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FREE VIBRATION OF UNIDIRECTIONAL FIBER REINFORCEMENT COMPOSITE ROTOR

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1. INTRODUCTION

Composites are widely used because they have desirable properties, which could not be achieved by either of the constituent materials acting alone. The reduction in weight is a primary criterion, so it is used in several structural components. The most common example for the composite material is reinforcing fibers embedded in a matrix. This form is effective because many materials are much stronger and stiffer in fiber form than in bulk form. E-glass, graphite and carbon are the most widely used advanced fibers. The case of a rotor-bearing system made of composite material is of considerable importance in several situations of engineering practice. Flexible rotor-bearing systems have been analyzed by many different mathematical methods [1–5]. The utilization of finite element models such as that introduced by Archer [2] in the area of rotor dynamics has yielded highly successful results. Several analysis techniques have been used in the past to predict the dynamic behavior of rotor systems. An approach that is highly suitable for modelling a complex rotor system is the finite element method [6, 7]. However, for large systems, the overall system matrices are quite large, which requires large computer memory.

The aim of this paper is to provide additional information on the use of composite rotors made of unidirectional fiber reinforcement composite materials, and this is supported by experimental results.

A comparative study on the free vibration characteristics between the unidirectional fiber reinforcement composite and traditional material rotors is presented. It is observed that the composite rotor system may give higher natural frequencies than those made of conventional materials due to the higher stiffness to mass ratio. Four composite materials are considered in this paper, graphite/epoxy, carbon/epoxy, boron/epoxy and E-glass/epoxy, for the fabrication of the rotor system. Typical design data of different composite materials for the first three natural frequencies are presented. Experimental work has been carried out on a composite rotor made of E-glass/epoxy with fiber volume fraction ($V_f = 0{\cdot}43$) having one steel disk at the mid-span. The system is supported by two ball bearings. A comparison between the theoretical results and experimental ones is presented.

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2. MATHEMATICAL FORMULATION

Figure 1 shows the schematic diagram of the simple composite rotor-bearing system under consideration. This rotor has a uniform circular shaft section with a rigid disk mounted at the center. This rotor system is discretized into two, first-divided, components, namely A and B. Then, these first-divided components are again divided into two second-divided components, namely A_1 and A_2 of component A, B_1 and B_2 of component B as shown in Figure 2 [8]. Component A is analyzed by using the described segments A_1 and A_2 and this discretization results in three nodal points with four degrees of freedom at each node point (two translations in the XZ and XY planes and the corresponding rotations). The generalized displacement vector $[q]$ is related by a modal transformation to the co-ordinate system vector $[\eta]$, the elements of which are the modal co-ordinates for the individual first-divided component modes. The details of this procedure are described in reference [8].

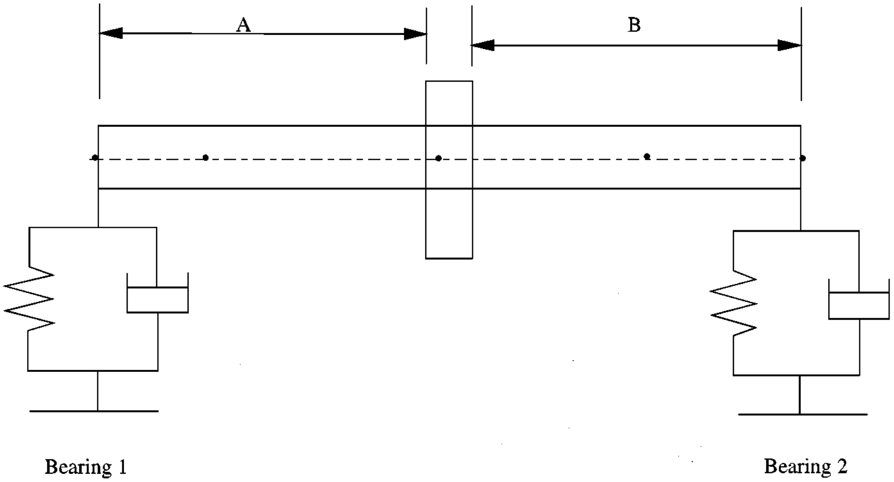

Figure 1. The mathematical model.

