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VIBRATION ANALYSIS OF THE CONTINUOUS BEAM SUBJECTED TO A MOVING MASS

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The dynamic behavior of the multi-span continuous beam traversed by a moving mass at a constant velocity is investigated, in which it is assumed that each span of the continuous beam obeys uniform Euler–Bernoulli beam theory. The solution to this system is simply obtained by using both eigenfunction expansion or the modal analysis method and the direct integration method in combination. The effects of the inertia and the moving velocity of the load on the dynamic response of the continuous beam are evaluated for three kinds of continuous beams having uniform span length.

1. INTRODUCTION

The dynamic behavior of beam structures, such as bridges on railways, subjected to moving loads or masses has been investigated for over a century. There are numerous reports available in the excellent monographs of Fryba [1, 2], and most of them treat a uniform simply supported beam of single span.

When the effect of the inertia of the load is accounted for, the problem is associated with serious difficulties even for the case of a single-span beam. Cai *et al.* [3] investigated the dynamic interactions between the vehicle and guideway of a maglev system by modelling the vehicle as a concentrated moving force and as a two-degree-of-freedom model. Michaltsos *et al.* [4] derived a closed-form solution for the single-span beam to a moving mass by approximating the total time derivative of the mass displacement with the partial derivative and by using as a first approximation the solution of the corresponding problem without the effect of the mass. Recently, Foda and Abduljabbar [5] studied the influence of the parameters of the system on the dynamic response of the single-span beam subjected to a moving mass using the method of dynamic Green function. Lee [6, 7] investigated the possibility of mass separation from the beam during the mass motion using the Lagrangian approach and the assumed mode method for the Euler and the Timoshenko beams of single span.

There are not so many reports on the dynamic problem of a multi-span continuous beam subjected to the moving load. Yang *et al.* [8] presented the useful impact formulas for vehicles moving over the simple and continuous beams. Lee

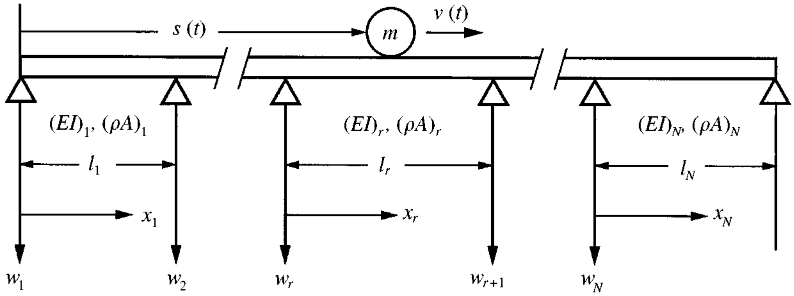

Figure 1. Illustration of an N-span continuous beam subjected to a moving mass.

[9] investigated the Euler beam on multiple supports with a moving mass using the assumed mode method. Chatterjee *et al.* [10] investigated the dynamic response of the multi-span continuous bridge under a moving vehicle modelled as a single unsprung or sprung mass, using the eigenstiffness method developed by Hayashikawa and Watanabe [11]. Henchi *et al.* [12] also presented the dynamic stiffness element method, followed by the modal fast Fourier transform approach, for the moving load problem of the multi-span continuous beam. Yang and Yau [13] developed the vehicle–bridge interaction element with both accuracy and efficiency in the analysis of railway bridges carrying high-speed trains.

Ichikawa *et al.* [14] investigated the dynamic response of a multi-span Euler–Bernoulli beam subjected to a moving load at time-dependent velocity using the method of the eigenfunction expansion or the modal analysis, and estimated the effects of acceleration or deceleration of a moving load on the dynamic amplification factor for a symmetric three-span continuous beam. Its solution method is simple and can be widely applied.

The present paper investigates the response of the multi-span Euler–Bernoulli beam subjected to a moving mass by using the method just described. The equation of transverse motion of each span is non-dimensionalized in a reasonable manner, and is transformed into the coupled ordinary differential equation of second order for the generalized co-ordinate whose solution is found through the direct integration method in the present paper. The effects of the inertia and the velocity of the moving load are evaluated numerically for three kinds of continuous beams of uniform span length.

2. FORMULATION

In the present paper, the following assumptions are made for the formulation of the vibration problem of a continuous beam subjected to moving mass as in Figure 1: (1) each span of the continuous beam obeys a Euler–Bernoulli beam theory and has linear elastic behavior; (2) the moving mass keeps contact with the continuous beam at all times; and (3) for the initial conditions, the moving mass is located at the left-hand end of the continuous beam.

The equation of the trnsverse vibration for each span is given by

$$(EI)_r \frac{\partial^2 w_r}{\partial x_r^4} + (\rho A)_r \frac{\partial^2 w_r}{\partial t^2} = f_r(x_r, t), \quad 0 \leqslant x_r \leqslant l_r, \quad r = 1, 2, \ldots, N, \tag{1}$$

in which the suffix r denotes the rth span, EI, ρ and A denote, respectively, the flexural rigidity, mass density and the cross-sectional area. Furthermore, w is the transverse deflection of each span, f is the time-varying external load distribution due to moving loads, x_r is the local co-ordinate along the axis of the rth span, and t is the time. The continuity and equilibrium conditions at the intermediate support points of the continuous beam require the following relations:

$$\left.\begin{aligned} & w_r(x_r = l_r, t) = 0, \\ & w_{r+1}(x_{r+1} = 0, t) = 0, \\ & \frac{\partial w_r}{\partial x_r}(x_r = l_r, t) = \frac{\partial w_{r+1}}{\partial x_{r+1}}(x_{r+1} = 0, t), \\ & (EI)_r \frac{\partial^2 w_r}{\partial x_r^2}(x_r = l_r, t) = (EI)_{r+1} \frac{\partial^2 w_{r+1}}{\partial x_{r+1}^2}(x_{r+1} = 0, t), \end{aligned}\right\} \quad r = 1, 2, \ldots, N-1. \tag{2}$$

