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ACOUSTIC VELOCITY MEASUREMENTS IN THE AIR BY MEANS OF LASER DOPPLER VELOCIMETRY: DYNAMIC AND FREQUENCY RANGE LIMITATIONS AND SIGNAL PROCESSING IMPROVEMENTS

J. C. VALIÈRE†, P. HERZOG‡, V. VALEAU AND G. TOURNOIS

Institut d'Acoustique et de Mécanique, Laboratoire d'Acoustique, U.M.R. 6613 *C.N.R.S.*, *Université du Maine*, 72085 *Le Mans Cedex* 9, *France*

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Although laser Doppler velocimetry (LDV) has been used for measurements in fluid mechanics and even high-level acoustics for a number of years, much uncertainty remains concerning its ability to measure the velocities encountered in the field of more usual linear acoustics. Specific aspects of the acoustic velocity have to be taken into account in order to achieve measurements, namely to deal with low levels or high frequencies, which requires some optimization of both the optics, the mechanical supports, and the signal processing. This paper presents a LDV set-up dedicated to acoustics in air, and presents the performances obtained using commercially available hardware. It also illustrates the main limitations, especially concerning the signal processing, and presents a new approach for the signal processing, optimized for a sine wave forced excitation of the acoustic field.

1. INTRODUCTION

The use of laser Doppler velocimetry (LDV) in acoustics could be the starting point of many new developments concerning aspects such as acoustic phenomena near sound sources, or close to boundaries, or in inhomogeneous media. Such a tool may be especially interesting when there are sharp changes in the behavior of the field, where the knowledge of a vector quantity like the velocity may give much more information than the one of a scalar quantity such as pressure or temperature.

Indeed several techniques exist for the measurement of acoustic velocity or intensity in air, but most of them rely on indirect evaluation which involves two or more pressure measurements. This method has several drawbacks, especially the size of the sensor, which both perturbs the field and leads to an average over a relatively large measurement volume. In addition, the frequency range is limited due to inaccuracies of the relative calibration of the microphones. Some other tools are used, such as hot-wire sensors, with the disadvantages of very non-linear

†Present address: LEA, UMR 6609 CNRS, BATH Av. du Recteur Pineau, 86022 Poitiers, France.
‡Present address: LEMA, Swiss Federal Institute of Technology, CH 1015 Lausanne, Switzerland.

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characteristics and an inadequate dynamic range, and even a sign ambiguity if used without flow. Conversely, the potential advantages of LDV are its theoretical "non-invasive" nature, its ability to track very low frequencies (as it can deal with steady flows), a small measuring volume giving good spatial resolution, and a linear response. However, conventional LDV has two main drawbacks: the first one is its complexity, which requires a substantial know-how for an efficient use, and the second one is that it is not aimed at the measurement of usual acoustic particle velocity, whose magnitude is much lower than the one in flows.

To date, a number of authors have dealt with the measurement of acoustic velocities using LDV. Taylor [1, 2] has developed a method based on the spectral analysis of the burst in the case of high velocities, and has proposed a calibration method for microphones using LDV. Vignola *et al.* [3] and Lee *et al.* [4] have worked on the use of LDV in dense fluids such as water. Davis and Hews-Taylor [5], Sharpe *et al.* [6] and Kunz and Vortmeyer [7] have measured complex acoustic impedance using LDV. Eckmann and Grotberg [8] have estimated acoustic velocities from signals superimposed upon flows. More recent papers by Hann and Greated [9] and Loizeau and Gervais [10] have also proposed signal processing improvements for measurements in acoustics.

Because the technology of LDV has become more affordable, and many theoretical works in acoustics still lack accurate measurements for experimental validation, the actual potential of LDV for low-level acoustics has been investigated. Potential applications include measurements around duct discontinuities, or close to vibrating surfaces, or the normal velocities close to an acoustic source in order to get its radiation impedance. A rough estimation of the needed range of performances for such applications would be a 60 dB dynamic range throughout the 50–5 kHz frequency band. Furthermore, in order to cope with the low-frequency noise inherent to velocity measurements, forced sine wave acoustic excitation with zero mean flow has been chosen. Synchronous demodulation ("lock-in") can therefore lead to accurate estimations of the amplitude and phase of the resulting acoustic response.

As described in Sections 2 and 3, the detection of acoustic velocity is quite difficult because of its small magnitude. Therefore, the choice of an equipment optimized to carry out such measurements is presented in Section 4, and a first calibration based on measurements in a wave guide is described in Section 5. Next, a signal processing approach based on instantaneous frequency tracking of the Doppler signal is covered in Sections 6–8, leading to a direct estimation of the velocity which constitutes the main topic of this paper.

2. SPECIFICITY OF ACOUSTIC VELOCITY

The acoustic field may be described as a spatial distribution of pressure and particle velocity, their amplitudes p and v being proportional for plane travelling waves ($p = \varrho c v$, where ϱ is the air density and c is the sound speed).

The sound pressure level Lp (SPL) is usually expressed in decibels as the ratio of p to the reference sound pressure $p_0 = 2 \times 10^{-5}$ Pa. For the acoustic dynamic range, the SPL values range from 0 to 130 dB, this maximum corresponding to 63 Pa. The corresponding acoustic velocities for acoustic plane waves range respectively from 50 to 150 mm/s. These values can be lower near rigid boundaries or in stationary fields where acoustic velocity almost vanishes. The last figures of interest concern the acoustic displacement, obtained by dividing the velocity by 2π times the frequency f. The acoustic frequency range is usually taken as 20 Hz–20 kHz, and the acoustic displacement thus ranges from 1·2 mm (130 dB at 20 Hz) to a theoretical 4×10^{-13} m (0 dB at 20 kHz). Although this last figure is not realistic because the microscopic nature of the gas should be taken into account in such a case, it makes one feel that acoustic phenomena may not be measured in the full acoustic ranges, and that perturbation of the field is likely to occur with almost all sensor principles. More reasonable specifications can then be derived by considering a normal laboratory environment: ambient noise of measurement rooms, or self-noise of microphones and preamplifiers, limit the dynamic range for low levels. Measured acoustic velocities may therefore range between a few μm/s and a few cm/s, values which are still low, but close to those observed in natural convection.

