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VIBRATORY SYSTEM SYNTHESIS FOR MULTI-BODY SYSTEMS BASED ON GENETIC ALGORITHM

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This paper presents the Euler–Newton formulation of oscillatory behaviour of multi-body systems interconnected by spring elements with three orthoganal stiffnesses and dampers and introduces the application of a genetic algorithm (GA) as an optimization tool to the problem of vibration synthesis. The state variables of the problem space are the mounting parameters such as two end positions, stiffnesses and angular orientation. It is demonstrated in the paper that in optimizing the vibratory characteristics of a system, the redundancy of variables does not cause a serious problem for the genetic algorithm. Furthermore, due to its flexibility, the algorithm imposes little restriction in the selection of the objective function.

1. INTRODUCTION

A number of different techniques have been developed in the past for modal synthesis of dynamic systems. The theme of most of the recent research work such as references [1–3] has been to use some form of iterative technique to modify the system dynamic characteristics. Kundra and Nakra [4] considered the effect of modifying stiffness and mass matrices in order to achieve the desired dynamics characteristics of the system. Wang [5] subsequently adopted a re-analysis formulation for optimization of dynamic behaviour.

This paper presents the general formulation of a general multi-body system flexibly supported by linear mountings and uses a genetic algorithm to optimize the systems vibratory behaviour. The methods proposed in this paper are extensively used in the VIBRATIO suite of vibration analysis software developed by Esat [6, 7] since 1985. A genetic algorithm is used to select a general objective function for dynamic synthesis of structures. Only a brief description of the theory of the genetic

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algorithm is given in this paper as it is by now a well-established optimization technique which was initially proposed by Holland [8] and later developed and documented by others such as Goldberg [9] and DeJong [10].

Two types of optimization problems are considered. The first problem involves reducing overall coupling of oscillation between selected directions. For this, the objective function is constructed from summation of all the non-diagonal terms of the global stiffness matrix. The formulation developed for decoupling could be used for selected modes of oscillation. In this case the objective function is constructed from the selected elements of the stiffness matrix. The second problem investigated involves reducing the amplitude of oscillation at a selected position on the system. This is a common problem where the oscillating mass is connected to the outside world through a connecting element. For example, an engine–gearbox connected to a drive-line through the universal joint. The genetic algorithm is successfully tested for both problems.

1.1. GENETIC ALGORITHM

Holland, [8] developed the genetic algorithm (GA) at the University of Michigan. In simple terms the algorithm represents a search strategy based on the mechanics of natural selection and reproduction in biological systems. In the genetic algorithm, like any other optimization technique, only those variables which have contributed to the objective function are identified. These variables are coded into strings (patterns) also known as chromosomes. Each pattern has a survival value or value of fitness which determines its effectiveness in the "survival of the fittest" gene, this is the value of the objective function due to the string. The probabilistic selection which is biased according to the fitness values of string produces the population for a "mating pool". Mating, which follows this, ensures that a fitter generation of string is resulted. The biasing random selection according to the level of fitness distinguishes the algorithm from methods based on random walk techniques.

DeJong [10] studied the use of genetic algorithm in general function optimization. He has shown that the ability of the GA to learn from the history and exploit the environment provides the basis of its effectiveness in general optimization. Exploitation of past experience does not feature in techniques based on hill-climbing, local gradient and simulated annealing.

Another powerful tool in a genetic algorithm is the concept of Schemata. A schema is a subset of chromosomes where some selected elements are common. This gives great flexibility to the algorithm in terms of focusing the search to specific area or emphasizing selected attributes.

The following steps form the bases of the genetic algorithm:

(A) The system variables are coded in a binary string.
(B) Randomly form an initial population of strings.
(C) Fitness values (value of objective function) are obtained for each string in the population.

(D) A number of strings are selected for mating. The selection process is biased randomly. Biasing is according to the fitness values of strings. Along with the strings selected, now *new* randomly created strings are added to the mating pool.
(E) A percentage of strings are selected for CROSSOVER (60–80%) and MUTATION (3–5%) from the mating pool. CROSSOVER is performed on the pairs randomly selected from the mating pool. Similarly, a selected proportion of the mating pool population is mutated. This selection is also random.

Crossover is the process whereby strings are randomly selected and the bits (genes) are exchanged at a random location to create offsprings. For example, two strings (1001 and 0011) crossed at bit 2 will give two new offsprings such as (0001 and 1011).

In mutation a bit is switched at a random site to include new search spaces, e.g. 1101 becomes 1100, the last bit switched.

A certain proportion of new strings may be added to the new generation of offsprings. The new strings are created by random string generation.

With this population the algorithm may proceed to the next step. This process is convergent [9]. These steps may be presented in a flow diagram as shown in Figure 1.

The following criteria stated by Goldberg [9] and Holland [8] should be met in order to exploit full efficiency of the genetic research.

(a) A gene should be represented with the smallest cardinality of alphabet.
(b) The genetic operators should produce legal solutions in each operation.
(c) The algorithm should be adaptive.