Introducing dimensionless variables $\xi_r = x_r/l_1$ and $T = \alpha t$, in which $\alpha = \sqrt{(EI)_1/(\rho A)_1}/l_1^2$, into equations (1) and (2) leads to

$$\frac{\partial^4 w_r}{\partial \xi_r^4} + a_r^4 \frac{\partial^2 w_r}{\partial T^2} = \frac{l_1^4}{b_r (EI)_1} \bar{f}_r(\xi_r, T), \quad 0 \leqslant \xi_r \leqslant c_r, \quad r = 1, 2, \ldots, N \tag{3}$$

and

$$\left.\begin{aligned} & w_r(\xi_r = c_r, T) = w_{r+1}(\xi_{r+1} = 0, T) = 0 \\ & w_r'(\xi_r = c_r, T) = w_{r+1}'(\xi_{r+1} = 0, T) \\ & b_r w_r''(\xi_r = c_r, T) = b_{r+1} w_{r+1}''(\xi_{r+1} = 0, T) \end{aligned}\right. \quad r = 1, 2, \ldots, N-1, \tag{4}$$

where

$$a_r^4 = \frac{(\rho A)_r (EI)_1}{(\rho A)_1 (EI)_r}, \quad b_r = \frac{(EI)_r}{(EI)_1}, \quad c_r = l_r/l_1, \; \bar{f}_r(\xi_r, T) = f_r(l_1 \xi_r, T/\alpha), \quad r = 1, 2, \ldots, N \tag{5}$$

and the symbol ($'$) denotes differentiation with respect to the dimensionless spatial variable.

For a steady state of free vibration, let $\bar{f}_r(\xi_r, T) = 0$ and $w_r(\xi_r, T) = \phi_r(\xi_r) e^{i\omega T}$ in which $\phi_r(\xi_r)$, i and ω denote the spatial function of the rth span, the imaginary unit and the dimensionless circular frequency on the basis of the first span respectively. Then, the following results are easily obtained for the simply supported continuous beam:

$$\phi_r(\xi_r) = \begin{cases} \sinh a_1 c_1 \lambda \sin a_1 \lambda \xi_1 - \sin a_1 c_1 \lambda \sinh a_1 \lambda \xi_1, & r = 1, \\ B_r F_r(\xi_r) + D_r G_r(\xi_r), & r = 2, 3, \dots, N, \end{cases} \tag{6}$$

where $\lambda = \sqrt{\omega}$, the coefficients B_r and D_r are the constants and the functions $F_r(\xi_r)$ and $G_r(\xi_r)$ are expressed by

$$F_r(\xi_r) = (\cos a_r c_r \lambda - \cosh a_r c_r \lambda) \sinh a_r \lambda \xi_r + \sinh a_r c_r \lambda (\cosh a_r \lambda \xi_r - \cos a_r \lambda \xi_r) \tag{7}$$

and

$$G_r(\xi_r) = (\cos a_r c_r \lambda - \cosh a_r c_r \lambda) \sin a_r \lambda \xi_r + \sin a_r c_r \lambda (\cosh a_r \lambda \xi_r - \cos a_r \lambda \xi_r). \tag{8}$$

The coefficients B_r and D_r may be expressed in the matrix form of

$$\begin{bmatrix} B_r \\ D_r \end{bmatrix} = \mathbf{U}_{r-1} \mathbf{U}_{r-2} \dots \mathbf{U}_1, \quad r = 1, 2, \dots, N, \tag{9}$$

where matrices $\mathbf{U}$ are given as follows:

$$\mathbf{U}_1 = \frac{1}{e_1} \tilde{\mathbf{U}}_1 \begin{bmatrix} -\dfrac{\sin a_1 c_1 \lambda \sinh a_1 c_1 \lambda}{a_2 b_2} \\ \sinh a_1 c_1 \lambda \cos a_1 c_1 \lambda - \sin a_1 c_1 \lambda \cosh a_1 c_1 \lambda \end{bmatrix}, \tag{10}$$

and

$$\mathbf{U}_k = \frac{\mathbf{1}}{e_k} \tilde{\mathbf{U}}_k \begin{bmatrix} \dfrac{a_k b_k h_{11}^k}{a_{k+1} b_{k+1}} & \dfrac{a_k b_k h_{12}^k}{a_{k+1} b_{k+1}} \\ h_{21}^k & h_{22}^k \end{bmatrix}, \quad k = 2, 3, \dots, N-1, \tag{11}$$

where

$$e_k = \frac{a_{k+1}}{a_k} (\cos a_{k+1} c_{k+1} \lambda - \cosh a_{k+1} c_{k+1} \lambda)(\sinh a_{k+1} c_{k+1} \lambda - \sin a_{k+1} c_{k+1} \lambda),$$

$$\tilde{\mathbf{U}}_k = \begin{bmatrix} \cos a_{k+1} c_{k+1} \lambda - \cosh a_{k+1} c_{k+1} \lambda & -\sin a_{k+1} c_{k+1} \lambda \\ -\cos a_{k+1} c_{k+1} \lambda + \cosh a_{k+1} c_{k+1} \lambda & \sinh a_{k+1} c_{k+1} \lambda \end{bmatrix},$$

$$k = 1, 2, \dots, N-1 \tag{12}$$

and

$$\left.\begin{aligned} h_{11}^k &= \cos a_k c_k \lambda \sinh a_k c_k \lambda \\ h_{12}^k &= \cos a_k c_k \lambda \sin a_k c_k \lambda \\ h_{21}^k &= \cos a_k c_k \lambda \cosh a_k c_k \lambda + \sinh a_k c_k \lambda \sin a_k c_k \lambda - 1 \\ h_{22}^k &= \sin a_k c_k \lambda \sinh a_k c_k \lambda - \cosh a_k c_k \lambda \cos a_k c_k \lambda + 1 \end{aligned}\right\} \quad k = 2, 3, \ldots, N-1. \tag{13}$$