3. PRINCIPLES OF LASER DOPPLER VELOCIMETRY IN ACOUSTICS

The basic principle of LDV in fluid is described in Figure 1. Two coherent laser beams are crossed and focused to generate an elipsoidal measuring volume, in which the electromagnetic interferences lead to apparent dark and bright fringes. A seeding particle passing through the probe volume scatters the light from both beams, leading to a diffused light which is detected by a photomultiplier. The electronic signal obtained, called a burst, is weighted by the Gaussian light intensity distribution across the beams section, and frequency modulated by the presence of the interference fringes. The resulting modulation frequency, usually called Doppler frequency (f_d), is given by the ratio of the component perpendicular to the fringes of the velocity (V) on the fringe spacing (i):

$$f_d = \frac{V}{i} = \frac{\sin(\theta/2)}{\lambda/2} V, \tag{1}$$

where λ is the laser wavelength and θ the beams angle (Figure 1). In the case of time-invariant particle velocity, the burst can be simply modelled as a sine wave weighted by a Gaussian envelope:

$$x(t) = K\mathrm{e}^{-(\beta V t)^2}\left(M + \cos\left(2\pi \frac{V}{i} t\right)\right), \tag{2}$$

with

$$\beta = \frac{\cos(\theta/2)}{r},$$

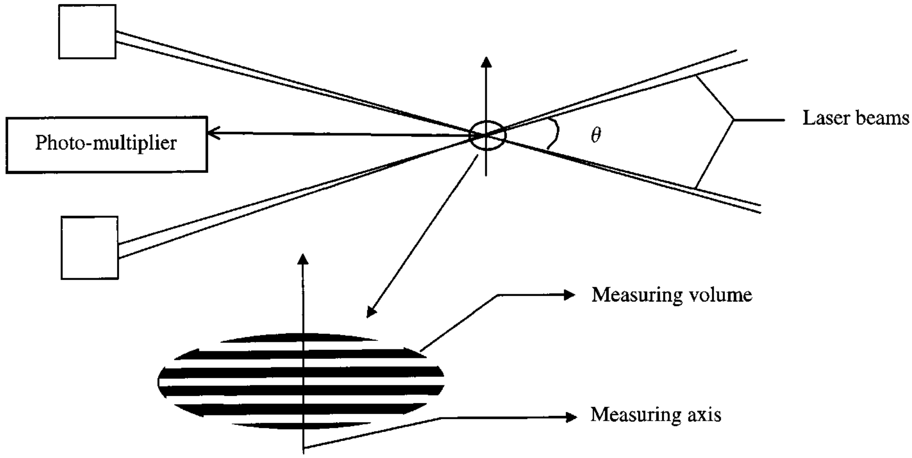

Figure 1. Basic set-up for LDV measurements.

where K results from the optical power and the particle scattering area section, and r is the radius of the focused laser beams (thus $1/\beta$ defines the width of the measuring volume).

In relation (2), the constant M has been introduced to describe the fact that, usually, the two beam intensities are different. This last phenomenon is responsible of an offset, also weighted by the Gaussian shape, called the burst pedestal. Another remark which can be derived from relation (2) is that the duration of the burst closely depends on the velocity.

In order to distinguish the velocity sign, the frequency of one beam is shifted using an acousto-optic modulator (Bragg cell), driven by a sine wave at constant frequency f_0. The consequence, in the measuring volume, is a sinusoidal shift of the interference fringes, so that a non-moving seeding particle located inside it leads to a sine wave signal. Thus, if a particle passes, the signal is shifted up or down around the Bragg frequency according to the velocity direction. Then equation (2) becomes

$$x(t) = K\mathrm{e}^{-(\beta V t)^2}\left(M + \cos\left(2\pi f_0 t + 2\pi \frac{V}{i} t\right)\right), \tag{3}$$

where f_0 is the carrier frequency (40 MHz for most Bragg cells). Because of the oscillating nature of acoustic waves, such a frequency-shifting technique must be included in our set-up.

Then, considering a particle moving with a varying velocity component normal to the fringes $v(t)$, the signal detected by the photomultiplier can finally be expressed as

$$x(t) = A(t)\left(M + \cos\left(2\pi f_0 t + 2\pi \int_0^t \frac{V(t)}{i}\,\mathrm{d}t\right)\right). \tag{4}$$

In this relation, the amplitude $A(t)$ depends on the particle position at the time t, and the frequency with which the particle crosses a fringe $(f_0 + v(t)/i)$ is now time dependent.

The velocity of a particle present in the measuring volume is assumed to follow the acoustic one (Section 4.3) and can then be written as

$$v(t) = V_{ac}\cos(2\pi f_{ac} t + \varphi_{ac}), \tag{5}$$

where f_{ac}, the acoustic frequency, is assumed to be perfectly known (since it results from a forced excitation), V_{ac} is the magnitude, and φ_{ac} the phase of the acoustic velocity.

If the probability of presence of at least one particle inside the probe volume is significant during some time, the signal delivered by the photomultiplier is found by introducing relation (5) into relation (4):

$$x(t) = A(t)\left(M + \cos\left(2\pi f_0 t + \frac{V_{ac}}{i f_{ac}}\sin(2\pi f_{ac} t + \varphi_{ac})\right)\right). \tag{6}$$

This burst signal is roughly a sine-wave frequency modulation of a carrier f_0. In the case of acoustics, a model of A(t) is unreachable because it depends both on the position of the particle inside the probe volume, and on the magnitude of the velocity. Nevertheless, M and $A(t)$ build together a modulated offset (burst pedestal) which behaves as a low-frequency noise, and thus may easily be filtered out. In the context of the forced excitation of a sinusoidal acoustic wave, the objective of a suitable signal processing is then exclusively to extract V_{ac} and φ_{ac} from the measured signal $x(t)$. As an example, Figure 2 shows a measured burst signal in the case of a 672 Hz sine-wave acoustic excitation, and a 20 mm/s velocity amplitude. The frequency modulation clearly appears, and the amplitude of the signal seems to be correlated with the frequency modulation.

4. DESIGN OF A SET-UP FOR ACOUSTICS PARTICLE VELOCITY MEASUREMENT

4.1. OPTICS AND SUPPORTS

As the velocities to be measured are very small compared to the usual ranges in fluid dynamics, the sensitivity needs to be as high as possible. This may be obtained easily by increasing the angle between incident beams, so that fringes get closer. This also gives a more compact measuring volume, which increases the spatial resolution of the set-up. A drawback is that it necessitates large lenses if used at small distances, and even separate optics for larger ones, as in the case of our installation. Another problem, emphasized by an increased beam angle, is that for measurements close to a boundary, one of the incident beams has to be almost parallel to the surface. The measuring axis is therefore not normal, but tilted of half the beams angle (see Figure 3). If this angle is significant, the measurement of the normal velocity component is perturbed by the tangential one, which may have a much higher amplitude. This led us to select a two-component LDV, in order to be able to measure the velocity in the two directions of a plane perpendicular to the boundary. However, this does not change the main aspects of the signal processing, and thus the following sections of this paper will only consider a single velocity component.

