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VIBRATION ANALYSIS OF BEAMS WITH GENERAL BOUNDARY CONDITIONS TRAVERSED BY A MOVING FORCE

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This paper contributes to the basic fundamental problem of vibration of elastic homogeneous isotropic beam with general boundary conditions traversed by moving loads. Closed-form solutions for the response of beams subjected to a single deterministic moving force are obtained. The moving force is assumed to move with accelerating, decelerating and constant velocity types of motion. Results show in detail different cases of boundary conditions, type of motion, and damping. Effects of variations of the corresponding parameters on the response of the beams are studied. Results presented in this paper are readily applicable for further investigation in this field.

1. INTRODUCTION

This paper is concerned with the transverse vibrations of homogeneous isotropic Euler–Bernoulli beams with general boundary conditions subjected to a constant force travelling with different types of motion. This problem is of great fundamental importance to many researchers because of its wide range of applications in many branches of engineering. Bridges on which vehicles or trains travel, piping systems subjected to two-phase flow, beams subjected to pressure waves, and machining operations where high axial speed may be employed can be modelled as moving forces on elastic beams with different boundary conditions. Based on the excellent book by Fryba [1] and the references therein, different papers that deal with various aspects of the moving load problem have emerged. Perhaps the most important aspect of these is the one that deals with the problem from the random or stochastic point of view due to inherent randomness in material properties, and nature and speed of the load [1–11]. Other aspects of the moving load problem may be found, for example, in references [12–15]. The solutions presented in these papers discuss the moving mass problem [12, 13], the moving load problem on

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beams resting on elastic foundation [3, 14], and the identification associated with the moving force problem [15]. In this paper, an attempt is made to address one of the basic fundamental issues related to the moving load problem. Closed-form solutions of the deterministic response of Euler–Bernoulli elastic beams with different boundary conditions subjected to a constant force travelling with accelerating, decelerating, and constant velocity types of motion are obtained. Results are presented for different damping cases. Comparisons with known solutions are also made. This work is considered as a continuation of the work presented in references [9–11]. The solution presented here is more general than the one shown in reference [11] in which the solution presented there is applied to a specific random force problem. The results arrived at in this paper are readily applicable to further studies in this field. For example, and as will be seen in a subsequent paper, the results are used to obtain the random response of the beams subjected to different random force models and ultimately arrive at better predictions for the service life of such beams.

2. ANALYTICAL FORMULATION

The problem to be considered is that of transverse vibrations of a uniform elastic beam of finite length originally at rest with different classical boundary conditions. The beam is acted upon by a constant force which moves from left to right in a uniform accelerating, decelerating, or uniform velocity types of motion. The problem is governed by the following differential equation:

$$EI \frac{\partial^4 v(x,t)}{\partial x^4} + \mu \frac{\partial^2 v(x,t)}{\partial t^2} + r_a \frac{\partial v(x,t)}{\partial t} + r_i \frac{\partial^5 v(x,t)}{\partial x^4 \partial t} = p(x,t), \tag{1}$$

where EI, μ, r_a, r_i and $v(x, t)$ denote, respectively, the flexural rigidity of the beam, the mass per unit length of the beam, the coefficient of external damping of the beam, the coefficient of internal damping of the beam, and the transverse deflection of the beam at point x and time t. The load $p(x, t)$ is written as [1]

$$p(x,t) = \delta(x - f(t))P_0, \tag{2}$$

where $\delta(\cdot)$ denotes the familiar Dirac delta function, P_0 denote the concentrated force of constant magnitude, and $f(t)$ denotes a function describing the motion of the force at time t defined as

$$f(t) = x_0 + ct + \tfrac{1}{2} at^2, \tag{3}$$

where x_0 is the point of application of the force, c is the initial speed, and a is the constant acceleration. This function describes a uniform decelerating or accelerating motion. The uniform velocity type of motion, of course, is given by

$$f(t) = ct. \tag{4}$$

In modal form, the transverse deflection of the beam is written as

$$v(x,t) = \sum_{k=1}^{\infty} X_k(x) Y_k(t), \tag{5}$$

where $X_k(x)$ are the normal modes of free vibration written as

$$X_k(x) = \sin \kappa_k x + A_k \cos \kappa_k x + B_k \sinh \kappa_k x + C_k \cosh \kappa_k x, \tag{6}$$

where the constants A_k, B_k, and C_k define the shape and amplitude of the beam vibration. They are evaluated by considering the boundary conditions associated with each beam. The constant κ_k is the frequency parameter associated with each beam. Here, the external and internal damping effects are assumed to be proportional, respectively, to the mass and stiffness properties of the beam. That is,

$$r_a = \gamma\mu, \qquad r_i = \beta EI, \tag{7, 8}$$

where γ and β are the proportionality constants. Substituting equations (5), (7), and (8) into equation (1), considering the orthogonality conditions of the normal modes, and carrying out the familiar operations, the differential equation of the kth mode of the generalized deflection or the modal response is expressed as