The frequency equation for the simply supported continuous beam, from which the values of λ are found, is as follows:

$$B_N F_N''(c_N) + D_N G_N''(c_N) = 0. \tag{14}$$

The solutions of equation (3) can be expressed as the series

$$w_r(\xi_r, T) = \sum_{n=1}^{\infty} \phi_{rn}(\xi_r) q_n(T), \quad r = 1, 2, \ldots, N, \tag{15}$$

where $\phi_{rn}(\xi_r)$ is the nth eigenfunction of the rth span and $q_n(T)$ is the corresponding generalized time-dependent co-ordinate to be determined. Using the relation of $\partial\phi_r/\partial\lambda = (\xi_r/\lambda)\,\partial\phi_r/\partial\xi_r$ and l'Hospital's rule leads to the following orthogonality relation of the eigenfunctions for the present system:

$$\sum_{r=1}^{N} a_r^4 b_r \int_0^{c_r} \phi_{ri}\phi_{rj}\,\mathrm{d}\xi_r = \frac{M_i}{4\lambda_i^4}\,\delta_{ij}, \tag{16}$$

where δ is the Kronecker delta and the coefficient M_i is expressed by

$$\begin{aligned} M_i = {} & b_1(\phi_{1i}''(0)\phi_{1i}'(0) - 3\phi_{1i}'''(0)\phi_{1i}(0)) + \sum_{r=1}^{N-1} b_r c_r \{(\phi_{ri}''(c_r))^2 - 2\phi_{ri}'(c_r)\phi_{ri}'''(c_r)\} \\ & + b_N\{(3\phi_{Ni}'''(c_N) + a_N^4 c_N \lambda_i^4 \phi_{Ni}(c_N))\phi_{Ni}(c_N) \\ & + (c_N\phi_{Ni}''(c_N) - \phi_{Ni}'(c_N))\phi_{Ni}''(c_N) - 2c_N\phi_{Ni}'(c_N)\phi_{Ni}'''(c_N)\}. \end{aligned} \tag{17}$$

This relation is indeed suitable for the computer implementation of the present analysis because of its clear and simple expression, so that the present method can be applied to the continuous beams with other combinations of boundary conditions with only minor changes. Thus, substitution of equation (15) into equation (3), and use of equation (16) lead to

$$\frac{\mathrm{d}^2 q_n}{\mathrm{d}T^2} + \lambda_n^4 q_n = \frac{4\lambda_n^4 l_1^4}{M_n (EI)_1} \sum_{r=1}^{N} \int_0^{c_r} \bar{f}_r(\xi_r, T)\phi_{rn}(\xi_r)\,\mathrm{d}\xi_r. \tag{18}$$

When considering the effect of the inertia of the moving mass m, the external forces $\bar{f}_r$ on the right-hand side of equation (18) should be regarded as the time-varying reaction forces acting at the point of contact. From the equation of vertical motion of the moving mass, the functions $\bar{f}_r$ can be expressed by

$$\bar{f}_r(\xi_r, T) = \frac{mg}{l_1}\left(1 - \frac{\alpha^2}{g}\frac{\mathrm{d}^2 y}{\mathrm{d}T^2}\right)\delta\left(\xi_r - \bar{s} + \sum_{i=1}^{r-1} c_i\right)\{H(\xi_r) - H(\xi_r - c_r)\},$$

$$r = 1, 2, \ldots, N, \tag{19}$$

where g is the gravitational acceleration, y the vertical displacement of the moving mass, $\bar{s}$ is defined to represent s/l_1 which indicates the dimensionless distance between the instantaneous position of the moving mass and the left-hand end of the continuous beam, δ is the Dirac delta function and H is the Heaviside step function. Since both y and $\bar{s}$ are functions of only time, substitution of equation (19) into equation (18) yields

$$\frac{\mathrm{d}^2\hat{q}_n}{\mathrm{d}T^2} + \lambda_n^4\hat{q}_n = \frac{4\lambda_n^4}{M_n}\left(1 - \varepsilon\frac{\mathrm{d}^2\hat{y}}{\mathrm{d}T^2}\right)\phi_n(\bar{s}), \tag{20}$$

where the introduced quantities are defined by the following expressions:

$$\hat{q}_n = \frac{q_n}{mgl_1^3/(EI)_1}, \quad \hat{y} = \frac{y}{mgl_1^3/(EI)_1}, \quad \varepsilon = m/(\rho A l)_1$$

and

$$\phi_n(\bar{s}) = \sum_{r=1}^{N}\left[\phi_{rn}\left(\bar{s} - \sum_{k=1}^{r-1} c_k\right)\times\left\{H\left(\bar{s} - \sum_{k=1}^{r-1} c_k\right) - H\left(\bar{s} - \sum_{k=1}^{r} c_k\right)\right\}\right].$$

It should be noticed that the dimensionless parameter ε denotes the mass ratio of the moving mass to the total mass of the first span, and that the quantity $mgl_1^3/(EI)_1$ means the scaling factor for the transverse displacement of the system under investigation.