2. EQUATIONS OF MOTION OF MULTI-BODY SYSTEMS

2.1. DEFINITIONS AND ASSUMPTIONS

In formulating and assembling the equations of motion of multi-body systems the following assumptions are made:

(A) It is generally assumed that a mounting (or spring) has zero length. This assumption is acceptable since mountings are relatively small compared with the body they support.
(B) Throughout the analysis it will be assumed that the stiffnesses of the springs in their principal axes of deflection remain uncoupled. Or in other words, a single physical mounting can be represented by three individual springs in three orthogonal directions.
(C) The amplitude of oscillation is small—no geometrical non-linearity is involved.
(D) Dynamic response characteristics of mountings either linear or non-linear are not considered. The system as presented is capable of dealing with certain types of non-linearity. Time-dependent effects are also excluded.
(E) Gyroscopic effects are assumed to be small.
(F) Damping is present in the system.

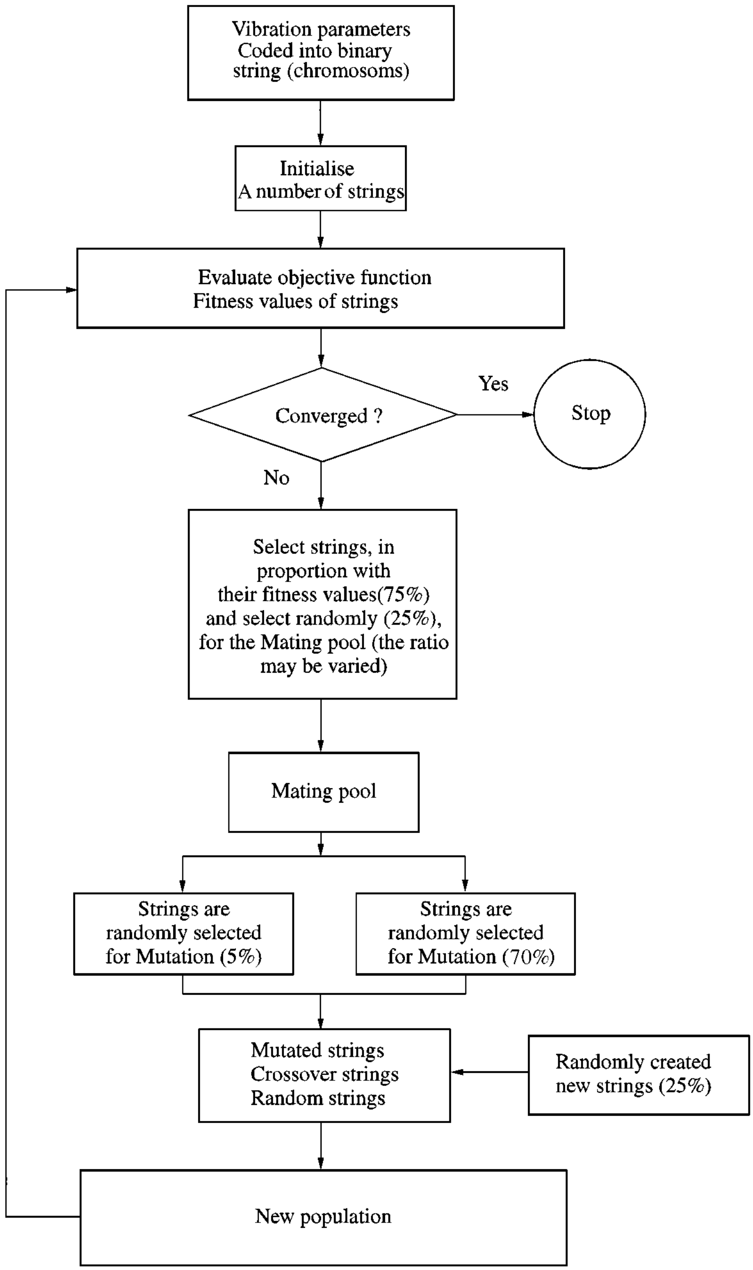

Figure 1. Flow diagram for simple genetic algorithm.

2.2. FORMULATION AND ASSEMBLY OF EQUATIONS

To formulate the equations of motion of a multi-body system, interaction of at least two bodies (Figure 2) should be considered. Let us assume that these bodies are designated as i and j. P_i and P_j are two points on these bodies as shown in Figure 2. In order to formulate the equations of motion, the internal forces acting

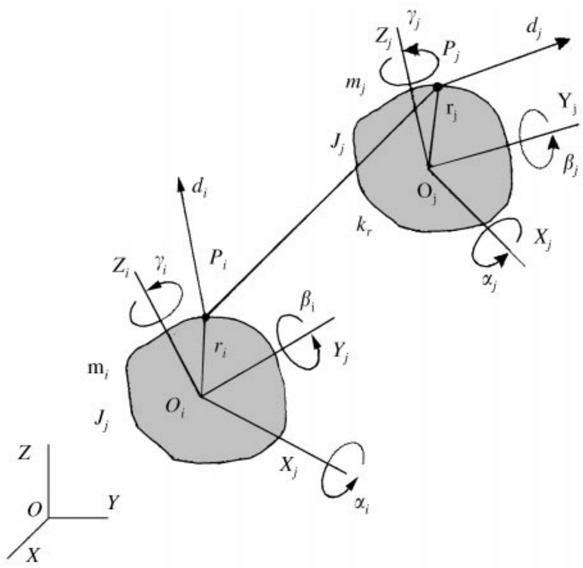

Figure 2. Bodies i and j connected by spring K_r.

on the individual bodies due to their motion relative to each other need to be expressed.