$$\ddot{Y}_k(t) + 2\omega_k\xi_k\dot{Y}(t) + \omega_k^2 Y_k(t) = \frac{Q_k(t)}{m_k}, \tag{9}$$

where

$$\omega_k = \kappa_k^2\sqrt{EI/\mu}, \qquad \xi_k = \frac{\alpha + \beta\omega_k^2}{2\omega_k}. \tag{10, 11}$$

$$Q_k(t) = \int_0^l X_k(x)p(x,t)\,\mathrm{d}x, \tag{12}$$

and

$$m_k = \int_0^l \mu X_k^2(x)\,\mathrm{d}x \tag{13}$$

are, respectively, the natural circular frequency of the kth mode, the dimensionless damping ratio of the kth mode, the generalized force of the beam associated with the kth mode, and the generalized mass of the beam associated with kth mode. By substituting equation (2) into equation (12) and the result into equation (9) to obtain:

$$\ddot{Y}_k(t) + 2\omega_k\xi_k\dot{Y}(t) + \omega_k^2 Y_k(t) = \frac{P_0}{m_k}X_k(f(t)). \tag{14}$$

The solution of equation (14) may be written as

$$Y_k(t) = \frac{P_0}{m_k}\int_0^t X_k(f(t))h_k(t-\tau)\,\mathrm{d}\tau, \tag{15}$$

where $h_k(t)$ is the impulse response function defined as

$$h_k(t) = \begin{cases} \dfrac{1}{\omega_{dk}}\mathrm{e}^{-\xi_k\omega_k t}\sin\omega_{dk}t, & t \geqslant 0, \\ 0, & t < 0, \end{cases} \tag{16}$$

in which

$$\omega_{dk} = \omega_k \sqrt{1-\xi_k^2} \quad (17)$$

is the damped circular frequency of the kth mode of the beam. Substituting equations (3), (6), and (16) into equation (15), carrying out the integration and substituting the result into equation (5), the deflection $v(x, t)$ of the beam is obtained as [16]

$$\begin{aligned} v(x,t) = \sum_{k=1}^{\infty} X_k(x) P^* \mathrm{Re}[& R_1 u_2 \mathrm{e}^{R_2}\{\mathrm{erf}(R_3 t + R_4) - \mathrm{erf}(R_4)\} \\ & - R_1 u_2 \mathrm{e}^{R_5}\{\mathrm{erf}(R_3 t + R_6) - \mathrm{erf}(R_6)\} \\ & - \sqrt{2} u_2 (B_k + C_k) \mathrm{e}^{R_7}\{\mathrm{erf}(-u_1 t \mathrm{i} + R_8) - \mathrm{erf}(R_8)\} \\ & + \sqrt{2} u_2 (B_k - C_k) \mathrm{e}^{R_9}\{\mathrm{erf}(u_1 t \mathrm{i} + R_{10}) - \mathrm{erf}(R_{10})\}], \end{aligned} \quad (18)$$

where

$$P^* = \frac{-P_0 \mathrm{e}^{-\xi_k \omega_k (t + c/a)}}{4 m_k \omega_{dk}},$$

$$u_1 = \sqrt{\frac{\kappa_k a}{2}},$$

$$u_2 = \sqrt{\frac{\pi}{\kappa_k a}},$$

$$R_1 = 1 - A_k + \mathrm{i}(1 + A_k),$$

$$R_2 = \frac{\xi_k \omega_k \omega_{dk}}{\kappa_k a} + \frac{\mathrm{i}}{2\kappa_k a}[\xi_k^2 \omega_k^2 - \omega_{dk}^2 + \kappa_k^2(2 a x_0 - c^2) + 2\kappa_k \omega_{dk}(c + at)],$$

$$R_3 = \tfrac{1}{2}\sqrt{\kappa_k a}\,(1 - \mathrm{i}),$$

$$R_4 = \frac{-1}{2\sqrt{\kappa_k a}}[\xi_k \omega_k + \omega_{dk} - \kappa_k c + \mathrm{i}(\xi_k \omega_k - \omega_{dk} + \kappa_k c)],$$

$$R_5 = \frac{-\xi_k \omega_k \omega_{dk}}{\kappa_k a} + \frac{\mathrm{i}}{2\kappa_k a}[\xi_k^2 \omega_k^2 - \omega_{dk}^2 + \kappa_k^2(2 a x_0 - c^2) - 2\kappa_k \omega_{dk}(c + at)],$$

$$R_6 = \frac{-1}{2\sqrt{\kappa_k a}}[\xi_k \omega_k - \omega_{dk} - \kappa_k c + \mathrm{i}(\xi_k \omega_k + \omega_{dk} + \kappa_k c)],$$

$$R_7 = \frac{-1}{2\kappa_k a}[\xi_k^2 \omega_k^2 - \omega_{dk}^2 - \kappa_k^2(2 a x_0 - c^2)] + \mathrm{i}\frac{\omega_{dk}}{\kappa_k a}[\xi_k \omega_k + \kappa_k(c + at)],$$

