following case is presented to verify the current method and experimental data [25] on three annular plates in air and completely immersed in water for both

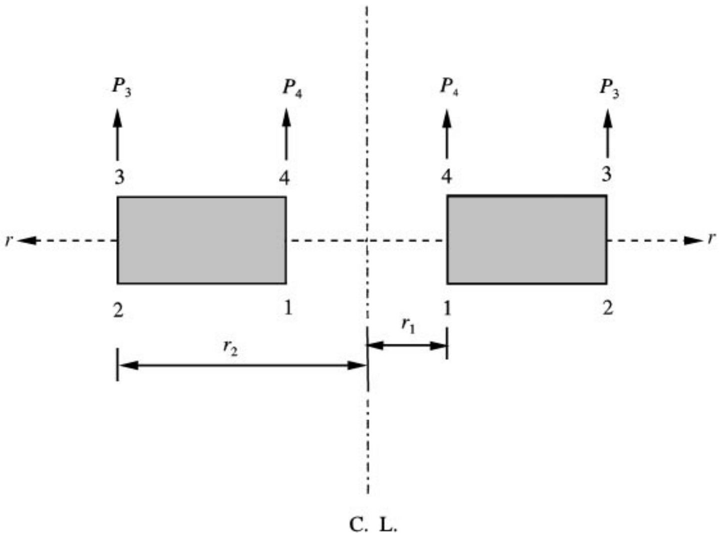

Figure 3. Ross's annular fluid element.

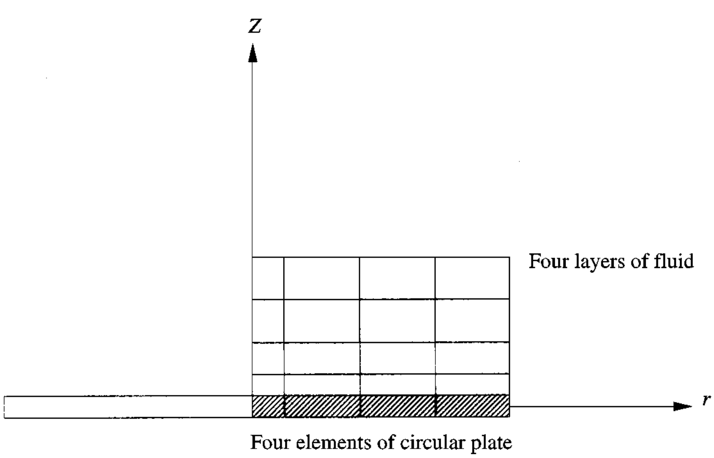

Figure 4. Mesh for circular plate in fluid.

free edges. As calculated by the proposed method, Table 7 compares the natural frequencies with the experimental data for the plates having a ratio b/a equal to 0·3, 0·15 and 0·5. The different characteristics are detected for mode shapes in air with $n = 0$ and with $n > 0$. Mode shapes in air with $n = 0$ correspond closely to the experimental data, while $n > 0$ has a slightly higher frequency. In addition, frequencies in water on both sides by the proposed method are lower than the experimental data. Moreover, the average natural frequency in water has roughly a value of 54% in air.

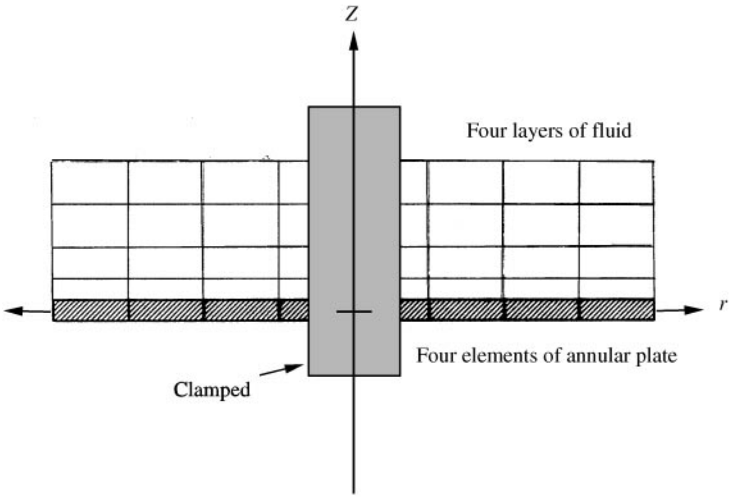

Figure 5. Mesh for annular plate in fluid.

TABLE 4

Plate #1 for circular plate (clamped edge)

Mode		Natural frequency in air (Hz)		Natural frequency in water on one side (Hz)	
m	*n*	Proposed method	Ross's FEM	Proposed method	Ross's FEM
0	0	165	165	66	60
0	1	642	643	355	339
0	2	1437	1455	927	944
0	3	2552	2612	1798	1956

Note: *m* denotes the number of nodal diameters; *n* denotes the number of nodal circles.