From the second assumption that the moving mass keeps contact with the continuous beam at all times, the non-dimensional displacement $\hat{y}$ is the right-hand side of equation (20) can be written by

$$\hat{y}(T) = \sum_{n=1}^{\infty}\phi_n(\bar{s})\hat{q}_n(T). \tag{21}$$

Since $\bar{s}$ is a function of dimensionless time T, we obtain the following relation:

$$\frac{\mathrm{d}^2\hat{y}}{\mathrm{d}T^2} = \left[\frac{\mathrm{d}^2\bar{s}}{\mathrm{d}T^2}\sum_{n=1}^{\infty}\frac{\mathrm{d}\phi_n(\bar{s})}{\mathrm{d}\bar{s}}\hat{q}_n(T) + \left(\frac{\mathrm{d}\bar{s}}{\mathrm{d}T}\right)^2\sum_{n=1}^{\infty}\frac{\mathrm{d}^2\phi_n(\bar{s})}{\mathrm{d}\bar{s}^2}\hat{q}_n(T)\right.$$

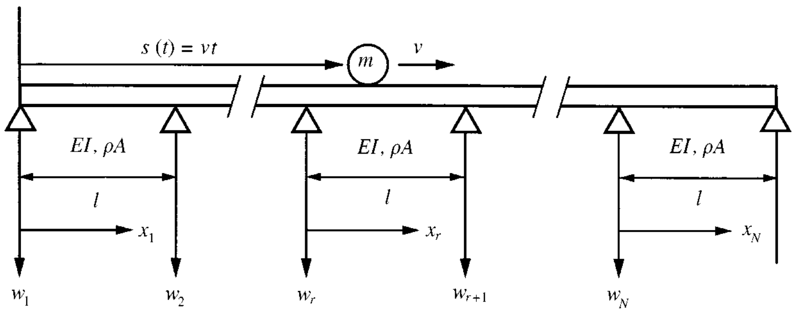

Figure 2. Illustration of the continuous beam of N equal spans subjected to a mass moving at constant velocity.

TABLE 1

The first six eigenvalues of continuous beams in Figure 2

Number of spans	Roots λ_j of equation (14)					
	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
1	π	2π	3π	4π	5π	6π
2	π	3·9266023	2π	7·0685827	3π	10·2101761
3	π	3·5564085	4·2975297	2π	6·7075956	7·4295413
4	π	3·3932313	3·9266023	4·4633244	2π	6·5454138

$$+ 2\frac{\mathrm{d}\bar{s}}{\mathrm{d}T}\sum_{n=1}^{\infty}\frac{\mathrm{d}\phi_n(\bar{s})}{\mathrm{d}\bar{s}}\frac{\mathrm{d}\hat{q}_n(T)}{\mathrm{d}T} + \sum_{n=1}^{\infty}\phi_n(\bar{s})\frac{\mathrm{d}^2\hat{q}_n(T)}{\mathrm{d}T^2}\Bigg]_{\bar{s}=\bar{s}(T)} \tag{22}$$

Thus, equation (20) yields the set of coupled ordinary differential equations of second order for the unknown time-dependent functions $\hat{q}_n$. They can be obtained by using the direct integration method [15] because closed solutions for them are unavailable except in the case of $\varepsilon = 0$ where the inertia of the moving mass is ignored. The central difference method [15] is used here since it has the simple procedure for computer implementation. The discrete time interval for integration, therefore, must be sufficiently small to ensure the stability and convergence of the solutions.

3. NUMERICAL EXAMPLES

The continuous beams having uniform span length of from 2 through 4 spans are considered in the numerical examples as shown in Figure 2, and it is also assumed that a moving mass starts to move at the left-hand end of the first span at $t = 0$ with the constant velocity, v. Hence, the dimensionless distance of the moving mass to the left-hand end of the continuous beam is given in a form of $\bar{s}(T) = \beta T$ in which β denotes the non-dimensional velocity parameter defined by $v/\alpha l$.