$$R_8 = \frac{-1}{\sqrt{2\kappa_k a}}[\omega_{dk} + \mathrm{i}(\xi_k \omega_k + \kappa_k c)],$$

$$R_9 = \frac{1}{2\kappa_k a}[\xi_k^2\omega_k^2 - \omega_{dk}^2 - \kappa_k^2(2ax_0 - c^2)] + \mathrm{i}\frac{\omega_{dk}}{\kappa_k a}[\xi_k\omega_k - \kappa_k(c + at)],$$

$$R_{10} = \frac{-1}{\sqrt{2\kappa_k a}}[\xi_k\omega_k - \kappa_k c + \mathrm{i}\omega_{dk}],$$

$$\mathrm{i} = \sqrt{-1},$$

$$\mathrm{erf}(h) = \frac{2}{\sqrt{\pi}}\int_0^h \mathrm{e}^{-t^2}\,\mathrm{d}t. \qquad (19\text{a–o})$$

The uniform velocity case does not follow from equation (18) because of the definition of the error function. The response of the uniform velocity case is obtained, however, by using equation (4) instead of equation (3) and following the same procedure that lead to equation (18). In this case, the response becomes

$$\begin{aligned} v(x,t) = \sum_{k=1}^{\infty} X_k(x)\frac{P_0}{2m_k\omega_{dk}}\Bigg[& \left\{\frac{(\xi_k\omega_k + q_5A_k)}{q_2} - \frac{(\xi_k\omega_k + q_6A_k)}{q_1}\right\}\cos(\kappa_k ct) \\ & + \left\{\frac{(q_5 - \xi_k\omega_k A_k)}{q_2} + \frac{(-q_6 + \xi_k\omega_k A_k)}{q_1}\right\}\sin(\kappa_k ct) \\ & + \frac{\omega_{dk}q_9}{q_4}\mathrm{e}^{\kappa_k ct} - \frac{\omega_{dk}q_{10}}{q_3}\mathrm{e}^{-\kappa_k ct} \\ & + \left\{\frac{(\xi_k\omega_k + q_6A_k)}{q_1} - \frac{(\xi_k\omega_k + q_5A_k)}{q_2}\right\}\mathrm{e}^{-\xi_k\omega_k t}\cos(\omega_{dk}t) \\ & + \left\{\frac{\omega_{dk}q_{10}}{q_3} - \frac{\omega_{dk}q_9}{q_4}\right\}\mathrm{e}^{-\xi_k\omega_k t}\cos(\omega_{dk}t) \\ & + \left\{\frac{(q_6 - \xi_k\omega_k A_k)}{q_1} + \frac{(q_5 - \xi_k\omega_k A_k)}{q_2}\right\}\mathrm{e}^{-\xi_k\omega_k t}\sin(\omega_{dk}t) \\ & + \left\{\frac{q_8q_{10}}{q_3} - \frac{q_7q_9}{q_4}\right\}\mathrm{e}^{-\xi_k\omega_k t}\sin(\omega_{dk}t)\Bigg], \end{aligned} \qquad (20)$$

where

$$q_1 = (\xi_k\omega_k)^2 + (\kappa_k c - \omega_{dk})^2, \qquad q_2 = (\xi_k\omega_k)^2 + (\kappa_k c + \omega_{dk})^2,$$

$$q_3 = \omega_{dk}^2 + (\xi_k\omega_k - \kappa_k c)^2, \qquad q_4 = \omega_{dk}^2 + (\xi_k\omega_k + \kappa_k c)^2,$$

$$q_5 = \kappa_k c + \omega_{dk}, \qquad q_6 = \kappa_k c - \omega_{dk}, \qquad q_7 = \xi_k\omega_k + \kappa_k c,$$

$$q_8 = \xi_k\omega_k - \kappa_k c, \qquad q_9 = B_k + C_k, \qquad q_{10} = B_k - C_k. \qquad (21\text{a–j})$$

Equations (18) and (20) give in analytical form the response of homogeneous isotropic damped beams with general boundary conditions subjected to single deterministic force travelling with different types of motion. The two equations allow a straightforward evaluation of the response for the case under consideration.