Motion of the origin of axes system i which is fixed to the body i is given by $\mathbf{r}_i = (x_i, y_i, z_i)$, and angular rotation of the axes is given by $\boldsymbol{\alpha}_i = (\alpha_i, \beta_i, \gamma_i)$. Similarly, the motion of body j is described by $\mathbf{r}_{j=}(x_j, y_j, z_j)$ and $\boldsymbol{\alpha}_j = (\alpha_j, \beta_j, \gamma_j)$ where (x_i, y_i, z_i) and (x_j, y_j, z_j) are the positions of centres of origin of the moving axes relative to the global axis and $(\alpha_i, \beta_i, \gamma_i)$ and $(\alpha_i, \beta_i, \gamma_i)$ are the Euler angles for body i and j respectively.

The combined translational and rotational motion of end points, $(a_i\ b_i\ c_i)$ and $(a_j\ b_j\ c_j)$, of the springs on each body, described in the axes of each body frame, is given by

$$\mathbf{d}_i = (x_i\ y_i\ z_i) + (\alpha_i, \beta_i, \gamma_i) \times (a_i\ b_i\ c_i), \tag{1}$$

$$\mathbf{d}_j = (x_j\ y_j\ z_j) + (\alpha_j, \beta_j, \gamma_j) \times (a_j\ b_j\ c_j), \tag{2}$$

where the relative displacement, $\mathbf{d}$, is given by

$$\mathbf{d} = \mathbf{d}_j - \mathbf{d}_i. \tag{3}$$

Reaction forces due to relative displacement on each body, respectively, are given by

$$\mathbf{F}_i = \mathbf{k}_r\mathbf{d}, \quad \mathbf{F}_j = -\mathbf{k}_r\mathbf{d}, \tag{4}$$

where $\mathbf{k}_r$ is the stiffness of spring number r between the two bodies.

The moments are given by

$$\mathbf{M}_i = \mathbf{r}_i \times \mathbf{F}_i \quad \text{and} \quad \mathbf{M}_j = \mathbf{r}_j \times \mathbf{F}_j, \tag{5}$$

where $\mathbf{r}_i = (a_i\ b_i\ c_i)$ and $\mathbf{r}_j = (a_j\ b_j\ c_j)$.

The equations of motion for mass i can be written as

$$\mathbf{m}_i\ddot{\mathbf{x}}_i + \mathbf{k}_r\mathbf{d}_i - \mathbf{k}_r\mathbf{d}_j = \mathbf{F}_i. \tag{6}$$

Now,

$$\mathbf{k}_r\mathbf{d}_i = \mathbf{k}_r(\mathbf{x}_i + \boldsymbol{\alpha}_i \times \mathbf{r}_i),$$

where $\boldsymbol{\alpha}_j = (\boldsymbol{\alpha}_j, \beta_j, \gamma_j)$ and $\mathbf{x}_i$ is a vector defining the displacement of the centre of gravity of mass i.

$$\mathbf{k}_r\mathbf{d}_i = \mathbf{k}_r\mathbf{x}_i + \mathbf{k}_r \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_i & \beta_i & \gamma_i \\ a_i & b_i & c_i \end{vmatrix},$$

where i, j, k are orthogonal unit vectors.

The above cross product can be converted into matrix form:

$$\mathbf{k}_r\mathbf{d}_i = \mathbf{k}_r\mathbf{x}_i + \mathbf{k}_r\mathbf{P}\boldsymbol{\alpha}_i,$$

where

$$\mathbf{P} = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$$

or

$$\mathbf{k}_r\mathbf{d}_i = \mathbf{k}_r\mathbf{x}_i + \mathbf{Lk}_{ir}\boldsymbol{\alpha}_i, \tag{7}$$

where $\mathbf{Lk}_{ir}$ matrix is the matrix obtained for mass i by converting cross product terms into matrix form, i.e.,

$$\mathbf{Lk}_{ir} = \mathbf{k}_r\mathbf{P}.$$

Equation of motion of body i can therefore be written as

$$\mathbf{m}_i\ddot{\mathbf{x}}_i + \mathbf{k}_r\mathbf{x}_i + \mathbf{Lk}_{ir}\boldsymbol{\alpha}_i - \mathbf{k}_r\mathbf{x}_j - \mathbf{Lk}_{jr}\boldsymbol{\alpha}_i = \mathbf{F}_i, \tag{8}$$

where matrix $\mathbf{Lk}_{jr}$ is obtained by using the position vector of spring r on mass j and is obtained in a similar way to matrix $\mathbf{Lk}_{ir}$.