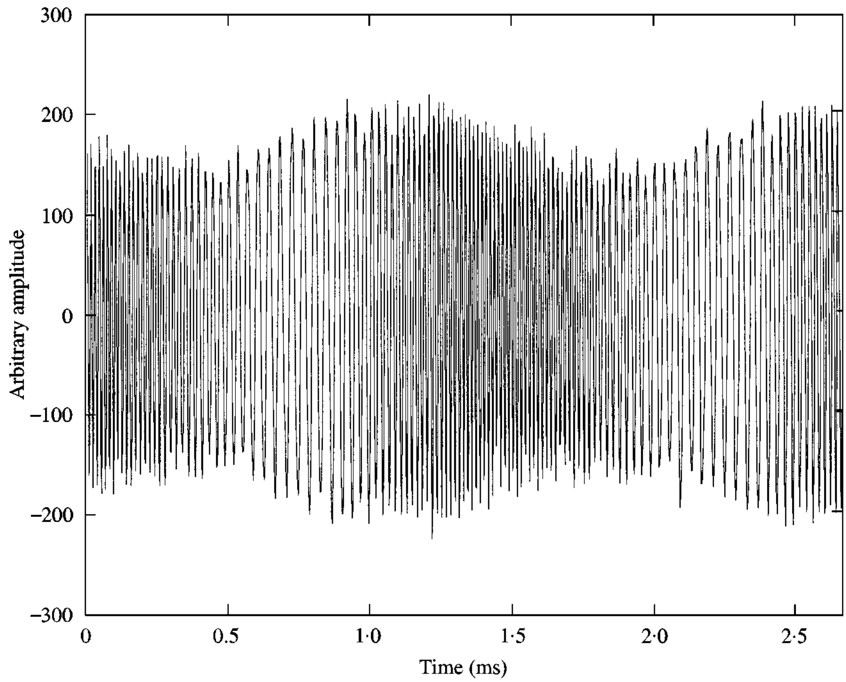

Figure 2. Sample of burst signal in the case of a 672 Hz acoustic sine wave with a 20 mm/s velocity amplitude.

The position of the photomultiplier may be chosen either facing the laser lenses (front scattering), or between them (back scattering). This choice is not obvious: back-scattering leads to a lower intensity of the light signal, thus requiring larger lens for the photomultiplier, or more laser power. Nevertheless, this solution was chosen in order to be able to measure velocities close to non-transparent surfaces, more common in usual acoustic problems. In this case, the angle between the beams cannot be increased too much, unless the back-scattered light becomes too weak. In the present set-up, this angle can be adjusted between about 15 and 80°.

To permit a scanning set-up, fiber optics seemed an evident choice. A supplementary reason for using them was that all noisy equipments had to be removed far away from the measurement zone, especially the laser and its power supply. They were therefore installed in a separate room, which required a fiber length of about 15 m. To be sure to get enough power at the measuring volume, without demanding too large proportion of the laser available power, a 1 W argon laser was then chosen. It does not seem however that this power is really needed for single-component measurements. An argon-ion laser was chosen as it emits mainly three colors, all usable for the simultaneous measurement of separate velocity components, although with decreasing output powers (green at 514·5 mm, blue at 488 nm, and purple at 476·5 nm).

For the validation presented further in this paper, the green color is used with a 30° beam angle, which leads to a fringe spacing $i = 0{\cdot}993\,\mu\text{m}$. The global output power of the laser is set to 100 mW and thus, for the green light, the power at the

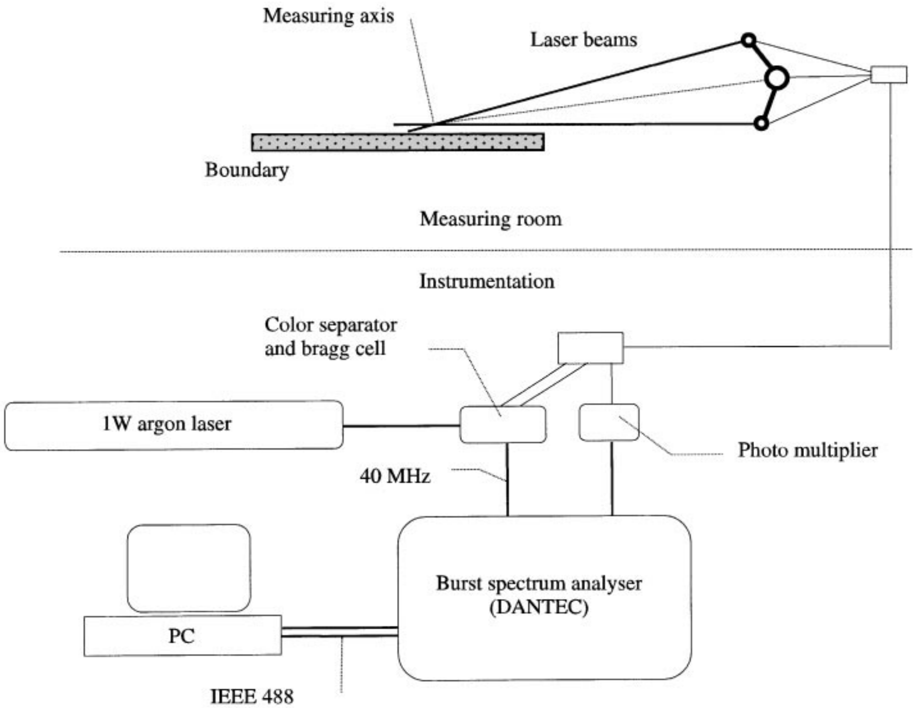

Figure 3. Schematic description of existing LDV set-up (here for a single velocity component).

fiber ends are roughly 20 mW each. For these laboratory measurements, the focal length is 60 cm.

4.2. BURST AND SIGNAL PROCESSOR

As for acoustics the main information is related to the time variation of the Doppler signal phase, or to its instantaneous frequency, only the higher frequencies of the signal spectrum are needed for its processing. For a target velocity comprised between 10 μm/s and 100 mm/s (40–120 sB SPL), the modulation (f_{mod}) around the Bragg frequency (f_0) ranges roughly from 10 Hz to 100 kHz (equation (1)). In order to get a good resolution, the frequency of the burst signal has to be down-shifted as much as possible, but taking into account the range of the frequency modulation ($f_0 + f_{\text{mod}}$), which must remain positive. Moreover, the very low frequencies, mainly created by the pedestal and the term $A(t)$ (equation (6)), have to be removed to preserve a good signal-to-noise ratio, thus limiting again the frequency down-shifting allowed before processing. Indeed, despite a strong high-pass filtering, some low-frequency noise remains. However, assuming a perfect rejection of the unwanted components, the signal to be processed may be expressed as

$$s(t) = A\cos\left(2\pi f_c t + \frac{V_{ac}}{i f_{ac}} \sin(2\pi f_{ac} t + \varphi_{ac})\right), \tag{7}$$

where f_c is the down-shifted carrier frequency ($f_{mod} < f_c \ll f_0$), and A the mean value of the $A(t)$ variation. The form of the signal in equation (7) leads to an equivalent analytical signal [11], obtained using the Hilbert transform

$$z(t) = A \exp j\left(2\pi f_c t + \frac{V_{ac}}{i f_{ac}} \sin(2\pi f_{ac} t + \varphi_{ac})\right). \tag{8}$$

The instantaneous frequency, f_i, is usually defined as

$$f_i(t) = \frac{1}{2\pi} \frac{\mathrm{d}}{\mathrm{d}t} [\arg(z(t))], \tag{9}$$

and can now be deduced from relation (8), leading to

$$f_i(t) = f_c + \frac{V_{ac}}{i} \cos(2\pi f_{ac} t + \varphi_{ac}). \tag{10}$$

As both f_c and i are known, the acoustic velocity can then be obtained from the instantaneous frequency of $z(t)$.