3. DISCUSSION OF RESULTS

The analysis arrived at in this paper is applied to homogeneous isotropic beams with the four classical boundary conditions. Hinged–hinged, fixed–fixed, fixed–free, free–fixed, fixed–hinged, and hinged–fixed beams are used to clarify the results. The beams are subjected to concentrated constant amplitude loads moving with uniformly accelerated, decelerated, or uniform velocity types of motion. Computations and results shown in the paper are obtained by using Mathematica. At this point, it is worth mentioning that some special cases related to this problem are presented in reference [1]. In the accelerated motion, a beam at rest is entered from the left-hand side at point $x_0 = 0$ by a concentrated force P_0 moving according to equation (3). The motion is assumed to be uniformly accelerated so that it reaches the speed c at point $x = l$. The instant t_1 at which the force arrives at the right-hand side of the beam and the acceleration a are written as

$$t_1 = \frac{2l}{c}, \qquad a = \frac{c^2}{2l}. \tag{22, 23}$$

In the decelerated motion, a beam also at rest is entered from the left-hand side at point $x_0 = 0$ by a concentrated force P_0 moving according to equation (3). The motion is assumed to be uniformly decelerated so that it stops at the right-hand side of the beam at point $x = l$. The instant t_2 at which the force stops and the deceleration a are written as

$$t_2 = \frac{2l}{c}, \qquad a = \frac{-c^2}{2l}. \tag{24, 25}$$

In the uniform velocity case, a beam also at rest is entered from the left-hand side at point $x_0 = 0$ by a concentrated force P_0 moving according to equation (4). The instant at which the force arrives at the right-hand side of the beam is

$$t_3 = \frac{l}{c}. \tag{26}$$

In the figures, the dimensionless dynamic deflection $v(x_{\max}, t)/v_0$ is shown for all beams versus the dimensionless time parameter $s = t/t_i$. Thus, when $s = 0$, the force is at the left-hand side of the beam $x = 0$, and when $s = 1$ the force is at the right-hand side of the beam $x = l$. Here v_0 and $x_{\max}$ denote the maximum static deflection and the point on the beam that corresponds to this deflection respectively. The deflection $v(x_{\max}, t)$ is obtained either from equation (18) or equation (20). In these figures, the effects of damping and speed are made clear. The effect of damping is represented by the dimensionless damping coefficient ξ. Three values are considered: $\xi = 0{\cdot}0$, $0{\cdot}1$, and $0{\cdot}2$. The effect of speed is represented by the dimensionless speed parameter α, which is defined as

$$\alpha = \frac{c}{c_{cr}}, \tag{27}$$

where c_{cr} is the critical speed, defined as [1, 2]

$$c_{cr} = \frac{\omega_1 l}{\pi}. \tag{28}$$

Results are obtained for $\alpha = 1{\cdot}0, 0{\cdot}5$ and $0{\cdot}25$. Figures 1(a–i) show the dimensionless dynamic deflection $v(x_{\max}, t)/v_0$ for the hinged–hinged beam where $x_{\max} = l/2$, the point of maximum static deflection, and v_0 is the maximum static deflection defined at $x_{\max}$ as

$$v_0 = \frac{P_0 l^3}{48EI}. \tag{29}$$

In all these figures, three levels of damping: $\xi = 0{\cdot}0$, $0{\cdot}1$, $0{\cdot}2$ are presented where each one of the figures represents different case and type of motion for the hinged–hinged beam. Figures 1(a–c) show the dimensionless dynamic deflection for three levels of accelerated motion, respectively, $\alpha = 1{\cdot}0, 0{\cdot}5, 0{\cdot}25$. Figures 1(d–f) and (g–i) are the same as Figures 1(a–c) but for the decelerated and uniform velocity types of motion respectively. In these figures; the effect of damping is clear for all cases where an increase in damping yields, in general, a decrease in the response. Differences in the dynamic deflections among the accelerated motion, Figures 1(a–c); the decelerated motion, Figures 1(d–f); and the uniform velocity types of

