STRUCTURAL DAMAGE DETECTION USING MODAL NORMS

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1. INTRODUCTION

Different approaches and criteria are used to sense and to localize a damage of flexible structures. Most of the methods take advantage of the dynamics behavior of the structural model [1–6]. This paper uses the modal and sensor norms, as defined in reference [7], to determine damage locations. The proposed approach allows localization of damaged elements of a structure, and provides the information about the impact of the damage on the natural modes of the damaged structure.

2. MODAL MODEL

A structural model is characterized by its mass, stiffness, and damping matrices, as well as by the locations of the sensors and actuators. It is represented by the following second

$$M\ddot{q} + D\dot{q} + Kq = Bu, \quad y = C_q q + C_v \dot{q}. \tag{1a,b}$$

In this equation q is the structural displacement vector of dimension n_d, u is the input vector of dimension r, y is the output vector of dimension s, and M, D, K are the mass, damping, and stiffness matrices, respectively, of dimensions $n_d \times n_d$. The input matrix B of dimensions $n_d \times r$, characterizes the actuator locations, the output displacement and rate matrices C_q and C_v of dimensions $s \times n_d$ characterize the displacement and rate sensor locations. The mass matrix is positive definite, and the stiffness and damping matrices are positive semidefinite; n_d is the number of degrees of freedom, r is the number of actuators, and s is the number of sensors.

The above equation is also obtained in the modal co-ordinates using the modal transformation. For a small proportional damping let ω_i be the ith natural frequency and ϕ_i be the ith natural mode, or mode shape. Define the matrix of natural

frequencies $\Omega = \text{diag}\,(\omega_1, \omega_2, \ldots, \omega_n)$. and the modal matrix $\Phi = [\phi_1\ \phi_2\ \ldots\ \phi_n]$, of dimensions $n_d \times n$, consists of n natural modes of a structure. A new variable, q_m, called the modal displacement vector is introduced, such that $q = \Phi q_m$. This transformation produces the modal mass matrix $M_m = \Phi^T M \Phi$, and the modal stiffness matrix $K_m = \Phi^T K \Phi$. Both matrices are diagonal. If the damping is proportional, the modal damping matrix $D_m = \Phi^T D \Phi$ is diagonal as well.

Left-multiplying equation (1) by Φ^T, and subsequently by M_m^{-1} gives the modal model

$$\ddot{q}_m + 2Z\Omega\dot{q}_m + \Omega^2 q_m = B_m u, \quad y = C_{mq}\,q_m + C_{mv}\,\dot{q}_m, \tag{2a,b}$$

where $Z = 0{\cdot}5\,M_m^{-1} D_m \Omega^{-1}$ is a diagonal matrix of the modal damping, and B_m is the modal input matrix, $B_m = M_m^{-1}\Phi^T B$, while $C_{mq} = C_q \Phi$ and $C_{mv} = C_v \Phi$ are the modal displacement and rate matrices, respectively.

The modal equations (2a, b) can be re-written as a set of n independent equations for each modal displacement

$$\ddot{q}_{mi} + 2\zeta_i \omega_i \dot{q}_{mi} + \omega_i^2 q_{mi} = b_{mi} u, \tag{3a}$$

$$y_i = c_{mqi} q_{mi} + c_{mvi} \dot{q}_{mi}, \quad i = 1, \ldots, n, \tag{3b}$$

where ζ_i is the ith diagonal entry of Z. In the above equations y_i is the system output due to the ith mode dynamics, while b_{mi} is the ith row of B_m, and c_{mqi} and c_{mvi} are the ith columns of C_{mq}, and C_{mv}, respectively. Define c_{mi} as the equivalent output matrix of the ith mode,

$$c_{mi} = c_{mqi}/\omega_i + c_{mvi}, \tag{4}$$

then $\|b_{mi}\|_2$ and $\|c_{mi}\|_2$ are the input and output gains of the ith mode, see reference [7], where $\|x\|_2$ denotes the Frobenius norm of x, i.e., $\|x\|_2 = \sqrt{\text{tr}(x^T x)}$. In the case of multiple sensors c_{mi} consists of s entries $c_{mij}, j = 1, \ldots, s$, that correspond to each sensor.