Equation of motion for body j can be written as

$$\mathbf{m}_j\ddot{\mathbf{x}}_j - \mathbf{k}_r\mathbf{x}_i - \mathbf{Lk}_{ir}\boldsymbol{\alpha}_i + \mathbf{k}_r\mathbf{x}_j + \mathbf{Lk}_{jr}\boldsymbol{\alpha}_i = \mathbf{F}_j. \tag{9}$$

Similarly, moment equation may be written as

$$\mathbf{J}_i\ddot{\boldsymbol{\alpha}}_i + \mathbf{r}_i \times (\mathbf{k}_r\mathbf{d}_i - \mathbf{k}_r\mathbf{d}_j) = \mathbf{M}_i. \tag{10}$$

Cross products may be eliminated by a suitable matrix operation,

$$\mathbf{J}_i\ddot{\boldsymbol{\alpha}}_i + \mathbf{kk}_{ir}\mathbf{d}_i - \mathbf{kk}_{ir}\mathbf{d}_j = \mathbf{M}_i, \tag{11}$$

where $\mathbf{kk}_{ir}$ is the matrix resulting from conversion of cross product into matrix form in a similar way to $\mathbf{Lk}_{ir}$.

Now $\mathbf{d}_i$ and $\mathbf{d}_j$ can be expanded as

$$\mathbf{J}_i\ddot{\boldsymbol{\alpha}}_i + \mathbf{kk}_{ir}\mathbf{x}_i + \mathbf{Lkk}_{iir}\boldsymbol{\alpha}_i - \mathbf{kk}_{ir}\mathbf{x}_j - \mathbf{Lkk}_{ijr}\boldsymbol{\alpha}_j = \mathbf{M}_i, \tag{12}$$

$$\mathbf{J}_j\ddot{\boldsymbol{\alpha}}_j - \mathbf{kk}_{jr}\mathbf{x}_i - \mathbf{Lkk}_{jir}\boldsymbol{\alpha}_i + \mathbf{kk}_{jr}\mathbf{x}_j + \mathbf{Lkk}_{jir}\boldsymbol{\alpha}_j = \mathbf{M}_j, \tag{13}$$

where again a conversion similar to that carried out in equation (7) is performed to obtain the matrices

$$\mathbf{Lkk}_{iir}, \mathbf{Lkk}_{ijr}, \mathbf{Lkk}_{iir} \quad \text{and} \quad \mathbf{Lkk}_{jjr}.$$

The suffix i appearing in the matrix notation used in the above matrices implies that the position vector of spring attached on mass i is used in obtaining that matrix. Similary, j implies the same with respect to mass j. For example, $\mathbf{Lkk}_{iir}$ is obtained by performing the cross product $\mathbf{r}_i \times \mathbf{Lk}_{ir}$ and converting the result into matrix form.

Assembling equations (8) and (12) into matrix form

$$\begin{bmatrix} \mathbf{m}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_i \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}}_i \\ \ddot{\boldsymbol{\alpha}}_i \end{Bmatrix} + \begin{bmatrix} \mathbf{k}_r & \mathbf{Lk}_{ir} \\ \mathbf{kk}_{ir} & \mathbf{Lkk}_{iir} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_i \\ \boldsymbol{\alpha}_i \end{Bmatrix} - \begin{bmatrix} \mathbf{k}_r & \mathbf{Lk}_{jr} \\ \mathbf{kk}_{ir} & \mathbf{Lkk}_{ijr} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_j \\ \boldsymbol{\alpha}_j \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_i \\ \mathbf{M}_i \end{Bmatrix}. \tag{14}$$

Similarly, equations (9) and (13) are assembled into

$$\begin{bmatrix} \mathbf{m}_j & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_j \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}}_j \\ \ddot{\boldsymbol{\alpha}}_j \end{Bmatrix} - \begin{bmatrix} \mathbf{k}_r & \mathbf{Lk}_{ir} \\ \mathbf{kk}_{jr} & \mathbf{Lkk}_{jir} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_i \\ \boldsymbol{\alpha}_i \end{Bmatrix} + \begin{bmatrix} \mathbf{k}_r & \mathbf{Lk}_{jr} \\ \mathbf{kk}_{jr} & \mathbf{Lkk}_{jjr} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_j \\ \boldsymbol{\alpha}_j \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_j \\ \mathbf{M}_j \end{Bmatrix}. \tag{15}$$

The above equations can be expanded for the generalized multi-body system by assembling the relevant stiffness and mass terms in their appropriate matrix locations.

3. OPTIMIZATION AND SELECTION OF AN OBJECTIVE FUNCTION

The generalized assembled equation of motion will form the equation of state for the system optimization. Since the genetic algorithm places no specific restriction on the objective function, the following possible objective functions may be selected for vibration optimization problem:

(A) Objective function for minimizing overall energy transfer between selected variables. In mathematical terms, this may be expressed as minimization of the sum of all non-diagonal terms of the global stiffness matrix. This objective function cannot be solved with meaningful results unless an additional constraint is imposed on the problem. This is obvious since a trivial solution exists when all the springs between bodies are removed and bodies are connected only to the ground. It is important not to misinterpret modal decoupling as a solution to this problem. In real life, the modal decoupling or expressing the problem in principal co-ordinates may not have any physical interpretation.