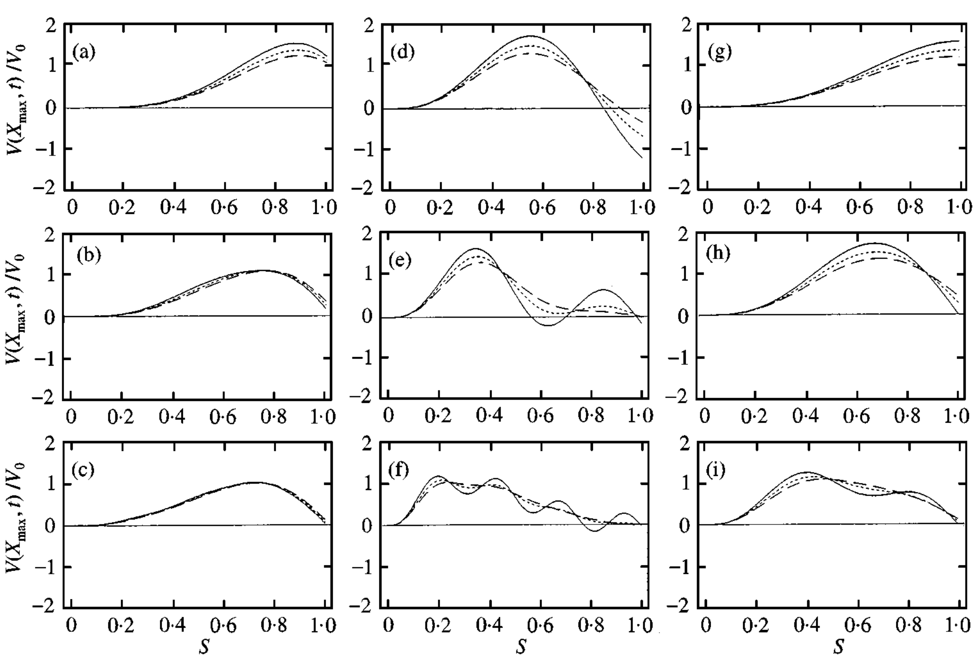

Figure 1. Dimensionless dynamic deflection versus the normalized time for a hinged–hinged beam, (a–c) accelerated motion, (a) $\alpha = 1{\cdot}0$, (b) $\alpha = 0{\cdot}5$, (c) $\alpha = 0{\cdot}25$; (d–f) decelerated motion, (d) $\alpha = 1{\cdot}0$, (e) $\alpha = 0{\cdot}5$, (f) $\alpha = 0{\cdot}25$; (g–i) uniform velocity, (g) $\alpha = 1{\cdot}0$, (h) $\alpha = 0{\cdot}5$, (i) $\alpha = 0{\cdot}25$; (– – –) $\xi = 0{\cdot}2$, (····) $\xi = 0{\cdot}1$, (—) $\xi = 0{\cdot}0$.

motion, Figures 1(g–i); are due to the kinematics involved. It is noticed that the decelerated motion has a higher dynamic effect than either the accelerated or uniform velocity types of motion. In the accelerated type of motion, it is also noticed that the maximum dynamic deflection is reached at much later time than the other two cases. In a similar manner, Figures 2(a–i) show the dimensionless dynamic deflection $v(x_{max}, t)/v_0$ for the fixed–fixed beam where $x_{max} = l/2$, and v_0 is the maximum static deflection defined at x_{max} as

$$v_0 = \frac{P_0 l^3}{192EI}. \tag{30}$$

The accelerated type of motion of this beam, Figures 2(a–c), shows a larger dimensionless dynamic deflection than the one of simply supported beam, Figures 1(a–c), for all values of α. A resemblance between the behavior of the two beams is quite clear in the decelerated type of motion, Figures 1(d–f) and 2(d–f). It is noticed that the dimensionless dynamic deflection of the fixed–fixed beam is larger than the one of the simply supported beam for $\alpha = 0{\cdot}5$ and $\alpha = 0{\cdot}25$. The opposite is true for $\alpha = 1{\cdot}0$. As for the uniform velocity type of motion both beams show similar behavior except for the case of $\alpha = 1{\cdot}0$ where it is noticed that the dimensionless dynamic deflection of the fixed–fixed beam is smaller than the one of the simply supported beam, Figure 2(g) and 1(g) respectively. The dimensionless dynamic deflection $v(x_{max}, t)/v_0$ for the fixed– free and free–fixed beams are shown in

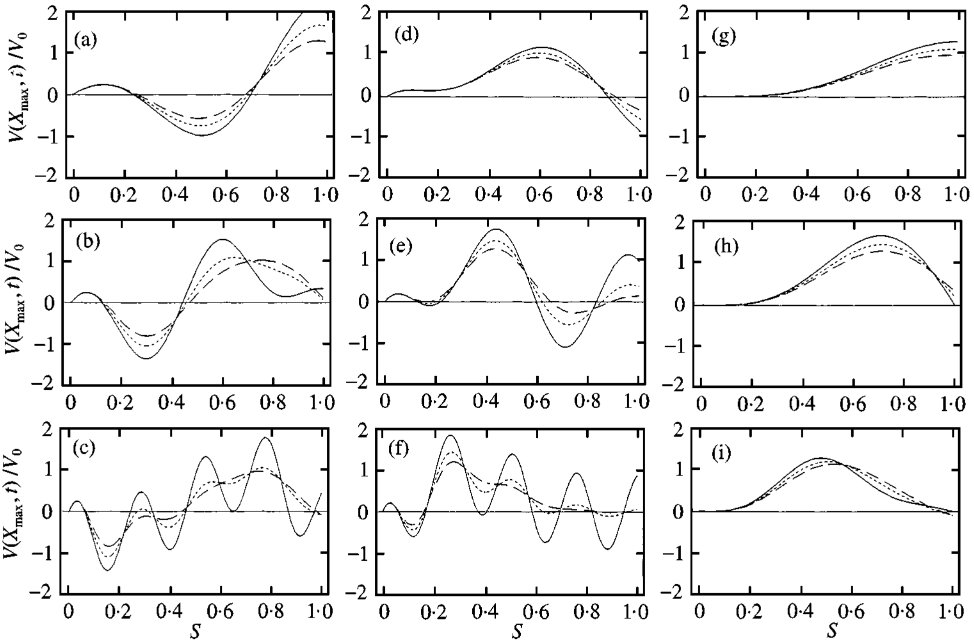

Figure 2. Dimensionless dynamic deflection versus the normalized time for a fixed–fixed beam; (a–c) accelerated motion, (a) $\alpha = 1{\cdot}0$, (b) $\alpha = 0{\cdot}5$, (c) $\alpha = 0{\cdot}25$; (d–f) decelerated motion, (d) $\alpha = 1{\cdot}0$, (e) $\alpha = 0{\cdot}5$, (f) $\alpha = 0{\cdot}25$; (g–i) uniform velocity, (g) $\alpha = 1{\cdot}0$, (h) $\alpha = 0{\cdot}5$, (i) $\alpha = 0{\cdot}25$; (- - -) $\xi = 0{\cdot}2$, (· · ·) $\xi = 0{\cdot}1$, (——) $\xi = 0{\cdot}0$.

