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EIGENVALUES OF RADIALLY SYMMETRIC MODES IN COMPOSITE SPHERICAL DOMAINS WITH A VERY SMALL CONCENTRIC CAVITY

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1. INTRODUCTION

Recently, and in a very ingenious manner, Wang has proved that the fundamental frequency coefficient of a circular annular membrane fixed at the outer radius "b" and at the inner radius "a", is the same eigenvalue as in the case of a solid circular membrane, when the inner radius of the annular membrane approaches zero [1]. Subsequently, it was shown by Laura and coworkers that the same rather unexpected conclusion holds true in the case of higher modes of vibrations [2, 3] and also in the case of composite membranes [4].

It is shown in the present study that, from a mathematical viewpoint, the same property holds when solving a Helmholtz differential — type system in the case of composite spherical domain when $a/c \to 0$, Figure 1.

2. GOVERNING DIFFERENTIAL SYSTEM

Referring to Figure 1 the following differential system will be considered, for the sake of generality:

Domain I ($a \leqslant r \leqslant b$):

$$\frac{d^2\psi_1}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\psi_1}{\mathrm{d}r} + \frac{\beta^2}{\delta_1}\psi_1 = 0. \tag{1}$$

Domain II ($b \leqslant r \leqslant c$):

$$\frac{\mathrm{d}^2\psi^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\psi_2}{\mathrm{d}r} + \frac{\beta^2}{\delta_2}\psi_2 = 0. \tag{2}$$

Boundary conditions:

$$\psi_1(a, t) = 0, \qquad \psi_1(b, t) = \psi_2(b, t),$$

$$k_1\frac{\mathrm{d}\psi_1}{\mathrm{d}r}(b, t) = k_2\frac{\mathrm{d}\psi_2}{\mathrm{d}r}(b, t), \qquad \psi_2(c, t) = 0, \tag{3}$$

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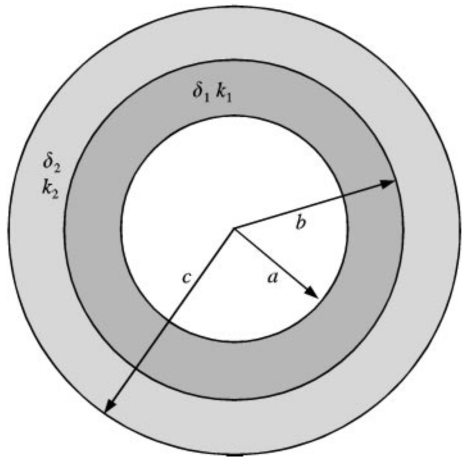

Figure 1. Composite, hollow, spherical domain.

TABLE 1

Variation of the eigenvalues Ω_n ($n = 1, 2, \ldots, 5$) as the inner cavity radius decreases in magnitude

a/c \ n	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	Solid composite sphere
1	6·42121	5·54464	5·46617	5·45842	5·45765	5·45756
2	9·94306	9·11525	9·04594	9·03908	9·03840	9·03832
3	15·38789	13·04455	12·85540	12·83689	12·83504	12·83483
4	20·81861	18·30155	18·04621	18·02084	18·01830	18·01802
5	24·70465	22·10779	21·91482	21·89572	21·89381	21·89360

where δ_i and k_i denote the system properties and β^2: eigenvalues of the configuration.

The solutions of equations (1) and (2) are

$$\psi_i(r) = \frac{A_i}{r}\sin\frac{\beta}{\sqrt{\delta_i}}r + \frac{B_i}{r}\cos\frac{\beta}{\sqrt{\delta_i}}r, \qquad (i = 1, 2). \tag{4}$$

It is convenient to define the eigenvalues in terms of a dimensionless parameter

$$\Omega = \frac{\beta b}{\sqrt{\delta_1}}.$$

3. NUMERICAL RESULTS

The numerical determinations have been greatly facilitated by the use of MATHEMATICA [5].

Table 1 depicts the first five eigenvalues for a particular mechanical configuration defined by $k_2/k_1 = 1{\cdot}4$; $\delta_2/\delta_1 = 5$ and $b/c = 0{\cdot}5$, as a function of the geometric ratio a/c. One observes that as a/c decreases in magnitude one approaches the eigenvalues of a solid composite spherical configuration (last column at the right). For $a/c = 10^{-5}$, at least four significant figures concide and for $n = 5$ the difference is of the order of 0·001%.

These mathematical conclusions are of interest in the classical diffusion theory* but are potentially applicable in some vibrational models of continuous media.

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*Certainly, when dealing with an unsteady thermoelastic situation, the presence of a cavity of small radius will generate severe stress concentration.