In practice, the processing should be able to provide an estimation of the instantaneous frequency using relatively short windows. This seems to imply either an FFT processor, or a correlator, both with interpolation capability. The unit must also provide the Bragg drive signal and the shifter signal from a single clock, in order to avoid any jitter between them. Lastly, the equipment required should be able to process the results using, as an external time reference, the synchronization output of the sine generator, so that further synchronous demodulation of the acoustic velocity signal remains possible.

A DANTEC Burst Spectrum Analyzer (BSA), based on FFT analysis with interpolation, is therefore used. It also supplies the Bragg cell and the photomultiplier, and is connected through an IEEE488 interface to a PC running the "Burstwar" software, using a "rotating machinery" feature to keep the time reference of the acoustic signal for further averaging. In parallel, the BSA shifter output allows to feed the A/D converter of a post-processing workstation ("Maxion" real-time acquisition system from Concurrent Computer Corporation) in order to develop custom algorithms for signal processing.

4.3. SEEDING

The seeding particles should be small enough to follow the acoustic movement, but large enough to scatter significantly the light. Several studies deal with these aspects, leading to criteria useful for selecting suitable seeding [1].

A specific problem is that there is almost no flow so the quantity of seeding needed is very small, and must be provided at a very low speed to avoid perturbation of the acoustic field. Moreover, the thermodynamic characteristics of the gas should not be modified too much by the presence of the particles. After testing a smoke generator of oil droplets (a modified lightshow unit), a fog generator based on water condensation (aerosol) was preferred, because of the better average size of the particles (1 μm). Nevertheless, in case of high acoustic levels (e.g. when studying non-linear systems), the fog condenses too quickly, and the measurements must be done using the smoke generator.

5. ACOUSTIC SET-UP FOR VALIDATION

The problem of validating LDV for use in acoustics is a difficult one, as there is no reliable method to evaluate the velocity exactly at the beams crossing point. In order to perform the validation, a cylindrical tube, for which an analytical description of the field may be derived with some accuracy, is used. Thus, the initial set-up is composed of a cylindrical tube made of PVC, except for the measuring section which is made of calibrated glass. The internal diameter of the tube is about 45 mm, so that the field is assumed to be described by plane waves up to 2–3 kHz (the eigenfrequency associated to the first next mode is about 4500 Hz). The measuring section is about 60 cm long, and two microphones permit to measure the acoustic pressure on the wall at its extremities (Figure 4). The field is created by a loudspeaker situated about 60 cm away from the measuring section. Seeding is introduced near the loudspeaker, and goes out at the other side of the measuring section, again 60 cm away. These distances are kept so that local perturbations related to the seeding inlets would not create significant evanescent modes throughout the measuring section. This set-up may be loaded by a 1 m wedge of absorbing material, so that the field is almost propagative, or closed by a rigid wall to get standing waves. This last case is interesting in order to get a larger dynamic range for the acoustic quantities, as standing waves permit, by selecting a suitable location along the tube, to get either a large velocity without stressing too much the loudspeaker, or a low velocity while keeping a good signal-to-noise ratio. The experiments described thereafter were thus conducted in this closed end configuration.

The beam optics are adjusted so that the beams plane includes the set-up axis, and that they cross almost exactly at the center. This way, the LDV measurement is supposed to give the plane waves velocity v, which can be calculated from the two measured pressures p_1 and p_2 using the following relation:

$$v(x) = \frac{1}{\xi \sinh(\Gamma \ell)} (p_1 \cosh(\Gamma x) - p_2 \cosh(\Gamma(\ell - x))), \tag{11}$$

where Γ is the complex propagation constant in the tube ($j\omega/c$ for a lossless fluid), ξ the characteristic impedance of the tube, ℓ the distance between the two microphones, and x the distance from the first microphone.

As the measured pressures are taken at the boundaries of the tube, any evanescent mode superimposed on the plane waves may lead to significant errors on the velocity estimation. The microphones themselves are a source of such coupling, so great care must be taken to respect the boundary condition at their position: they must be carefully flush mounted, without any leakage. The microphone capsules used for the set-up are electret ones, with a diameter of about 5 mm and an equivalent volume small enough to avoid significant perturbation. Their signals are amplified by custom preamplifiers which feed the inputs of two

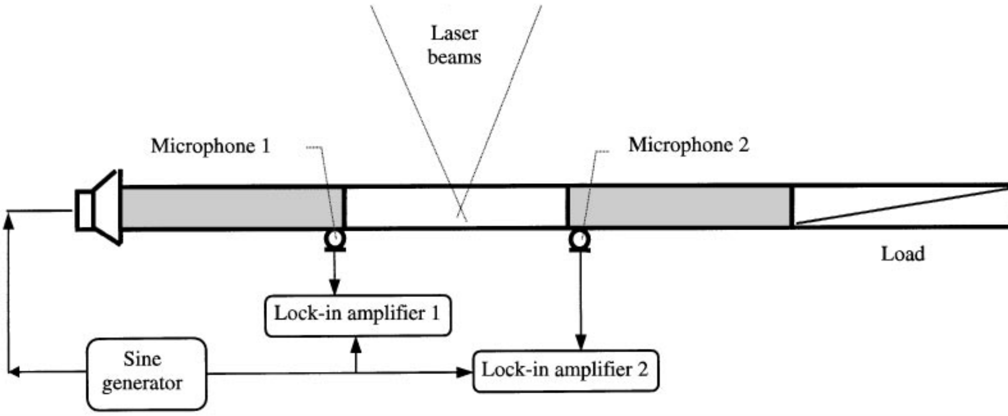

Figure 4. Schematic of acoustic set-up used for validating the LDV measurements.

lock-in amplifiers whose phase references are directly taken from the sine generator supplying the loudspeaker.

The relative calibration of the two microphones was performed using a reference source at 1 kHz. A full absolute calibration was not performed for the current tests, as this was not considered necessary to provide initial comparisons.

6. EXTENSIONS OF SIGNAL PROCESSING

6.1. SIGNAL PROCESSING STRATEGY

Two different cases must be considered for the signal processing, depending on the magnitudes of the particles displacement and velocity.