(B) Minimizing coupling between two selected motions (between i and j) of the multi-body system. This, in mathematical terms, may be expressed as

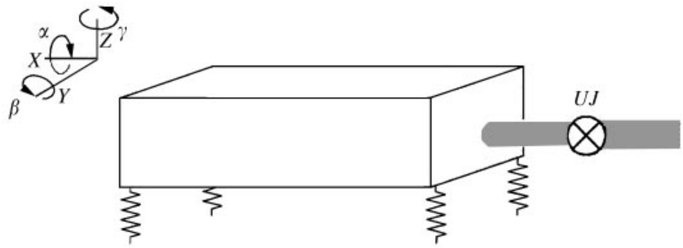

Figure 3. Minimizing the amplitude of oscillation at the universal joint on a drive shaft.

minimization of the terms at the matrix positions *ij* and *ji* corresponding to the rows of the equations of motion. The trivial solution described above (i.e., elimination of stiffness elements between the selected motions of bodies) exists for this objective function. However, there are practical solutions as well. For example, it is known that finding symmetry for mounting arrangement may decouple translational motion from the rotational motion. Generally, in industry, mounting arrangements are designed to ensure that their "centre of stiffness" coincides with the centre of mass. This method is sometimes known as "Remote Centre Compliance". The method is applicable for single-body systems, but its practical value is limited. For multi-body systems this method becomes completely ineffective.

(C) Minimizing amplitude of oscillation at a given frequency. This is the most common engineering statement of vibration synthesis.

This may be expressed for the motion of a body in general as

$$\min\{(x + \theta_y r_z - \theta_z r_y)^2 + (y + \theta_z r_x - \theta_x r_z)^2 + (z + \theta_x r_y - \theta_y r_x)^2\},$$

where $(x\ y\ z)$ is the co-ordinate of the centre of the origin of the axes of the system

$(\theta_x\ \theta_y\ \theta_z)$ is the angular rotation of the spring,

$(r_x\ r_y\ r_z)$ is the position of the endpoint of the spring.

This type of vibration isolation is required in order to minimize vibration transfer from the body with vibration source and the frame or link connecting the body to the rest of the system. As an example, in an engine set-up situation vibration is transferred to the drive shaft system through a universal joint (UJ), Figure 3. In this case, the objective would be to minimize the amplitude of oscillation on the drive shaft.

3.1. ANALYSIS AND RESULTS

In order to formulate the optimization problem, the problem variables need to be selected. It is assumed that the variables vary from an initial nominal value to a maximum value by prescribed increments. To describe the levels of increments a three-bit string is used, giving 7 levels of increments.

The variables used in the synthesis are those that define the mountings. There are six co-ordinates, three at each end of the spring. If one end is fixed to the ground then these are set to zero and the number of variables are reduced to three. The other variables include spring stiffnesses in three orthogonal directions and three Euler angles describing the orientation of the principal axes of the spring. The end positions of the spring may give an indication of the orientation of the spring. In practical situations this information is not used. It is generally assumed that springs may take any orientation without any reference to its end positions. This is a valid assumption since if the spring length is assumed to be zero the angular orientation of springs will have no effect on the positions of its endpoints. The positions of the ends of springs are measured relative to the body axes on which they are connected. Therefore, they appear as two distinct vectors. When each end position vector relative to it's respective body frame is added to the position vector of the centre of axes of their respective bodies in the global frame then the result should be the same. This is because spring lengths are taken to be zero. This assumption is used in reducing the number of independent variables. If the centres of bodies are known then springs may be described only in terms of a single vector relative to the centre of mass of body giving its absolute position. It is sufficient to know the spring end position relative to one of the bodies only. With reference to Figure 4, we therefore have

$$\mathbf{r}_{gi} + \mathbf{r}_{ki} = \mathbf{r}_{gj} + \mathbf{r}_{kj}, \tag{16a}$$

where $\mathbf{r}_{gi}$ and $\mathbf{r}_{gj}$ are the position vectors of the centre of gravity of masses i and j respectively. $\mathbf{r}_{ki}$ and $\mathbf{r}_{kj}$ are the position vectors of the kth stiffness on masses i and j respectively.

3.1.1. *Summary of the optimization problem*

(a) *Control variables*

$\mathbf{r}_{ki}$, $\mathbf{r}_{gi}$, $\mathbf{r}_{kj}$, $\mathbf{r}_{gj}$	(defined in Figure 4)
$\boldsymbol{\alpha}_i$ $\boldsymbol{\alpha}_j$	angular rotation of the body i and j axis respectively

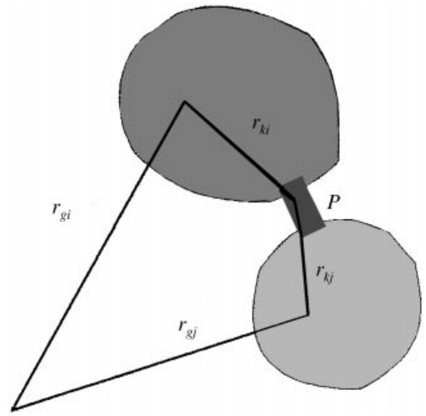

Figure 4. Spring-configuration.