3. MODAL NORMS

The H_2 norm of a system is used to characterize the system itself, along with its modes, and its sensors. Let $G(\omega)$ be a transfer function of a system. The H_2 norm of the system is defined as

$$\|G\|_2^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \text{tr}(G^*(\omega)\,G(\omega))\,d\omega. \tag{5}$$

In modal co-ordinates the equations of a flexible structure are uncoupled, see reference (3). In this case the norms of the ith mode and jth sensor can be approximately determined from the following equations, cf. reference [7]:

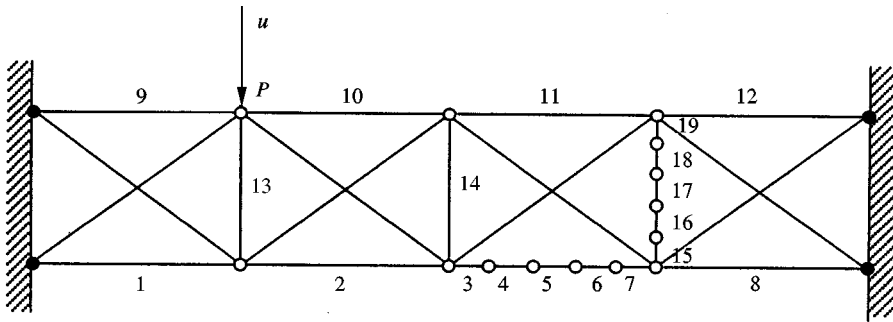$$\|G_{ij}\|_2 \cong \|b_{mi}\|_2 \|c_{mij}\|_2 / 2\sqrt{\zeta_i \omega_i}. \tag{6}$$

Figure 1. The beam structure: fixed nodes are filled, numbers refer to the sensors.

Define the H_2 norm of the jth mode, $\|G_{mi}\|_2$, as the root-mean-square sum over all sensors [7]:

$$\|G_{mi}\|_2^2 = \sum_{j=1}^{S} \|G_{ij}\|_2^2, \quad i = 1, \ldots, n. \tag{7}$$

Define also the H_2 norm of the jth sensor, $\|G_{sj}\|_2$, as the root-mean-square sum over all modes,

$$\|G_{sj}\|_2^2 = \sum_{i=1}^{n} \|G_{ij}\|_2^2, \quad j = 1, \ldots, s. \tag{8}$$
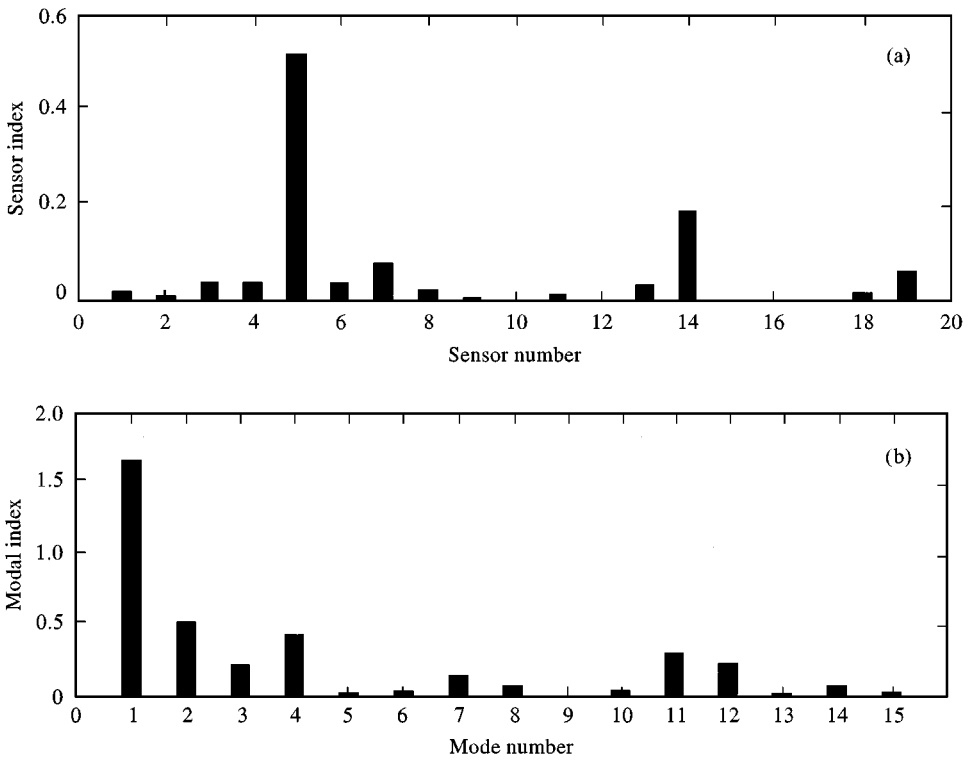