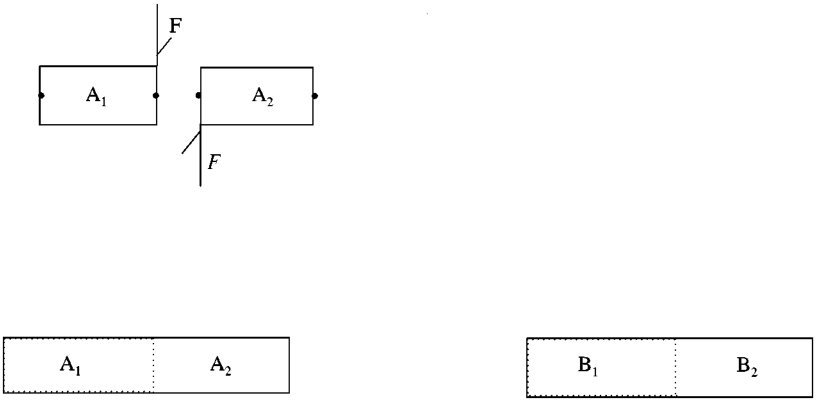

Figure 2. The divided sub-component of the rotor.

The total structural matrices are determined by assembly of the stiffness and mass matrices and force vectors of component A, and the corresponding matrices of component B. The structure equation of motion is obtained as

$$[M]\lceil\ddot{\eta}\rceil + [K]\lceil\eta\rceil = \lceil 0\rceil. \tag{1}$$

A computer program based on the procedure described in reference [8], is used to obtain the natural frequencies for the composite rotor-bearing system.

3. EXPERIMENTAL INVESTIGATION

In the study presented here, a unidirectional E-glass fiber-reinforced epoxy resin composite rotor shaft was tested. The type of epoxy resin was clear light liquid (CY-205) and the hardener clear liquid (HT-951). The epoxy resin was mixed with the hardener (10:1) by weight. The fiber volume fraction of the composite is 0·43. The composite shaft was molded by a plastic tube (0·02 m diameter). The casting was cured at room temperature.

For unidirectional fiber-reinforced composite materials, according to the model presented in reference [9, 10], the results for the longitudinal Young's modulus and the densities of the composite materials (E_{11} and ρ_{11}, respectively), may be written as

$$\rho_{11} = \rho_f V_f + \rho_m V_m, \tag{2}$$

$$E_{11} = (E_f V_f + E_m V_m) + \{4 V_f V_m (\nu_f - \nu_m)^2 / (V_m/K_f + V_f/K_m + 1/G_m)\}, \tag{3}$$

where $K_f = E_f/3(1 - 2\nu_f)$, $K_m = E_m/3(1 - 2\nu_m)$, and E_f, V_f, ν_f, and K_f are the elastic modulus, volume fraction, the Poisson ratio and plane strain bulk modulus for the fiber material, respectively, E_m, V_m, ν_m, and K_m are the elastic modulus, volume fraction, and the Poisson ratio and strain bulk modulus for the matrix material, respectively, and G_m is the shear modulus for the matrix.

Experimental work was carried out to verify the theoretical model. The rotor consists of one disk from St. 37, diameter 0·15 m, and thickness 0·02 m. A composite material shaft is made from E-glass/Epoxy, with mechanical properties and dimensions, $E_{11} = 33{\cdot}3$ GPa, $\rho_{11} = 1773{\cdot}74\ \mathrm{kg/m^3}$, diameter 0·02 m, and length 0·48 m. The fiber volume fraction for the shaft is $V_f = 0{\cdot}423$. The rotor was excited by an impact hammer through a relatively soft cap. The response signal was measured by an accelerometer. The frequency response signal was obtained by using a dual-channel analyzer.