$\mathbf{k}_x$, $\mathbf{k}_y$, $\mathbf{k}_z$ spring stiffnesses in directions x, y and z respectively
$\mathbf{r}_{gi}$, $\mathbf{r}_{gj}$ position vectors of the centre of gravity of bodies i and j respectively

(b) *Constraints*

The following are the constraints of the problem:

$$\mathbf{r}_{gi} + \mathbf{r}_{ki} = \mathbf{r}_{gj} + \mathbf{r}_{kj},$$

$$r_{kil} \leqslant r_{ki} \leqslant r_{kiu},$$

$$r_{kjl} \leqslant r_{ki} \leqslant r_{kju},$$

where r_{kil} and r_{kiu} are the lower and upper limits of r_{ki} and r_{kjl} and r_{kju} are the lower and upper limits of r_{kj}.

$$k_{xl} \leqslant k_x \leqslant k_{xu},$$

$$k_{yl} \leqslant k_y \leqslant k_{yu},$$

$$k_{zl} \leqslant k_z \leqslant k_{zu},$$

where r_{xl}, r_{xu}, r_{yl}, r_{yu}, r_{zl}, r_{zu} are the lower and upper limits of stiffnesses in the x, y and z directions.

(c) *Objective Functions*

(i) For the case of minimization of selected cross coupling the objective function is

$$J = \min \sum_{r=1}^{4} \sum_{i=1}^{3} \sum_{j=1}^{3} \mathbf{Lkk}_{ijr}.$$

(ii) For the case of minimization of the amplitude of oscillation the objective function is

$$\min\{(x + \theta_y r_z - \theta_z r_y)^2 + (y + \theta_z r_x - \theta_x r_z)^2 + (z + \theta_x r_y - \theta_y r_x)^2\}.$$

3.1.2. *An example run*

An implementation of data format in the software suite VIBRATIO [6, 7] is shown in Table 1.

Table 1 gives data fragment for a two-mass system. For the first example given below, only mass number one is employed. For the second example both the bodies are used. All the variables are assumed to have seven discrete levels or able to attain eight possible values. The initial and increment values for each variable may be adjusted. For both of the examples given below, the following numbering scheme is used

Spring and the centre of gravity positions

$x = 500$ mm binary 0 0 0 increment $= 100$
$x = 1200$ mm binary 1 1 1
y and z are also set in the same manner.

TABLE 1

A two-mass problem
Number of mounting stiffness types = 1

Mount no	kx	ky	kz				
1	10	30	70				
GX	GY	GZ					
0	0	0					
MASS	IXX	IYY	IZZ				
1000	100	200	300				
Constr	IXY	IXZ	IYZ				
	0	0	0				
Mount no	X	Y	Z	ALPH	BETA	GAM	TYPE
1	1000	1000	—	0	0	0	1
			1000				
2	1000	—	—	0	0	0	1
		1000	1000				
3	—	—	—	0	0	0	1
	1000	1000	1000				
4	—	1000	—	0	0	0	1
	1000		1000				
GX	GY	GZ					
0	0	1500					
MASS	IXX	IYY	IZZ				
1200	90	150	200				
Constr	IXY	IXZ	IYZ				
	0	0	0				
Mount no	X	Y	Z	ALPH	BETA	GAM	TYPE
1	1000	1000	—	0	0	0	1
			1000				
2	1000	—	—	0	0	0	1
		1000	1000				
3	—	—	—	0	0	0	1
	1000	1000	1000				
4	—	1000	—	0	0	0	1
	1000		1000				

Note: kx, ky, kz, spring stiffnesses in the x, y and z local directions; IXX, IYY, IZZ moments of inertia relative to the centres of gravity; IXY, IXZ, IYZ, cross moments of inertia relative to the centre of gravity; ALPH, BETA, GAM, Euler angles (α, β, γ); GX, GY, GZ, gravitational acceleration in the x, y and z directions.

The heading "type" refers to spring definition. "Option" and "constr" are not relevant for this.

Stiffness values

$kx = 1$ kN binary 0 0 0 increment = 10

$kx = 71$ kN binary 1 1 1

ky and kz are set in the same manner.

Euler angles

$a = 0°$ binary 0 0 0 increment $= 7°$

$a = 49$ binary 1 1 1

b and g are set in the same manner.

3.1.2.1. *Minimizing selected cross coupling. A one-mass problem.* It is assumed that mounting configuration is described in terms of 36 variables, three positions, three stiffnesses and three Euler angles for each of the four mountings. Each variable is assumed to be defined in seven levels as described above, therefore each binary number has three bits. Therefore, the total length of the binary string is 118 bits.