In the case of high velocities and/or low frequencies, the particles displacement is larger than the probe volume, and the signal is a succession of bursts, randomly separated in time by “drop out” intervals when all particles are flowing outside the measuring volume. As the Gaussian envelope of the individual bursts realizes a natural time window, and the instantaneous frequency is varying slowly compared to the window duration, the signal processing may be performed by using classical spectral estimation. Fourier transform, actually done by the BSA, zero-crossing detection, or correlation, could be adopted for such cases. This kind of method, however, requires a high data rate (number of bursts per second) to allow a good estimation of the instantaneous frequency, and are therefore suitable in the context of fluid mechanics, aero-acoustics or high level acoustics.

In the second case, when a particle excursion is small enough so that it stays during several acoustic periods inside the probe volume, the signal processing can be applied to a long, uniformly sampled sequence of data starting just after the burst detection, and whose duration may be very long if no convection effect removes the particles from the measuring volume. As shown in the next sections, time-frequency techniques can therefore be applied to the burst signal, in order to extract continuously the instantaneous frequency versus time.

6.2. PRINCIPLE OF EXISTING PROCESSOR/SOFTWARE

For the time being, the burst processing is provided by the BSA itself, as stated above. This processing consists of FFT spectra calculated from separate windows of data acquired when bursts are detected. Each of these spectra is searched for an emerging frequency component, and some criteria are applied to check that these may correspond to the "instantaneous" frequency linked to a velocity component. If these criteria are verified, the velocity is computed using constants related to the optics. The velocity data are then transferred to the PC, together with the time of arrival referenced to the last clock signal (using "rotating machinery" mode). This way, the software can accumulate several periods of the audio signal on a single graph, which reconstitutes the shape of the velocity sine wave over the many bursts involved. All these events are also distributed into discrete classes of arrival time, and averaged over each of these classes, leading to a single, evenly sampled, period of signal suitable for further processing, e.g. by synchronous demodulation. A drawback of this method is that it averages together a random number of samples not corresponding exactly to the same instant in the period, thus leading to an estimation error increasing as the number of classes decreases.

6.3. IMPROVED PROCESSING USING CONVENTIONAL HARDWARE

Our first effort aimed at reducing such errors in order to get the best from the existing equipment. A least-squares method has therefore been developed to extract the amplitude and phase of the sine wave directly from the data sent by the BSA, including the "exact" (randomly spaced) arrival time. In the present implementation however, the "Burstwar" software was used to process the data for validation, and to export them as ASCII files. This last method suffers from the fact that the actual velocity is usually not a pure sine wave, e.g., because of distortions occurring at the source or during the propagation. As it has no way to get rid of eventual harmonics, this estimator of the fundamental component might be biased for distorted signals. We did not however notice significant problems during our experiments.

A more severe limitation occurs when the acoustic velocity varies rapidly with time. As the BSA searches for a spectrum maximum, it requires the velocity to be almost constant during one window, and does not track the actual velocity evolution within the window duration. In fact, the BSA can be seen as an instantaneous frequency detector based on a windowed Fourier transform, with no overlap between windows. This transform is not optimum for varying signal, which suggests to look at other time-frequency tools to build a more suited detector.

6.4. DIRECT TIME-FREQUENCY ANALYSIS: THE CROSS-WIGNER-VILLE DETECTOR

To be able to test other processings, the shifted signal from the BSA was used and fed to the acquisition workstation described above. Different algorithms were tested, first on simulated signals, and then on a few samples of real bursts [12].

Among various methods, the most suited was found to be the Cross-Wigner–Ville detector (CWV), which is based on a modified kernel for the Wigner–Ville transform. To build this detector, we start from the analytic signal $z(t)$, (equation (8)), constructed from the real signal from the A/D converter and completed by an imaginary part obtained by Hilbert transform. The instantaneous frequency (IF) is detected using an iterative algorithm, based on three steps, as shown by Boashash and O'Shea [13]:

(1) An initial value of the IF, f_{i0}, is set and an unit amplitude reference windowed signal, $r(t)$, is built as a linear sweep:

$$r(t) = h(t)\exp\left(\mathrm{j}2\pi\left(f_{i0} + \frac{\alpha^2}{2}t\right)\right),$$

where $h(t)$ is a window centered on $t = 0$ and α the slope of the frequency sweep. f_{i0} has to be as close as possible to the IF in order to obtain fast convergence, so we use the value estimated during the previous window.

(2) The CWV transformation is calculated between the reference and the windowed signal $z(t)$, as

$$CWV(t, f) = \int_{-\infty}^{+\infty} z(t+\tau)r^*(t-\tau)\mathrm{e}^{-\mathrm{j}2\pi f\tau}\,\mathrm{d}\tau \tag{12}$$

and a peak detection determines a new value of the current estimated IF.

(3) The procedure is repeated from step (1), supplying the new value of IF into the kernel of the CVW, until the resulting IF estimate stays close to the "kernel" value. This last resulting value is then taken as the IF of $z(t)$.

This detector could extract successfully simulated acoustic signals, even with low signal/noise ratios. It proved to be the best detector in the case of short windows, but as it assumes a linear frequency law during the window analysis, it is still limited to signals with a low modulation index.

6.5. LIMITS OF THE DIRECT ANALYSIS PRINCIPLE

The limits of the CVW estimation presented above are in fact common to all methods having no assumption about the long-term properties of the signal. As the acoustic frequency increases, the number of points inside a period decreases for a given sampling frequency. It is therefore necessary to decrease the number of windows per acoustic period to keep a constant frequency resolution for each of them, or to keep this number constant by reducing the window length, leading to a degraded resolution. Obviously, a limit occurs when increasing the frequency or decreasing the velocity.

The limit can be expressed from the following simple considerations: to be able to evaluate correctly the sine wave parameters, one needs at least two values per period (Shannon theorem). The minimum detectable velocity δ_v can then be expressed as a function of the acoustic frequency f_{ac} from the uncertainty principle given in a general case by

$$\sigma_t \sigma_f \geqslant \frac{1}{4\pi}, \tag{13}$$

where σ_t is the temporal spread of the signal and σ_f the frequency resolution.

In order to respect the Shannon criteria, $\sigma_t = 1/2f_{ac}$ is the minimum window length. On the other hand, σ_f, the resolution, can be related to the minimum velocity δ_v as (equation (1))

$$\sigma_f = \frac{\delta_v}{i} = \frac{\delta_v \sin(\theta/2)}{\lambda/2}. \tag{14}$$

As a result, the minimum velocity and the acoustic frequency can be related as follows:

$$\delta_v \geqslant \frac{\lambda}{4\pi \sin(\theta/2)} f_{ac}. \tag{15}$$

This last relation should only be taken as a rough estimate showing the limits of a processor having no *a priori* knowledge of the periodic nature of the acoustic signal.