Figure 3, shows the experimental frequency response function for rotor under test. The stiffnesses of the bearings are considered to be 44E6 N/m [11]. The experimental natural frequencies are compared with theoretical ones, as shown in Table 1.

From the results of Table 1, there is a good agreement between the theoretical and the experimental results, within an error of approximately 6%, which verifies the mathematical model.

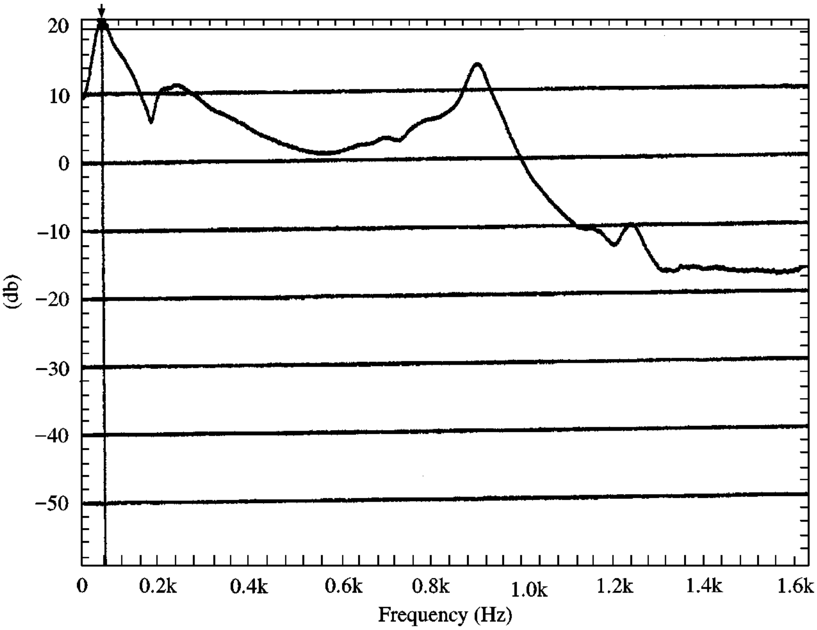

Figure 3. Frequency response function (steel disk and E-glass/epoxy shaft).

TABLE 1

The theoretical and the experimental natural frequencies (steel disk and E-glass/epoxy *shaft)*

	F_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_4 (Hz)
Theoretical	45	200	874	1208
Experimental	42	190	870	1200

The verified mathematical model was used to obtain the natural frequencies for composite rotors consisting of disk and a shaft having the same composite material. Table 2 shows the percentage change in the natural frequencies with respect to the steel values. Table 3 presents the natural frequencies of a composite rotor (disk and shaft have different materials). The two tables give the designer a wide range for the natural frequencies of rotors having the same dimensions with different composite materials. The dimensions are the same as those of the tested rotor; only the length of the shaft becomes 0·7 m.

The minus or plus signs indicate the decrease or increase percentage in the natural frequencies with respect to steel. It is observed that the percentage is almost the same for the first three natural frequencies for each material.

TABLE 2

The natural frequencies (Hz) *for a rotor* (*shaft and disk having the same material*), *and its percentage change with respect to the steel*

Material	f_1	%	f_2	%	f_3	%
Steel	42·2	0	276·7	0	576·7	0
E-glass/epoxy	18·5	− 56	122·2	− 55·8	258·3	− 55·2
Boron/epoxy	60·8	+ 44	400	+ 44·5	838·3	+ 45·3
Carbon/epoxy	68·3	+ 62	448·3	+ 62	941·7	+ 63·2
Graphite/epoxy	73·5	+ 74	475	+ 71·6	1000	+ 73·4

TABLE 3

The natural frequencies (Hz) *for a rotor* (*shaft and disk having different composite materials*)