The objective function for this case is selected to be,

$$J = \min \sum_{r=1}^{4} \sum_{i=1}^{3} \sum_{j=1}^{3} \mathbf{Lkk}_{ijr}. \tag{16b}$$

The main cause of coupling in this problem is the mountings being away from the centre of the mass (Figure 5). Therefore, an inertial force in O_x or O_y direction will generate motion in α or β direction. By comparing the results of Tables 2 and 3 an improved decoupling in y relative to β and x relative to α motion before and after optimization can be seen.

3.1.2.2. *Minimizing amplitude of oscillation at a point. Two mass problem.* The data given in Table 1 is used in constructing the state vector of optimization. In this case, the number of variables for each spring is the same as before but the system is supported by eight springs and therefore 72 variables are required to describe all the spring parameters. The system is shown in Figure 6. Another three variables are needed to describe the centre of mass of body 2. Therefore, the problem requires 75 variables. In this case the length of the binary vector is reduced from 3 to 2. The minimum values of variables and increments are the same as before.

In this case the objective function is selected to be,

$$J = \min\{(x_{g2} + \theta_{2y}r_z - \theta_{2z}r_y)^2 + (y_{g2} + \theta_{2z}r_x - \theta_{2x}r_z)^2 + (z_{g2} + \theta_{2x}r_y - \theta_{2y}r_x)^2\}. \tag{17}$$

In order to calculate the time-dependent variables appearing above, the system equations should be solved. In general, the steady state solutions of the equations of

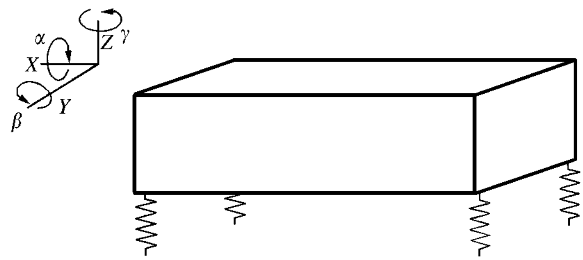

Figure 5. Single-mass problem.

TABLE 2

Single-mass problem before optimization

Relative eigenvector values for mass = 1

X	Y	Z	ALPHA	BETA	GAMA
Frequency in X = 0·94 Hz (56. CPM)					
1·0000	0·0000	0·0000	0·0000	0·1278	0·0000
Frequency in Y = 1·45 Hz (87. CPM)					
0·0000	1·0000	0·0000	– 0·3064	0·0000	0·0000
Frequency in Z = 2·66 Hz (160. CPM)					
0·0000	0·0000	1·0000	0·0000	0·000	0·0000
Frequency in ALPHA = 10·12 Hz (607. CPM)					
0·0000	0·0306	0·0000	1·0000	0·0000	0·0000
Frequency in BETA = 6·38 Hz (383. CPM)					
0·0256	0·0000	0·0000	0·0000	– 1·0000	0·0000
Frequency in GAMA = 3·68 Hz (221. CPM)					
0·0000	0·0000	– 0·0001	0·0000	0·0000	1·0000

The parameters are the same as in Table 1.

TABLE 3

Single-mass problem after optimization

Relative eigenvector values for mass = 1

X	Y	Z	ALPHA	BETA	GAMA
Frequency in X = 1·00 Hz (60. CPM)					
1·0000	0·0000	0·0000	0·0000	0·0435	0·0000
Frequency in Y = 1·71 Hz (103. CPM)					
0·0000	1·0000	0·0000	– 0·1289	0·0000	0·0000
Frequency in Z = 2·66 Hz (160. CPM)					
0·0000	0·0000	1·0000	0·0000	0·0000	0·0000
Frequency in ALPHA = 8·59 Hz (516. CPM)					
0·0000	0·0129	0·0000	1·0000	0·0000	0·0000
Frequency in BETA = 6·00 Hz (360. CPM)					
– 0·0087	0·0000	0·0000	0·0000	1·0000	0·0000
Frequency in GAMA = 3·68 Hz (221. CPM)					
0·0000	0·0000	0·0000	0·0000	0·0000	1·0000

The parameters are the same as in Table 1.

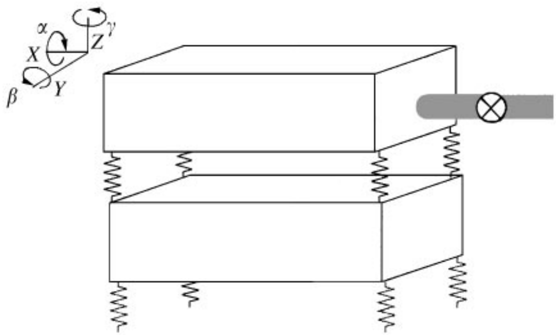

Figure 6. Two-mass problem.

motion are to be minimized. The system equations under steady state sinusoidal excitation is given as

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F}_1 \sin \omega t + F_2 \cos \omega t, \tag{18}$$

where $\ddot{\mathbf{X}}$, $\dot{\mathbf{X}}$, $\mathbf{X}$, are, respectively, the acceleration, velocity and displacement vectors of the assembled equation of motion, $\mathbf{F}_1$ and $\mathbf{F}_2$ are vectors containing amplitudes of the forcing exitation, **M** is the assembled mass matrix, **C** is the assembled damping matrix, and **K** is the assembled stiffness matrix, seeking a solution in the following form:

$$\mathbf{X} = \mathbf{A} \sin \omega t + \mathbf{B} \cos \omega\, t, \tag{19}$$

where **A** and **B** are vectors containing amplitudes of the output displacement.