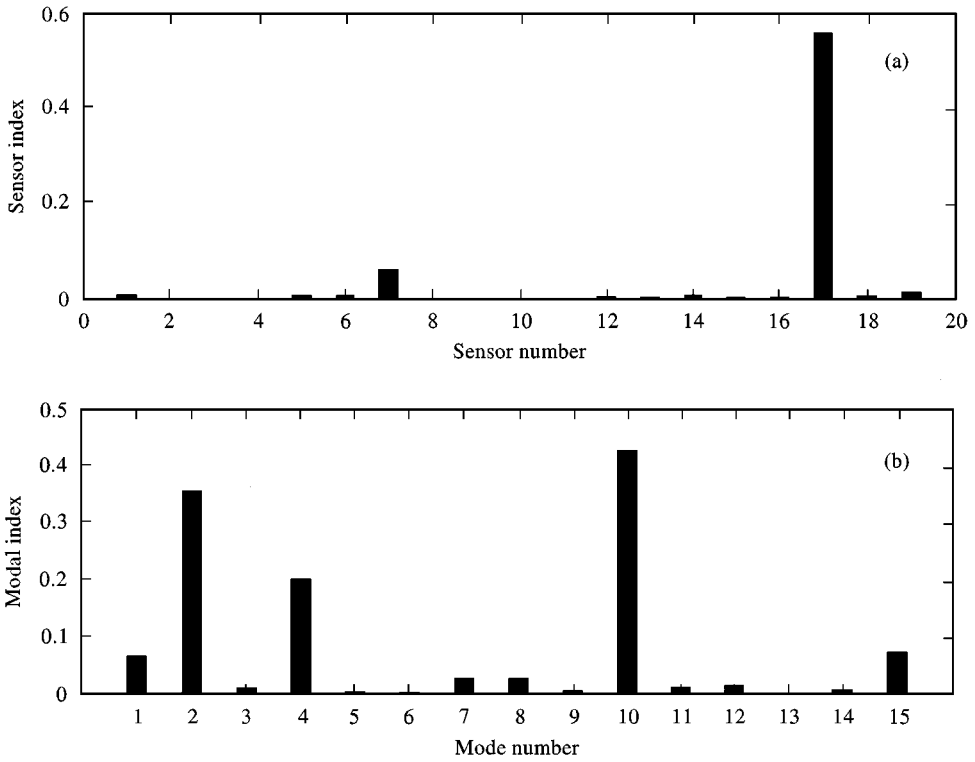Figure 2. Sensor and modal indices for the beam structure, damage case 1.

Figure 3. Sensor and modal indices for the beam structure, damage case 2.

The H_2 norm of the total system is approximately the root-mean-square sum over either all its modal norms, or all its sensor norms, that is

$$\|G\|_2^2 \cong \sum_{i=1}^{n} \|G_{mi}\|_2^2 \quad \text{or} \quad \|G\|_2^2 \cong \sum_{j=1}^{s} \|G_{sj}\|_2^2. \qquad (9)$$

The modal and sensor norms are used in the following damage detection procedure.

4. DAMAGE DETECTORS

Denote the jth sensor norm of a healthy structure by $\|G_{shj}\|_2$, *and* the jth sensor norm of a damaged structure by $\|G_{sdj}\|_2$. The jth sensor index of the structural damage is defined as a weighted difference between the jth sensor norm of a healthy and damaged structure, i.e.,

$$\sigma_{sj} = |\|G_{shj}\|_2^2 - \|G_{sdj}\|_2^2|/\|G_{shj}\|_2^2. \qquad (10)$$

The sensor index reflects the impact of the structural damage on the jth sensor.

Similarly, denote the ith mode norm of a healthy structure by $\|G_{mhi}\|_2$, and the ith mode norm of a damaged structure by $\|G_{mdi}\|_2$. The ith mode index of the structural damage is defined as a weighted difference between the ith mode norm of

a healthy and damaged structure, i.e.,

$$\sigma_{mi} = |\,\|G_{mhi}\|_2^2 - \|G_{mdi}\|_2^2\,|/\|G_{mhi}\|_2^2. \tag{11}$$

The ith mode index reflects impact of the structural damage on the ith mode.

The question arises how to measure the sensor and modal norms. It follows from equation (6) that the norm is determined from the system natural frequencies, modal damping ratios, and the modal input and output gains. The gains, on the other hand, are determined from the natural mode shapes at the actuator and sensor locations. Thus the measurements of natural frequencies, modal damping, and the modal displacements at the actuator and sensor locations of healthy and damaged structures allows for the localization of a structural damage. Note that the natural mode shapes characterize the structure itself since they are determined from the modal matrix Φ. Thus, they depend neither on the input time history, u(t), nor on the actual system deformations, q(t).

5. EXAMPLE

A beam structure with fixed corners as in Figure 1 is analyzed. The cross-section area of the steel beams is of $1\ cm^2$. Two damage cases are considered. First, as a 20% reduction of the stiffness of the beam No. 5, and the second case as a 20% reduction of the stiffness of the beam No. 17. The structure was more densely divided near the damage locations to better reflect the stress concentration. Nineteen strain-gage sensors are placed at the beams 1–19. A vertical force applied at node P excites the structure.

For the first case the sensor and the modal indices are shown in Figures 2(a,b). The sensor indices in Figure 2(a) indicate that the sensor No. 5, located at the damaged beam suffered the most changes. The modal indices in Figure 2(b) show that the first mode was heavily affected by the damage.

The sensor and modal indices for the second case are shown in Figure 3(a,b). Figure 3(a) shows the largest sensor index at location No. 17 of the damaged beam. The modal indices in Figure 3(b) show that the tenth and the second modes were mostly affected by the damage.

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