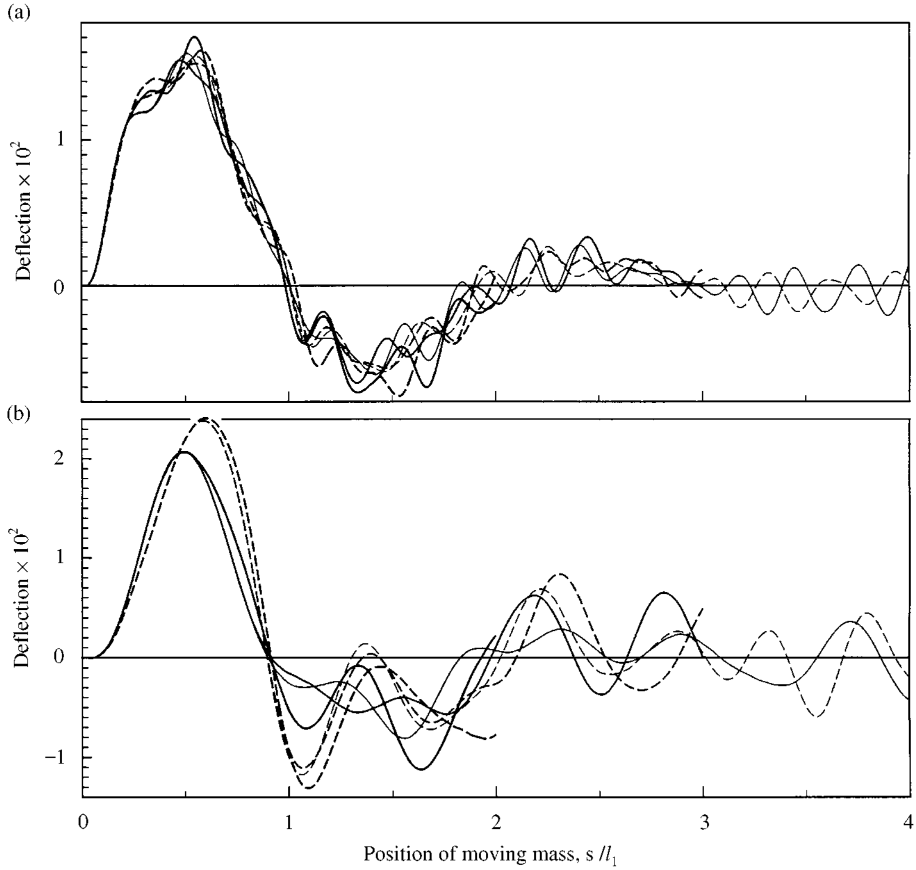

Figure 3. Normalized deflections at the midpoint of the first span for two different values of the mass ratio ε when (a) $\beta = 0{\cdot}5$ and (b) $\beta = 1{\cdot}2$; ——, $\varepsilon = 0$; – – –, $\varepsilon = 0{\cdot}4$.

The calculated eigenvalues λ_k corresponding to the first six natural modes are listed in Table 1 including those for a single-span beam. The present results are in perfect agreement with those in reference [16]. The series solution of equation (15) converges rapidly, and it is confirmed that the lowest 12 terms give sufficient results for all the calculated cases in the present paper.

The history curves of the midpoint deflection, which is normalized by the scaling factor of mgl^3/EI, on each span of the continuous beams are shown in Figures 3–6 for four combinations of the mass ratio and the velocity parameter. The equivalent velocities corresponding to $\beta = 0{\cdot}5$ and $1{\cdot}2$ are, respectively, $v = 35$ and 85 m/s when the flexural rigidity $EI = 1{\cdot}96 \times 10^9$ N m^2, mass per unit length $\rho A = 1{\cdot}0 \times 10^3$ kg/m and $l = 20$ m in Figure 2. The abscissa of these plots can be considered the instantaneous position of the moving mass on the continuous beams. The influence of the inertia of the moving mass on the dynamic response of the continuous beam is small in the case of $\beta = 0{\cdot}5$, whereas it is large for $\beta = 1{\cdot}2$ and the inertia of the moving mass seems to have greater effects upon the latter spans of the continuous beam than the first span.

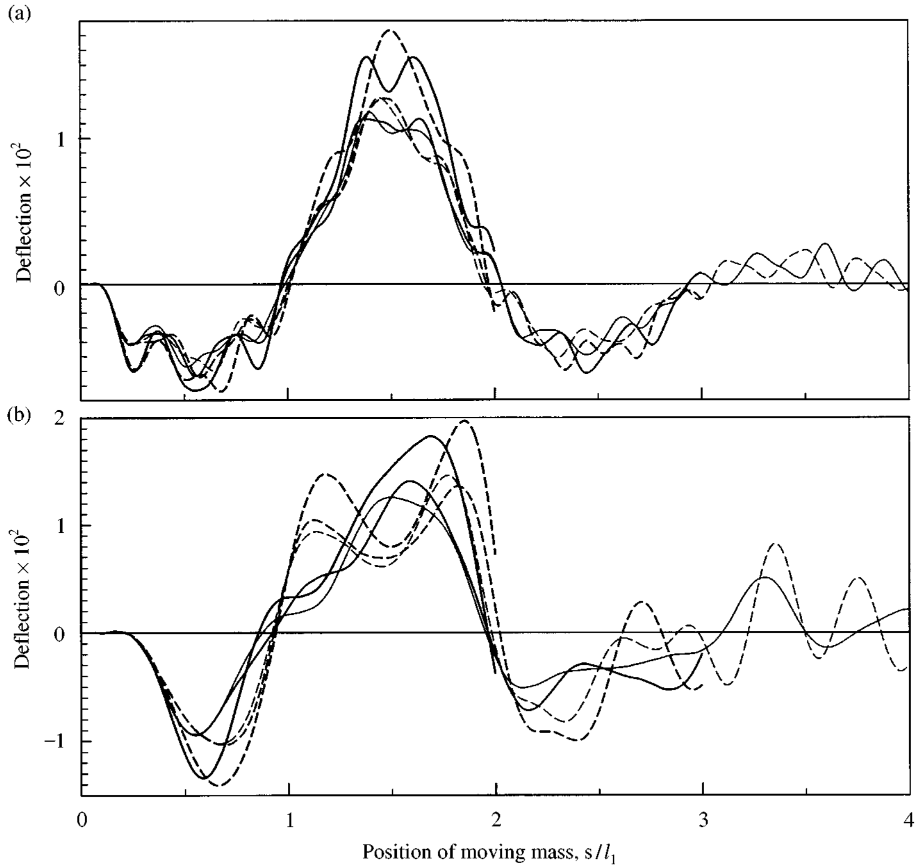

Figure 4. Normalized deflections at the midpoint of the second span for two different values of the mass ratio ε when (a) $\beta = 0{\cdot}5$ and (b) $\beta = 1{\cdot}2$; ——, $\varepsilon = 0$; – – –, $\varepsilon = 0{\cdot}4$.