7. NEW SIGNAL PROCESSING APPROACH

7.1. ADAPTED KERNEL TRANSFORMATION

A further development in the processing of burst signals is to take into account the known analytic form of the velocity. A different method which evaluates the parameters of the sine wave directly from the burst signal has therefore been developed, allowing one to use longer windows by assuming a suited form for the time-frequency law [14]. The so-called time-frequency synchronous detector (TFSD) is thus written as

$$\mathrm{TFSD}(t, f) = \int_{-\infty}^{\infty} (s(t+\tau)s^*(t-\tau))^{K2\pi f_{ac}\tau \sin(2\pi f_{ac}\tau)}$$
$$\times \left(s\left(t+\tau+\frac{1}{4f_{ac}}\right) s^*\left(t-\tau-\frac{1}{4f_{ac}}\right)\right)^{K2\pi f_{ac}\tau \cos(2\pi f_{ac}\tau)} \mathrm{e}^{j2\pi f\tau}\, \mathrm{d}\tau. \tag{16}$$

When the input signal is

$$s(t) = \mathrm{e}^{\,\mathrm{j}(V_{ac}/i f_{ac})\sin(2\pi f_{ac}t + \varphi_{ac})}, \tag{17}$$

it can be proved that equation (16) gives the following result:

$$\delta(f - KV_{ac}\cos(2\pi f_{ac}t + \varphi_{ac})). \tag{18}$$

The arbitrary constant K is set in order to be able to improve the dynamic range of the measurement. Expression (18) is the exact formulation of the IF law in the time-frequency plane. A peak detection thus leads to the estimated signal

$$\tilde{v}(t) = KV_{ac}\cos(2\pi f_{ac}t + \varphi_{ac}), \tag{19}$$

which is proportional to the acoustic velocity. These last relations do not take into account the shifting of the acoustic frequency resulting from the BSA processing (see section 4.2, equation (7)). The analytical signal provided to the TFSD algorithm (equation (17)) must therefore be built by shifting the BSA output back to 0 Hz, using a carrier frequency f_c corresponding to the difference of the Bragg and shifter frequencies used inside the BSA.

The estimation of the magnitude and the phase of the velocity from the BSA shifter output can then be described as the four following steps:

(1) The analytical signal is calculated from the sampled BSA shifter output.
(2) It is shifted down to zero frequency (using carrier frequency f_c).
(3) Expression (16) is used to get frequency values at evenly distribute intervals.
(4) A synchronous demodulation extracts the magnitude and phase of the obtained IF law.

In step (3), a moving window is used, the center of which is the time reference for the estimation of the instantaneous velocity, and over which are performed the following three operations:

— the real exponents of equation (16) are computed using a value for K depending on an estimation of the velocity amplitude (for a large velocity K is set to one, and it must be increased when velocity becomes smaller),
— expression (16) is calculated for this window,
— the corresponding velocity is obtained by a peak detection.

As this synchronous transformation uses an analytical model of the signal, it is theoretically independent of the window length. The calculation can therefore be made over several acoustic periods, thus permitting a better detection in the case of a low signal to noise ratio.

The main drawback of this method is the need for the phase of the analytical signal, used for calculating the real exponents of the equation. Phase unwrapping algorithms have to be used despite their sensitivity to noise. Nevertheless, our experience with the processing of real data is that the IF extraction is still robust even when phase unwrapping errors occur.

8. RESULTS

8.1. DESCRIPTION OF THE VALIDATED RESULTS OBTAINED USING BSA AND CWV

The first experiments were aimed at comparing the velocities deduced from the microphone measurements, and those obtained by post-processing the output from the Burst Spectrum Analyzer of DANTEC through the use of the CWV, as described in sections 5 and 6. A summary of the results at various frequencies and levels is shown in Figure 5. The black points represent "validated results", i.e., conditions for which a good agreement was obtained between the LDA measurement and the velocity deduced from the two pressures measured in the

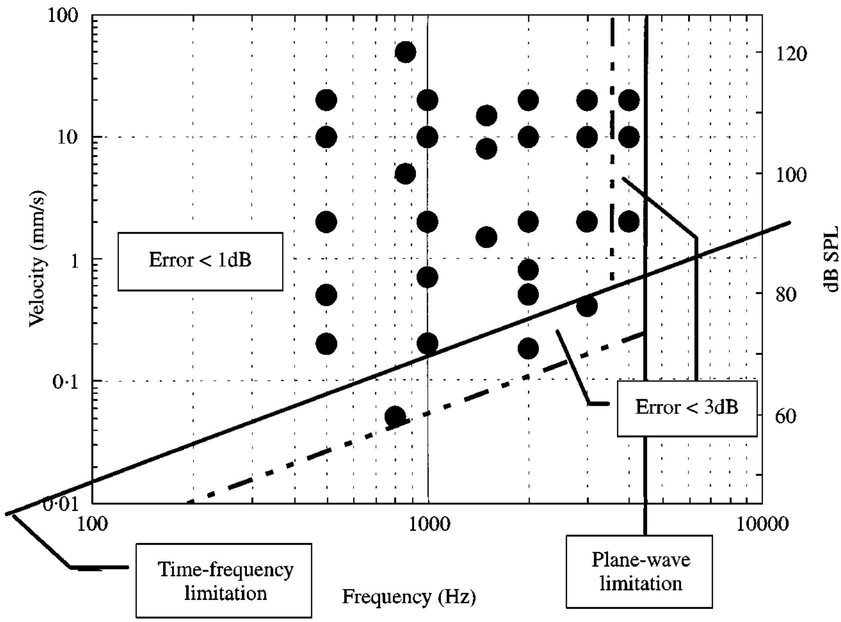

Figure 5. Frequency-level map showing the comparison between LDV and pressure measurements. The black points represent measurements showing a good agreement between both results.

guide. The velocity magnitude is expressed as an equivalent pressure level in free field (SPL dB), in order to facilitate the comparison with more usual measurements. Although we only mentioned our own results in Figure 5, it must be emphasized that several authors [8, 15] have also used LDV, and their measurements occurred in velocity and frequency ranges which are consistent with the analysis in the next section, deduced from our measurements.

In the experimental set-up, the transverse dimension of the acoustic waveguide leads to a cut-off frequency for the first transverse mode around 4500 Hz. The theoretical expression (equation (11)) used to deduce the velocity from the two pressure measurements assumes a single plane mode inside the guide. This expression is therefore no longer valid when the second mode may be present, that is for frequency close to or above its cut-off frequency. From a practical standpoint, we assumed this limit to be about 3000 Hz, which is the reason for the vertical dotted line in Figure 5; the vertical solid line corresponds to the theoretical cut-off frequency of the second (first non-planar) mode in the waveguide. Still in Figure 5, the solid line increasing with frequency displays the limitation estimated from the time-frequency incertitude expressed by equation (15).

The upper-left part of the figure contains points for which the two measurements agree within 1 dB. For the points included between the two solid lines and the nearby dotted ones, the discrepancy between the two measurements (LDV and microphone measurements) was less than 3 dB. No satisfactory comparison has been done for the conditions concerned by the lower-right areas of the figure, beyond the two dotted and solid lines. These two simple boundaries therefore seem to define a validity domain for our LDV measurements. However, one should keep in mind that the discrepancies occurring for frequencies above 3 kHz may occur either because of the LDV itself, or because of the limitations of the experimental set-up. On the other hand, one may suspect that expression (15) gives a good evaluation of the limitation of the measurement by LDV at low levels, as the agreement between the two estimations of the velocity degrades rapidly for points of the map situated below this line, while the pressure measurements are still valid.