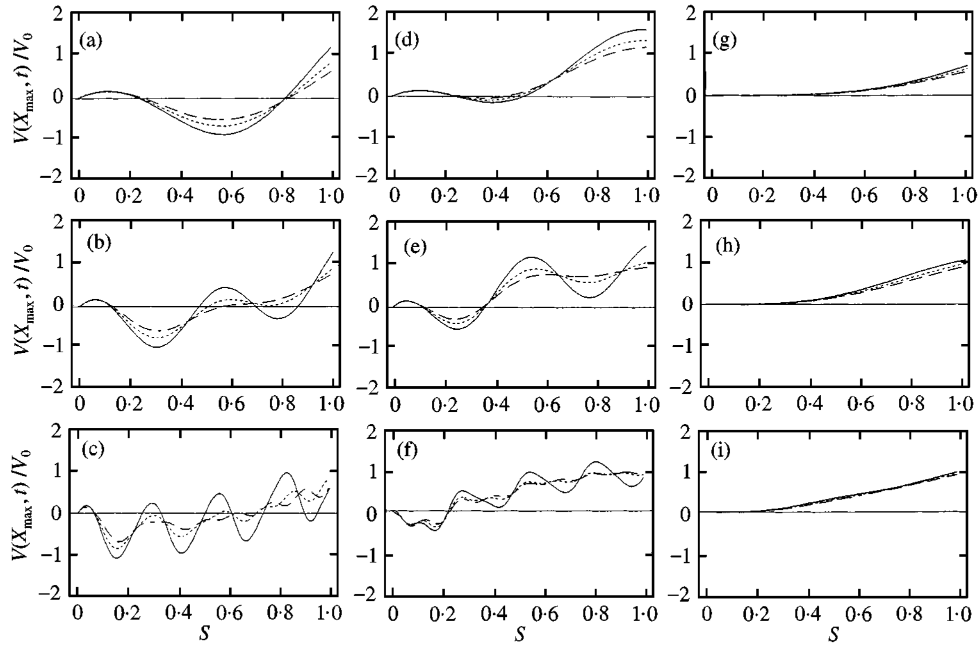

Figure 3. Dimensionless dynamic deflection versus the normalized time for a fixed–free beam; (a–c) accelerated motion, (a) $\alpha = 1{\cdot}0$, (b) $\alpha = 0{\cdot}5$, (c) $\alpha = 0{\cdot}25$; (d–f) decelerated motion, (d) $\alpha = 1{\cdot}0$, (e) $\alpha = 0{\cdot}5$, (f) $\alpha = 0{\cdot}25$; (g–i) uniform velocity, (g) $\alpha = 1{\cdot}0$, (h) $\alpha = 0{\cdot}5$, (i) $\alpha = 0{\cdot}25$; (- - -) $\xi = 0{\cdot}2$, (· · ·) $\xi = 0{\cdot}1$, (——) $\xi = 0{\cdot}0$.

Figures 3(a–i) and 4(a–i) respectively. The difference between the two beams is that in the former the force enters the beam from the fixed end while in the latter the force enters the beam from the free end. The results are obtained at the free end where v_0 is given as

$$v_0 = \frac{P_0 l^3}{3EI}. \tag{31}$$

The cases presented in these two groups of figures are the same as those presented in Figures 1(a–i) and 2(a–i). As seen from Figures 3(a–f) and 4(a–f) respectively, the two beams behave in quite an opposite manner for the accelerated and decelerated types of motion. For example, the maximum value of the dynamic deflection for the decelerated type of motion in the case of fixed–free beam and for $\alpha = 1{\cdot}0$, $0{\cdot}5$; Figures 3(d–e), is larger than its counterpart of the free–fixed beam (Figures 4(d–e)). When $\alpha = 0{\cdot}25$ the maximum value of the dynamic deflection is smaller in the case of fixed–free beam (Figure 3(f)), than in the case of free–fixed beam (Figure 4(f)). The opposite is true for the accelerated type of motion. In the uniform velocity type of motion, it is clear that the relative dynamic deflection is higher when the force enters the beam from the free end (Figures 4(g–i)), than when the force enters the beam from the fixed end (Figures 3(g–i)). Cases that are shown in Figures 5(a–i)–6(a–i) are the same as those presented Figures 1(a–i)–4(a–i). In Figures 5(a–i),