As pointed out by Lee [6, 7, 9], there is a possibility that the moving mass may separate from the beam during motion. Then, the second assumption in the present paper will not be valid for the succeeding motion after the separation. The mass separation can be determined by observing the sign of the contact force; the mass becomes free from the beam when the sign of the contact force changes the positive to the negative. From equations (19), the contact force F_c in the course of motion has an expression as follows:

$$\frac{F_c}{mg/l} = 1 - \varepsilon \frac{\mathrm{d}^2 \hat{y}}{\mathrm{d}T^2}, \tag{23}$$

where mg/l is the scaling factor for the contact force and the right-hand side is to be evaluated using equation (22). The calculated minimum values of F_c during motion are shown in Figure 7 for both the values of β ranging from 0 to 1·5 and four different values of the mass ratio. Consequently, it is verified that the mass separation does not occur in Figures 3–6. As seen from Figure 7, the minimum

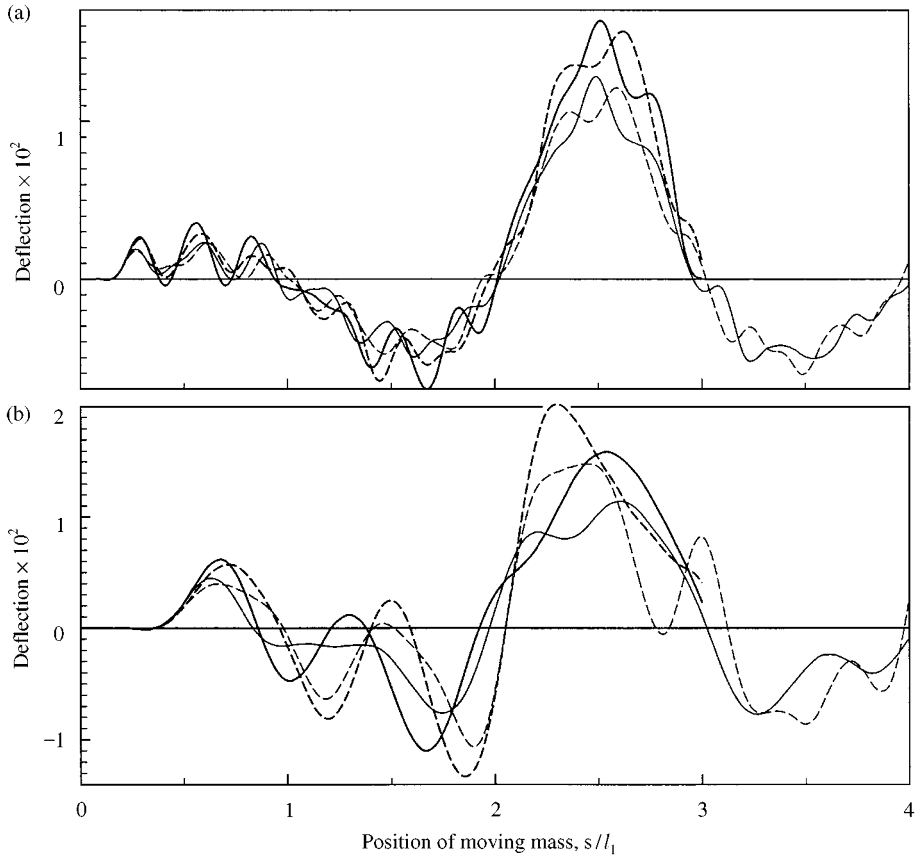

Figure 5. Normalized deflections at the midpoint of the third span for two different values of the mass ratio ε when (a) $\beta = 0{\cdot}5$ and (b) $\beta = 1{\cdot}2$; ——, $\varepsilon = 0$; - - -, $\varepsilon = 0{\cdot}4$.

contact force does not decrease monotonously with β, so that it is necessary to evaluate the contact force in each case for determination of the possible separation.

The influence of the velocity of the moving mass on the amplification of displacement at the middle point of each span is evaluated under the same conditions as Figure 7. Since the purpose of the present study is to clarify the effect of the inertia of the moving mass, the amplification factor is defined as the ratio between the maximum dynamic deflection in the moving mass problem and that in the corresponding moving force problem, namely, $\varepsilon = 0$. The computed results are shown in Figures 8–11—the mass separation during motion is excluded here because of a great difficulty in fully considering the possible separation shown in Figure 7. From these results, the following points can be made:

(1) When the value of β exceeds by about 0·5 the amplification factors for almost all the spans of the continuous beams seem to change its behavior and their local peaks show an increasing tendency with the velocity. The former property appears clearly for the first span particularly.