The performances of the present "classical" LDV set-up are already sufficient for some applications involving high-velocity levels, as experiments in non-linear acoustics applied to thermoacoustics [10, 16], or propagation in waveguides, for which investigations are in progress. Considering plane waves with pressure levels ranging between 60 and 120 dB SPL (good usual laboratory conditions), Figure 5 however shows that only the low-frequency band up to roughly 500 Hz is accurately estimated by such LDV velocity measurements. On the other hand, most applications in the field of audible acoustics would require a frequency range extending up to at least 5 kHz. Additional efforts are therefore required to measure lower velocity levels, especially between 500 and 5000 Hz. This emphasizes the necessity for more specific signal processing, as the one proposed in Section 7.

8.2. RESULTS OBTAINED USING THE TIME-FREQUENCY SYNCHRONOUS DETECTOR APPROACH

A comparison is now made between the extraction of the IF by means of the CVW and the TFSD, both used as a post-processing of the BSA shifter output, as previously described. The experimental acoustic set-up is the same closed tube as described in section 5. Figures 6(a) and (b) are the extracted IF from the CWV and from the TFSD, respectively, for a 3000 Hz acoustic velocity of about 2·5 mm/s. Assuming that the actual acoustic velocity is a sine wave, the results obtained from TFSD seems to follow the IF law more accurately than those from CVW, despite of important low frequency disturbances which might be related to an unsteady air flow throughout the measuring volume. At first glance, this seems to indicate that the TFSD is less sensitive to noise, probably because it makes use of larger windows than the CVW. Figures 7(a) and (b) describe the same experiment, but with a lower velocity of approximately 0·5 mm/s, for which the CWV leads to a very poor estimation of the velocity. On the other hand, the TFSD still detects a visible sine wave at the excitation frequency, even if the resulting signal is corrupted with a significant level of noise. In both cases, a further synchronous demodulation can still extract the velocity amplitude and phase, although the accuracy of this estimation is quite poor when using the CVW.

The next experiment consisted of scanning one-half wavelength along the axis of the closed acoustic guide, the working frequency being about 3000 Hz. Figures 8(a), (b) and (c) compare the velocities deduced from the acoustic pressures (dotted line) to those estimated by the BSA, the CVW, and the TFSD respectively (solid lines). For the two last methods, the measurements have required fewer seeding than for the BSA measurement.

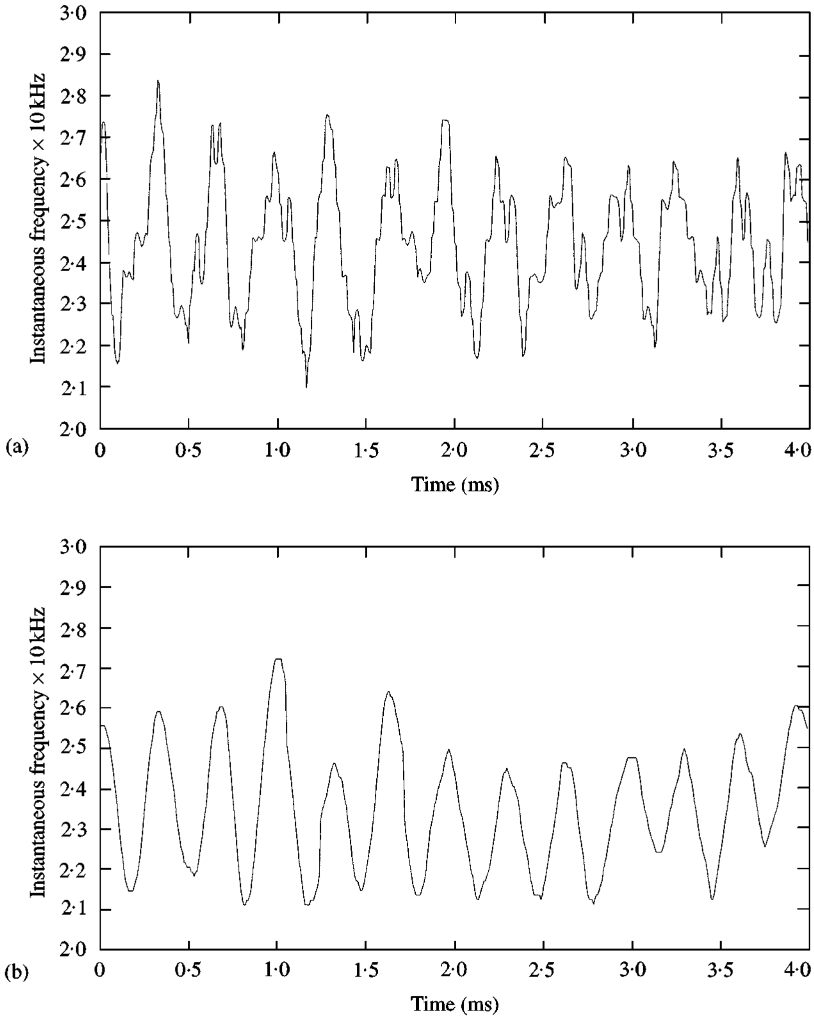

Figure 6. (a) IF extraction from the burst signal, for a 3000 Hz acoustic sine wave at 2·5 mm/s, by means of CWV technique. (b) IF extraction from the burst signal, for a 3000 Hz acoustic sine wave at 2·5 mm/s, by means of TFSD technique.

All three LDV measurements show a systematic position shift compared to the velocity estimated from pressures. Although we have no evidence for that, we suspect that it could be caused by a calibration error of the two microphones (phase mismatch). Compared to other methods, the BSA estimation also shows a slight underestimation of the velocity magnitude, but this is hardly significant as the results obtained until now are not numerous enough.

In all these comparisons between BSA and other processings, we noticed that the ideal amount of seeding was lower for the CVW and the TFSD post-processing, than for the BSA alone. This fact is very positive for acoustics, as the presence of particles seems to change significantly the properties of the medium.