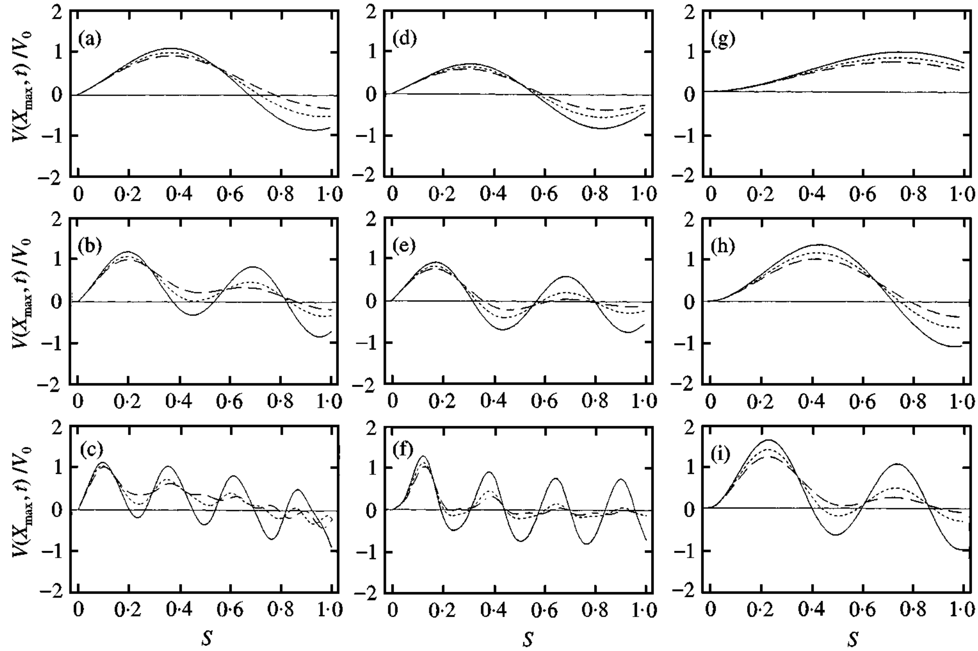

Figure 4. Dimensionless dynamic deflection versus the normalized time for a free–fixed beam; (a–c) accelerated motion, (a) $\alpha = 1{\cdot}0$, (b) $\alpha = 0{\cdot}5$, (c) $\alpha = 0{\cdot}25$; (d–f) decelerated motion, (d) $\alpha = 1{\cdot}0$, (e) $\alpha = 0{\cdot}5$, (f) $\alpha = 0{\cdot}25$; (g–i) uniform velocity, (g) $\alpha = 1{\cdot}0$, (h) $\alpha = 0{\cdot}5$, (i) $\alpha = 0{\cdot}25$; (- - -) $\xi = 0{\cdot}2$, (· · ·) $\xi = 0{\cdot}1$, (——) $\xi = 0{\cdot}0$.