Disk	Shaft	Symbol	f_1	f_2	f_3
E-glass/epoxy	Boron/epoxy	Eg/Bor	61·7	401·7	840
Graphite/epoxy	Boron/epoxy	Gr/Bor	63·3	408·3	845
Boron/epoxy	Carbon/epoxy	Bor/Cr	65·5	440	936·7
E-glass/epoxy	Carbon/epoxy	Eg/Cr	66·3	443·3	938·3
Graphite/epoxy	Carbon/epoxy	Gr/Cr	69	450	943·3
E-glass/epoxy	Graphite/epoxy	Eg/Cr	70	468	995
Boron/epoxy	E-glass/epoxy	Bor/Eg	18·3	121·5	256·7
Boron/epoxy	Graphite/epoxy	Bor/Gr	68·7	465	933·3
Carbon/epoxy	Boron/epoxy	Cr/Bor	63·3	406·7	845
Carbon/epoxy	E-glass/epoxy	Cr/Eg	19·2	123·7	258·3
Carbon/epoxy	Graphite/epoxy	Cr/Gr	71·1	473.3	998.3
Graphite/epoxy	E-glass/epoxy	Gr/Eg	19·3	124·3	260

4. RESULTS AND CONCLUSIONS

The results in Table 2 are plotted in Figure 4. All parts of the rotor (disk and shaft) are made from the same material. E-glass/epoxy gives the lowest values of the natural frequencies, because E_{11} for the material is lower than that of the other composite materials. For the graphite, boron, and carbon, there is a slight difference in the natural frequency values, but all their values are higher than the values of the steel rotor. The designer can select from among steel, E-glass, and one of the other composite materials to adjust the natural frequencies as desired, guided by the percentage change in Table 2.

Figure 5 presents the first six results in Table 3, and Figure 6 the rest to show the effect of changing the composite material between the disk and the shaft on the natural frequencies of the rotor. It is noted that a great reduction in the natural frequencies occurs when using E-glass/epoxy as a material for the shaft. But when it

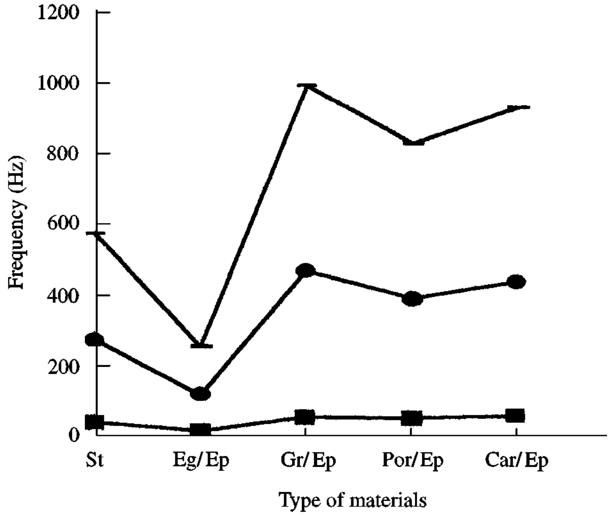

Figure 4. Composite rotor natural frequencies (disk and shaft has same material). —■—, First mode; —●—, second mode; ——, third mode.

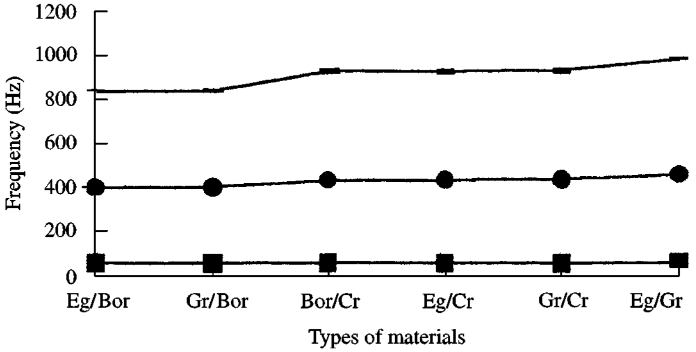

Figure 5. Composite rotor natural frequencies (disk and shaft have different materials) (the first six results of Table 3). —■—, First mode; —●—, second mode; ——, third mode.