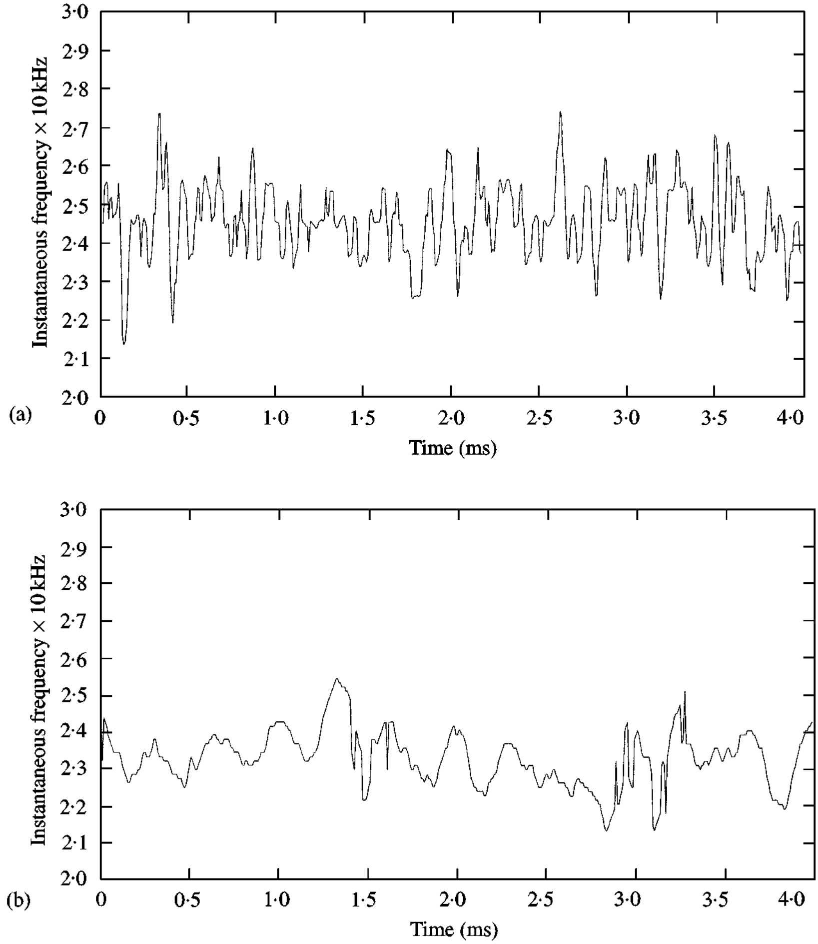

Figure 7. (a) IF extraction from the burst signal, for a 3000 Hz acoustic sine wave at 0·5 mm/s, by means of CWV technique. (b) IF extraction from the burst signal, for a 3000 Hz acoustic sine wave at 0·5 mm/s, by means of TFSD technique.

Simulations, carried out on a larger frequency range, have shown that the TFSD has the potential to estimate acoustic velocities up to 10 kHz with an accuracy better than 5%. This indicates that the synchronous approach is an efficient way to extract the velocity from LDV signals. Nevertheless, the tracking at lower levels remains difficult, except when using higher values for the K parameter in equation (16). A better tuning of this parameter could thus increase the dynamic range of the detection method, whose properties are currently being investigated more deeply.

9. CONCLUSION

This work has proved the ability of commercial equipment to achieve acoustic velocity measurements with a dynamic range comprised between 60 and 120 dB SPL for plane waves, and has validated it for the 50–500 Hz frequency range.

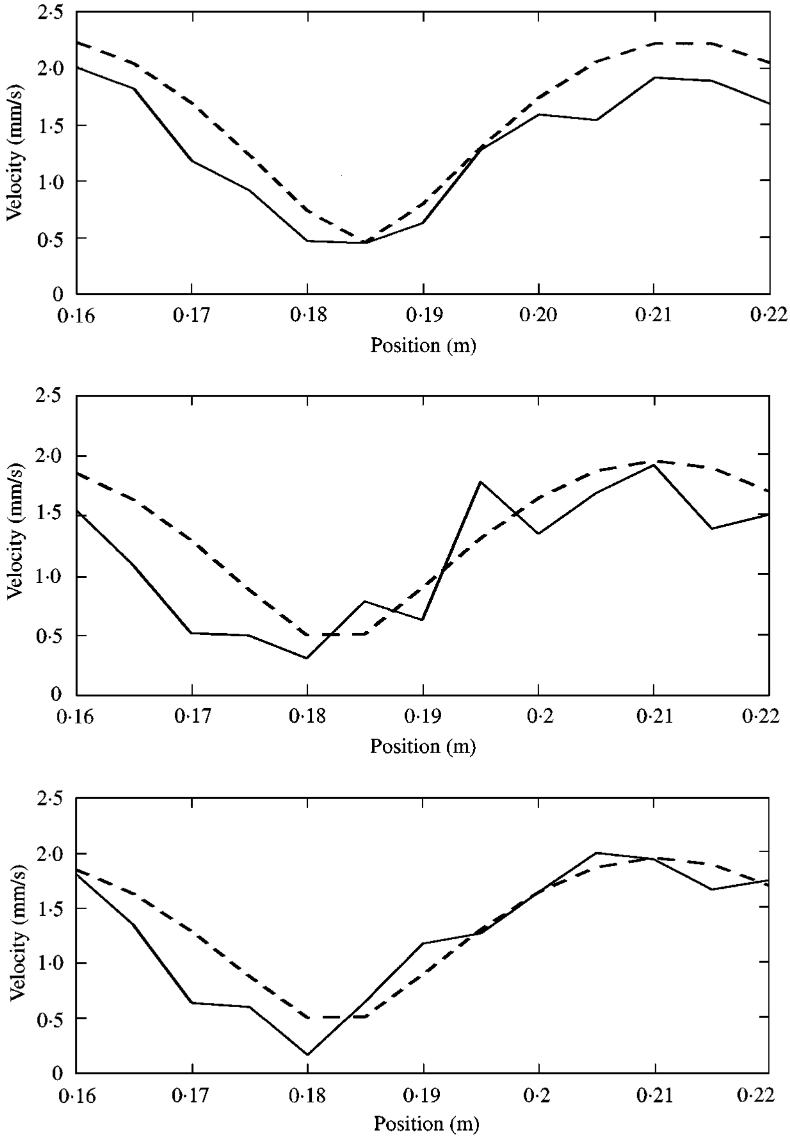

Figure 8. (a) Measurement of velocity along a closed guide using the BSA (solid line) or acoustic signals (dashed line). (b) Measurement of velocity along a closed guide using the CWV (solid line) or acoustic signals (dashed line). (c) Measurement of velocity along a closed guide using the TFSD (solid line) or acoustic signals (dashed line).

Moreover, measurements have been found to be feasible at least up to 3000 Hz, which is the limit of our experimental set-up, but only for higher acoustic levels.

This paper also shows that there seem to be a theoretical limit for the signal processing tools based on conventional time-frequency analysis, as the velocity resolution achievable by such direct analysis would reach a limit proportional to the acoustic frequency.

Proposed methods, introducing a synchronization with the acoustic signal, are likely to improve both the minimum reachable velocity, and the maximum frequency, by allowing a longer term estimation of the velocity parameters. However, this has to be better validated by further measurements, and will require reference acoustic measurements less dependent on microphones calibration. Anyway, these methods already permit to use fewer seeding, thus limiting the perturbation of the acoustic field.

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