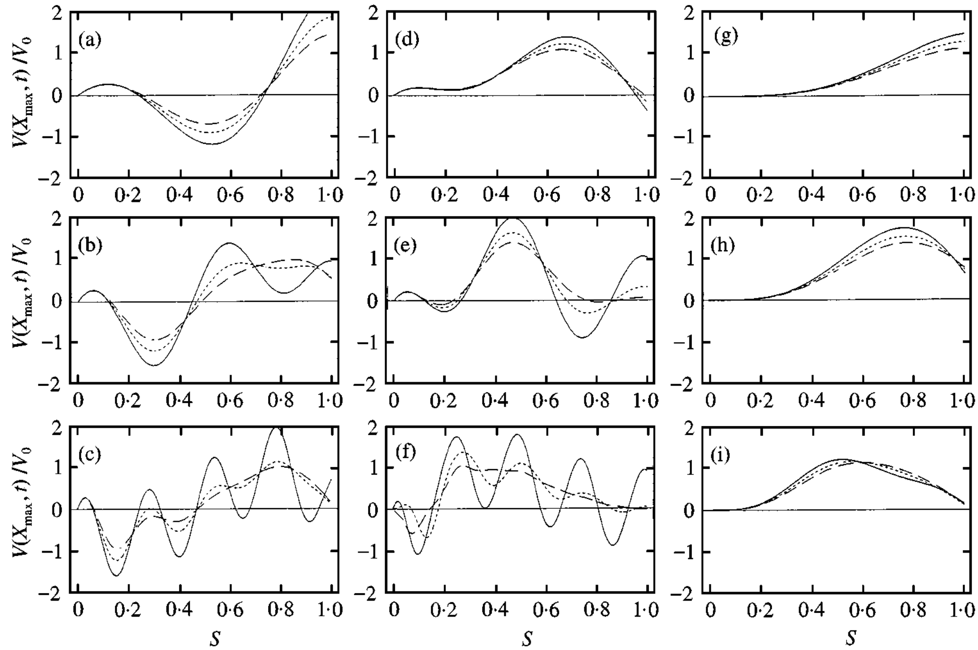

Figure 5. Dimensionless dynamic deflection versus the normalized time for a fixed–hinged beam; (a–c) accelerated motion, (a) $\alpha = 1{\cdot}0$, (b) $\alpha = 0{\cdot}5$, (c) $\alpha = 0{\cdot}25$; (d–f) decelerated motion, (d) $\alpha = 1{\cdot}0$, (e) $\alpha = 0{\cdot}5$, (f) $\alpha = 0{\cdot}25$; (g–i) uniform velocity, (g) $\alpha = 1{\cdot}0$, (h) $\alpha = 0{\cdot}5$, (i) $\alpha = 0{\cdot}25$; (- - -) $\xi = 0{\cdot}2$, (· · ·) $\xi = 0{\cdot}1$, (——) $\xi = 0{\cdot}0$.

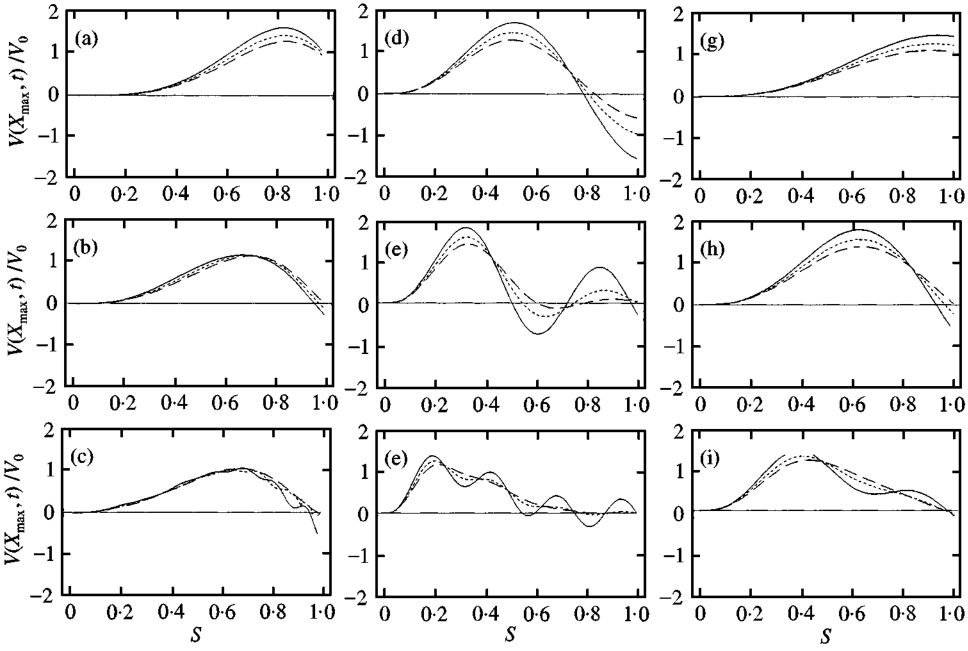

Figure 6. Dimensionless dynamic deflection versus the normalized time for a hinged–fixed beam; (a–c) accelerated motion, (a) $\alpha = 1{\cdot}0$, (b) $\alpha = 0{\cdot}5$, (c) $\alpha = 0{\cdot}25$; (d–f) decelerated motion, (d) $\alpha = 1{\cdot}0$, (e) $\alpha = 0{\cdot}5$, (f) $\alpha = 0{\cdot}25$; (g–i) uniform velocity, (g) $\alpha = 1{\cdot}0$, (h) $\alpha = 0{\cdot}5$, (i) $\alpha = 0{\cdot}25$; (- - -) $\xi = 0{\cdot}2$, (· · · ·) $\xi = 0{\cdot}1$, (——) $\xi = 0{\cdot}0$.

the dynamic deflection $v(x_{\max}, t)/v_0$ is shown for the fixed–pinned beam where $x_{max} = 0{\cdot}55l$, and v_0 is the maximum static deflection defined at x_{max} as

$$v_0 = \frac{P_0 l^3}{48\sqrt{5}EI}. \tag{32}$$

Similarities between Figures 5(a–i) and 2(a–i) are clear. Figures 6(a–i) show the dynamic deflection $v(x_{max}, t)/v_0$ for the pinned–fixed beam where $x_{max} = 0{\cdot}45\,l$, and v_0 is the maximum static deflection defined at x_{max} as

$$v_0 = \frac{P_0 l^3}{48\sqrt{5}EI}. \tag{33}$$

Similarities between Figures 6(a–i) and 1(a–i) are also clear.

4. CONCLUSIONS

The deterministic dynamic response for homogeneous isotropic elastic beams with general classical boundary conditions traversed by a moving constant force was discussed in detail for different cases. The effects of boundary conditions, type of motion, and damping on the response of the beams were studied. The results arrived at in this paper are readily applicable to further studies in this field.

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