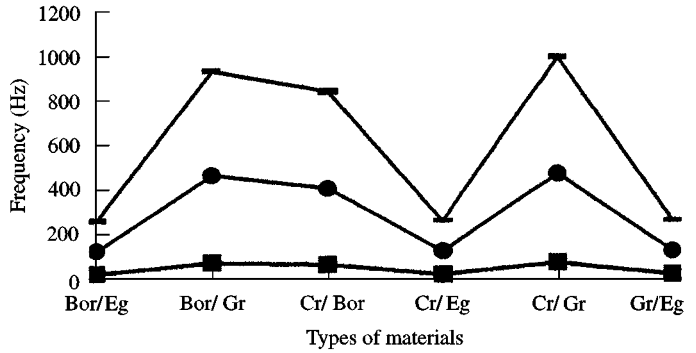

Figure 6. Composite rotor natural frequencies (disk and shaft have different materials) (the second six results of Table 3). —■—, First mode; —●—, second mode; ——, third mode.

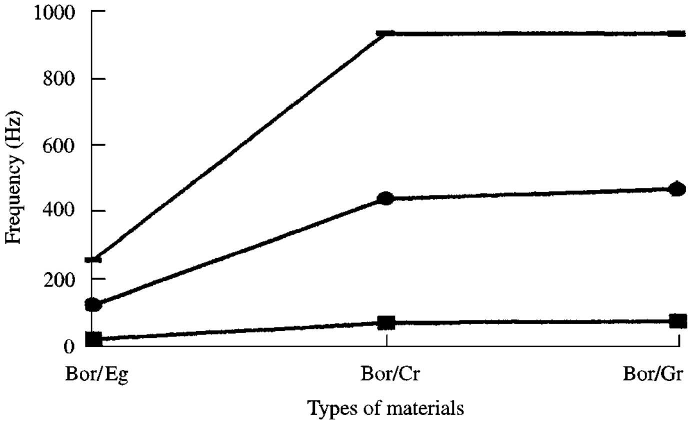

Figure 7. Composite rotor natural frequencies (disk material boron). —■—, First mode; —●—, second mode; ——, third mode.

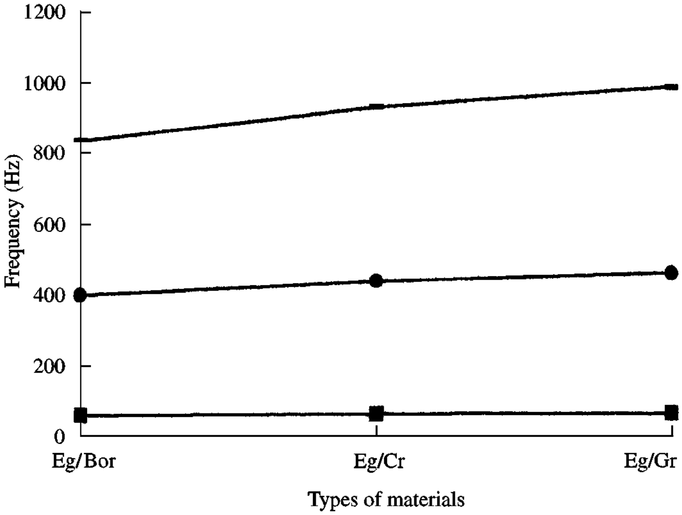

Figure 8. Composite rotor natural frequencies (disk material E-glass). —■—, First mode; —●—, second mode; ——, third mode.

is used as a material for the disk, it does not give a remarkable change in the natural frequencies with respect to the other materials. In Figure 7, boron/epoxy is taken as a reference material for the disk and the shaft material is changed. In Figure 8, E-glass/epoxy is chosen as a reference material for the disk. The results obtained from Figures 4–6 are also observed in Figures 7 and 8.

In general, the results indicate specific ways in which the system may be designed for higher stiffness to mass ratio. Steel rotors can be replaced by composite ones with the same dimensions to adjust the natural frequencies as desired from the dynamic point of view. Using E-glass/epoxy as a shaft material reduces the natural frequencies of the rotor.

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