The steady state solution may be obtained in the matrix form as

$$\begin{bmatrix} (\mathbf{K} - \omega^2 \mathbf{M}) & -\omega \mathbf{C} \\ \omega \mathbf{C} & (\mathbf{K} - \mathbf{M}\omega^2) \end{bmatrix} \begin{Bmatrix} \mathbf{A} \\ \mathbf{B} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{Bmatrix}, \tag{20}$$

where **A** and **B** are the amplitudes of motion variables at frequency *w*. Therefore, the objective function given above may be constructed from the selected elements of **A** and **B**.

The response which is presented as a time domain solution before and after optimization is given in Figures 7(a) and (b). These two examples are selected arbitrarily. Forcing excitation and frequency are kept the same before and after optimization but the two examples represent different forcing and frequency selections.

4. COMMENTS AND CONCLUSIONS

Some early results from our work on application of GA to vibration synthesis is presented. The initial results of this investigation are encouraging. A possible constraint in the application of the genetic algorithm to vibration synthesis can occur if the objective function cannot be expressed functionally in terms of the

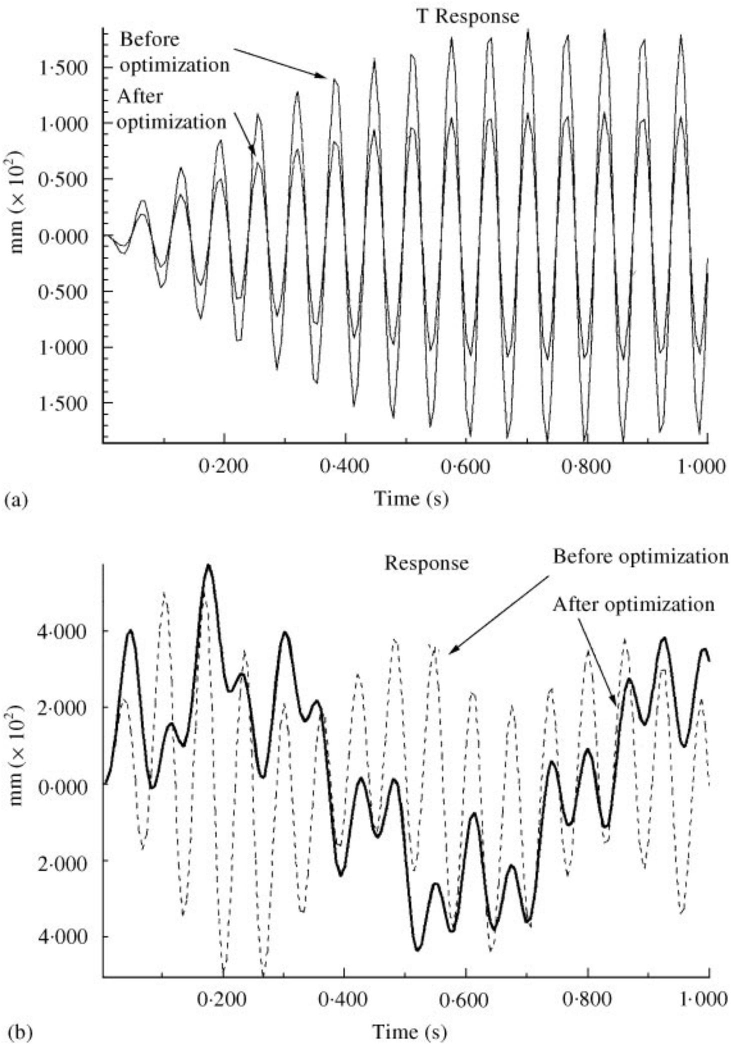

Figure 7. The time-domain solutions before and after optimization.

optimization variables. This situation may arise if, for example, the design variables are used in some commercial analysis package from which the objective function is obtained. Here, GA is applied without any explicit information that the objective function is really a function of the design variables. As the analysis was confined to simple functions, it was ensured that a functional relationship existed between the objective function and design variables.

The tests carried out in this investigation are mostly conclusive, although in some situations the results are not easy to interpret. A distinct improvement is observed in decoupling modes, demonstrated by eigenvalue analysis. The response analysis carried out for two different forcing functions gave less conclusive results. The first example shown in Figure 7(a), demonstrates a clear reduction in amplitude. The result obtained in the second example is not conclusive, the amplitude at the selected frequency, is clearly reduced but the overall response of the system which is superimposed with another frequency has not been reduced in its maximum amplitude. This is possibly due to rough discretization which was used in this analysis. It is anticipated that a finer sampling of the variables would lead to improved results. In order to reduce the computation time it may also be possible to reduce the number of variables by assuming reasonable values for some of the variables and increasing the length of the binary vector for the rest of the variable.

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