TABLE 5

Plate #1 for circular plate (simply supported)

Mode		Natural frequency in air (Hz)		Natural frequency in water on one side (Hz)	
m	*n*	Proposed method	Ross's FEM	Proposed method	Ross's FEM
0	0	80	80	31	27
0	1	479	480	269	249
0	2	1196	1206	780	781
0	3	2231	2310	1585	1781

Note: *m* denotes the number of nodal diameters; *n* denotes the number of nodal circles.

TABLE 6

Plate #2 for annular plate (outside/inside, free/clamped)

Mode		Natural frequency in air (Hz)			Natural frequency in water on one side (Hz)		
m	n	Proposed method	Ross's FEM	Experimental	Proposed method	Ross's FEM	Experimental
0	0	83	83	78	15	14	17
0	1	489	492	420	124	148	146
0	2	1430	1450	1255	485	570	*
0	3	2839	2896	2465	1153	1277	*

Note:* Denotes the absence of experimental data. m denotes the number of nodal diameters; n denotes the number of nodal circles.

TABLE 7

Plate #3, #4 and #5 annular plate (outside/inside, free edges)

Mode		Natural frequency in air (Hz)		Natural frequency in water on both sides (Hz)	
			$b/a = 0{\cdot}3$		
m	n	Proposed method	Experimental	Proposed method	Experimental
2	0	182·14	182·46	82·92	92·97
0	1	310·14	300·02	155·28	173·32
3	0	455·39	455·64	232·60	252·30
4	0	808·75	806·98	442·27	472·58
0	2	1869·44	1832·91	1148·29	1188·77
1	2	2182·45	2126·98	1356·52	1380·74
			$b/a = 0{\cdot}15$		
0	4	810·63	*	442·68	455·10
			$b/a = 0{\cdot}5$		
0	4	782·16	*	427·84	448·20

Note:* Denotes the absence of experimental data. m denotes the number of nodal diameters; n denotes the number of nodal circles.

4. CONCLUSIONS

Fluid heavily influences the natural frequencies of an annular plate, having decreased vacuum frequency by roughly 50% for the fundamental mode. The results are summarized as follows.

The natural frequencies in fluid are generally influenced by (NAVMI) factors. For a mode's certain nodal diameter (m), the NAVMI factor decreases with an increasing number of nodal circles (n). For a mode's certain nodal circle(n), the NAVMI factor decreases with an increasing number of nodal diameters (m). Moreover, for a certain mode, the NAVMI factor decreases with an increasing inner radius/outer radius ratio. On the effect of plate boundary condition, it is found that the clamped–clamped edge case has a larger value than the free–free edge case for the natural frequency in air and in water. The natural frequency of the free–free edge case having contact with water on both sides has roughly a value of 5–40% clamped–clamped edge case.

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APPENDIX A: NOMENCLATURE

a	outer radius of annular plate
$A(\eta)$	non-dimensionalized variable $(\eta B(\eta))$
b	inner radius of annular plate
c	ratio between the inner and outer radii (b/a)
D	flexible rigidity of annular plate
E	Young's modulus
f_{mn}	natural frequency of the generic mode in air (Hz)
f_{Fmn}	natural frequencies of plate in fluid on one side (Hz)
f^*_{Fmn}	natural frequencies of plate in fluid on both sides (Hz)

$\dot{g}_{mn}(t)$	derivative of g_{mn} with respect to time
h	thickness of the annular plate
I_m, K_m	modified Bessel functions of the first and second kinds
J_m, Y_m	Bessel functions of the first and second kinds
L	fluid domain
m	number of nodal diameter
n	number of nodal circles
r	radial co-ordinate
S_1, S_3	fluid in contact with an infinite rigid wall through the surface
S_2	fluid in contact with the plate through the surface
T_F^*	reference kinetic energy of fluid
T_p^*	reference kinetic energy of annular plate
u	spatial distribution of the velocity potential $u(r, \theta, z)$
U	velocity potential $U(r, \theta, z, r, t)$
V_p	maximum potential energy of annular plate
w	transverse displacement of annular plate $w(r, \theta, t)$
$W_{mn}(r)$	$A_{mn}J_m(\lambda_{mn}r/a) + B_{mn}Y_m(\lambda_{mn}r/a) + C_{mn}I_m(\lambda_{mn}r/a) + D_{mn}K_m(\lambda_{mn}r/a)$ where $A_{mn}, B_{mn}, C_{mn}, D_{mn}$ are the mode shape constants
$W_{mn}(r, \theta)$	deflection shape function of the generic mode $W_{mn}(r)\cos(m\theta)$
β_{mn}	added virtual mass incremental (AVMI) (T_F^*/T_p^*)
Γ_{mn}	non-dimensionalized added virtual mass incremental factor (NAVMI)
∇^2	Laplacian operator
η	non-dimensionalized variable $(a\xi)$
λ_{mn}	frequency parameter
v	the Poisson's ratio
ρ	non-dimensionalized variable (r/a)
ρ_p	mass density of annular plate