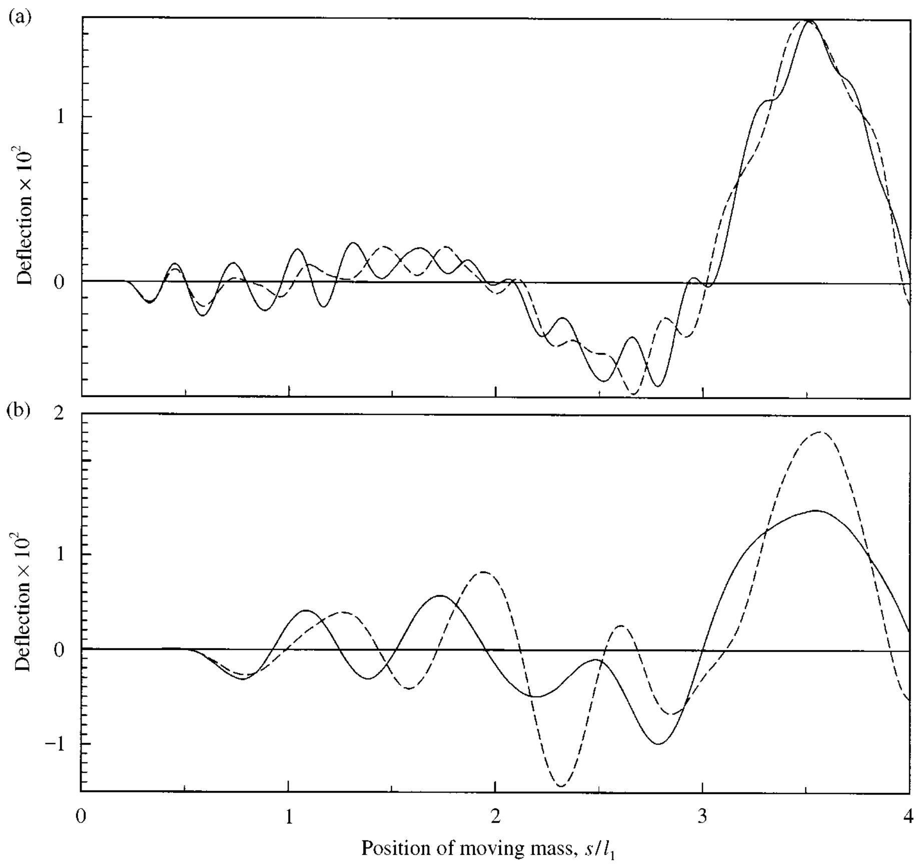

Figure 6. Normalized deflections at the midpoint of the fourth span for two different values of the mass ratio ε when (a) $\beta = 0{\cdot}5$ and (b) $\beta = 1{\cdot}2$; ——, $\varepsilon = 0$; - - -, $\varepsilon = 0{\cdot}4$.

(2) Without reference to both the mass ratio and the total number of spans, the amplification factor of the first span quite differs from those of the second and the successive spans in that the first span reaches a certain value strongly dependent on the mass ratio and changes gradually in the range of β larger than 0·6; the other spans show considerable variations in the same range.

(3) The amplification factor is more than unity generally and will become very large in its magnitude for the multi-span continuous beam. However, it does not necessarily increase with the mass ratio.

(4) In the calculated range of β, the inertia of the moving mass does not significantly affect the amplification factor when the value of the mass ratio is less than 0·1.

4. CONCLUSION

The dynamic behavior of a multi-span continuous beam subjected to a moving mass with a constant velocity has been investigated. The method for analyzing the

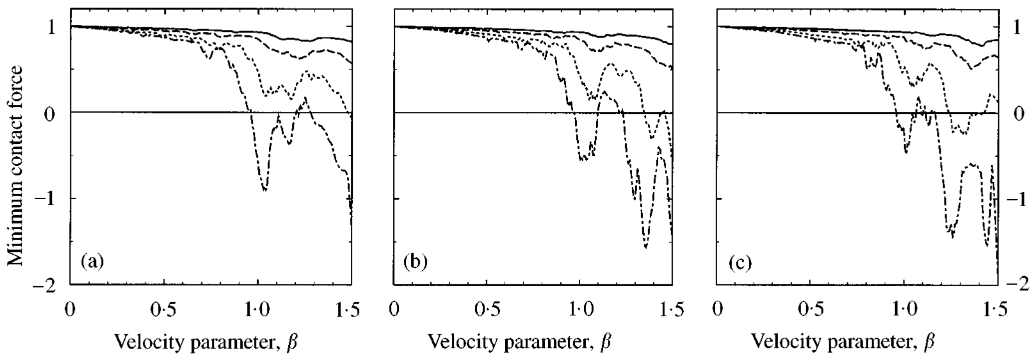

Figure 7. Variation of minimum values of the normalized contact force during motion with the velocity parameter β for four different values of the mass ratio ε where (a) two-span beam, (b) three-span beam, and (c) four-span beam: ——, $\varepsilon = 0{\cdot}1$; – – –, $\varepsilon = 0{\cdot}2$; -----, $\varepsilon = 0{\cdot}4$; – - – -, $\varepsilon = 0{\cdot}6$.

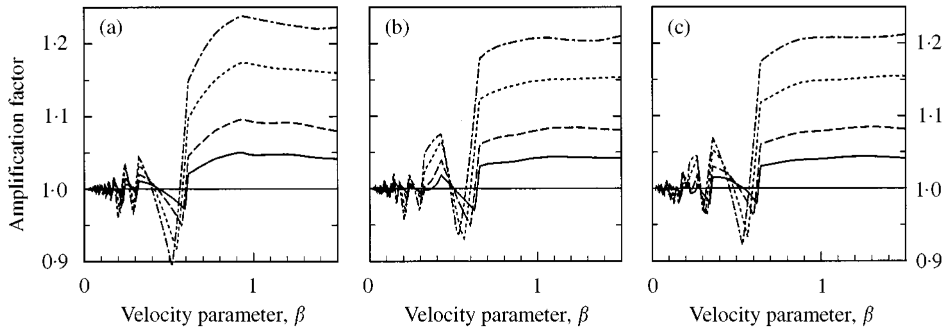

Figure 8. Variation of amplification factor at the middle point of the first span with the velocity parameter β for four different values of the mass ratio ε where (a) two-span beam, (b) three-span beam, and (c) four-span beam: ——, $\varepsilon = 0{\cdot}1$; – – –, $\varepsilon = 0{\cdot}2$; -----, $\varepsilon = 0{\cdot}4$; – - – -, $\varepsilon = 0{\cdot}6$.

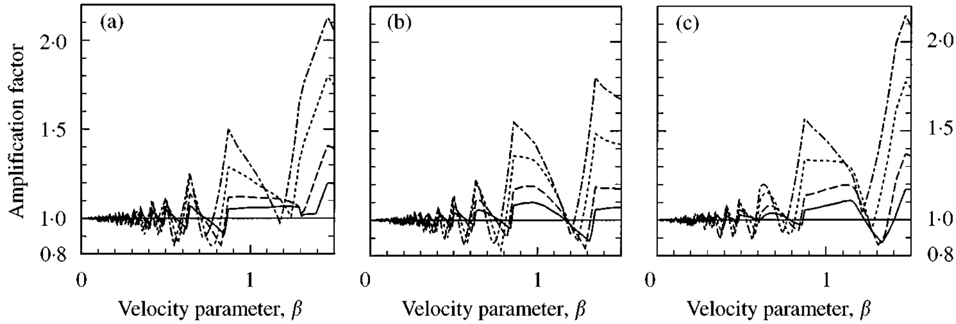

Figure 9. Variation of amplification factor at the middle point of the second span with the velocity parameter β for four different values of the mass ratio ε where (a) two-span beam, (b) three-span beam, and (c) four-span beam: ——, $\varepsilon = 0{\cdot}1$; – – –, $\varepsilon = 0{\cdot}2$; -----, $\varepsilon = 0{\cdot}4$; – - – -, $\varepsilon = 0{\cdot}6$.

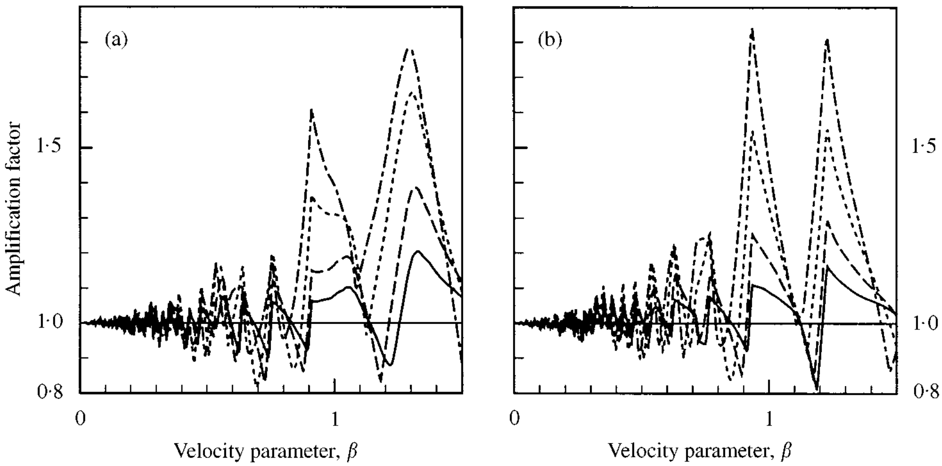

Figure 10. Variation of amplification factor at the middle point of the third span with the velocity parameter β for four different values of the mass ratio ε where (a) three-span beam, and (b) four-span beam: ——, $\varepsilon = 0{\cdot}1$; – – –, $\varepsilon = 0{\cdot}2$; -----, $\varepsilon = 0{\cdot}4$; – – - -, $\varepsilon = 0{\cdot}6$.

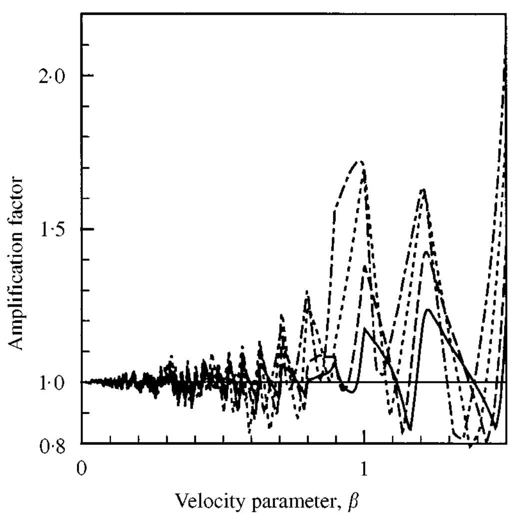

Figure 11. Variation of amplification factor at the middle point of the fourth span with the velocity parameter β for four different values of the mass ratio ε: ——, $\varepsilon = 0{\cdot}1$; – – –, $\varepsilon = 0{\cdot}2$; -----, $\varepsilon = 0{\cdot}4$; – – - -, $\varepsilon = 0{\cdot}6$.

present problem is the eigenfunction expansion or modal analysis accompanied by the direct integration method, and it can also easily include other effects such as non-uniformity of the continuous beam, various combinations of boundary conditions and the speed variation of the moving mass. Numerical calculations have been conducted to clarify the effects of two important parameters, the mass ratio of the moving mass to the first span and the velocity of the moving mass, on the dynamic response and the amplification factor of the continuous beams having uniform span length.

Conclusions drawn from present analysis are as follows: (1) with the multi-span continuous beam, the inertia of the moving mass has greater influences on the second and the successive spans than the first span; (2) the amplification factor for almost all the spans of the multi-span continuous beam appears to change its characteristics when the dimensionless velocity parameter β is larger than about 0·5, which is markedly recognized for the first span; and (3) the amplification factor will become very large in its magnitude for the multi-span continuous beam, but it does not necessarily increase with the value of the mass ratio.

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