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A GENERALIZATION OF BODINE'S PROBLEM TO THE CASE OF PLATES OF POLAR ORTHOTROPY

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1. INTRODUCTION

The problem of free vibrations of an isotropic circular plate with a free edge and a concentric circular support was studied by Bodine in a well-known publication [1]; see Figure 1.

Since composite materials have become quite popular in the last four decades and Bodine's problem is of interest in several technological situations, the present analytical investigation was undertaken in order to determine the fundamental frequency coefficient of the structural system shown in Figure 1 when the plate material is cylindrically anisotropic [2].

The fundamental frequency is determined for a wide range of values of the intervening geometric and constitutive parameters. For the sake of generality, the presence of a central, concentrated mass is taken into account and the influence of the parameter M/M_p, where M is the concentrated mass and M_p the total plate mass, is investigated.

2. APPROXIMATE SOLUTION BY MEANS OF THE OPTIMIZED RAYLEIGH–RITZ METHOD

In the case of normal, axisymmetric, modes of transverse vibration the problem is described by the well-known functional [2, 3]

$$J[W] = \frac{1}{2}\iint\left\{D_r\left(\frac{\mathrm{d}^2W}{\mathrm{d}r^2}\right)^2 + D_\theta\left(\frac{1}{r}\frac{\mathrm{d}W}{\mathrm{d}r}\right)^2 + 2D_r\,\upsilon_\theta\frac{\mathrm{d}^2W}{\mathrm{d}r^2}\left(\frac{1}{r}\frac{\mathrm{d}W}{\mathrm{d}r}\right)\right\}r\mathrm{d}r\,d\theta$$
$$-\frac{\omega^2}{2}\left(\rho\iint h\,W^2 r\,\mathrm{d}r\,\mathrm{d}\theta + MW^2(0)\right) \qquad (1)$$

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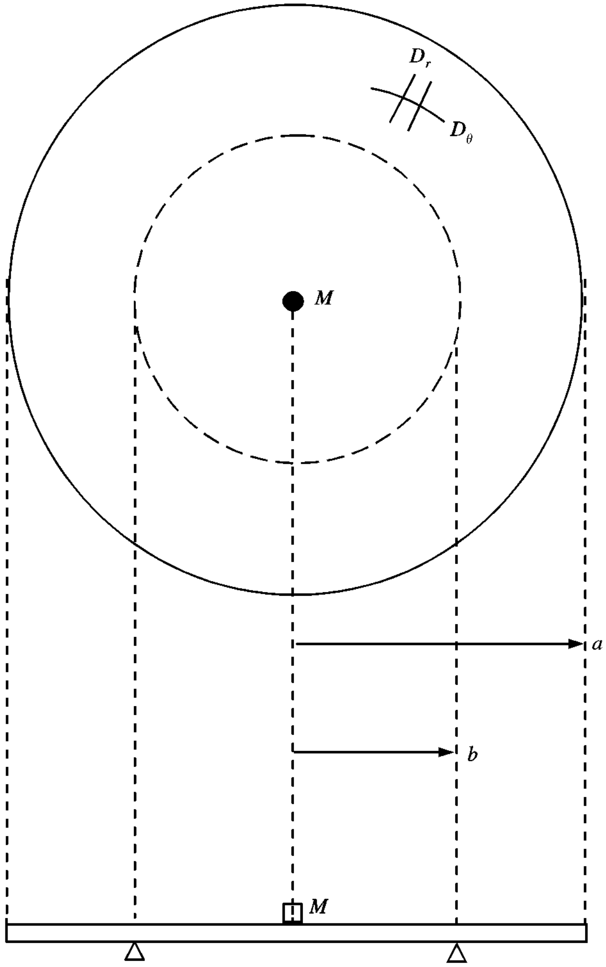

Figure 1. Plate of polar orthotropy executing transverse vibrations.

and appropriate boundary conditions. With regard to them, it is a legitimate procedure to use the geometric condition

$$W(b) = 0 \tag{2}$$

and the natural condition at $r = a$ which expresses that the amplitude of the radial bending moment at the plate edge is null:

$$\left.\frac{\mathrm{d}^2 W}{\mathrm{d}r^2} + \frac{\upsilon_\theta}{r}\frac{\mathrm{d}W}{\mathrm{d}r}\right|_{r=a} = 0. \tag{3}$$

The condition that the Kelvin–Kirchhoff force is zero at $r = a$ is not taken into taken into account since this simplifies the algorithmic procedure considerably on one hand and on the other the solution will be valid in the limiting situation, where $b = a$ which corresponds to the simply supported plate configuration.

TABLE 1

Values of Ω_1 as a function of D_θ/D_r and b/a for $M/M_p = 0$ and 0·1

$\frac{M}{M_p}$	$\frac{b}{a}$	v_θ	D_θ/D_r 0·5	0·75	1	1·25	1·5	1·75	2
0	0	0·30	2·906	3·362	3·752	4·099	4·415	4·708	4·982
		1/3	*2·931*	*3·385*	*3·773*	*4·119*	*4·434*	*4·725*	*4·998*
0	0·1	0·30	3·072	3·525	3·912	4·258	4·573	4·865	5·139
		1/3	*3·100*	*3·550*	*3·935*	*4·279*	*4·593*	*4·885*	*5·157*
0	0·2	0·30	3·415	3·881	4·279	4·635	4·959	5·260	5·542
		1/3	*3·449*	*3·911*	*4·307*	*4·661*	*4·984*	*5·283*	*5·564*
0	0·3	0·30	3·938	4·435	4·859	5·237	5·581	5·899	6·198
		1/3	*3·980*	*4·473*	*4·894*	*5·269*	*5·611*	*5·928*	*6·225*
0	0·4	0·30	4·699	5·254	5·724	6·140	6·518	6·867	7·194
		1/3	*4·752*	*5·303*	*5·769*	*6·182*	*6·558*	*6·905*	*7·230*
0	0·5	0·30	5·749	6·411	6·964	7·449	7·887	8·290	8·666
		1/3	*5·819*	*6·475*	*7·023*	*7·505*	*7·940*	*8·341*	*8·715*
0	0·6	0·30	6·909	7·734	8·425	9·031	9·578	10·079	10·546
		1/3	*6·994*	*7·807*	*8·499*	*9·101*	*9·645*	*10·144*	*10·608*
0	0·7	0·30	7·277	8·193	8·969	9·656	10·280	10·856	11·394
		1/3	*7·359*	*8·269*	*9·040*	*9·723*	*10·344*	*10·917*	*11·453*
0	0·8	0·30	6·361	7·156	7·828	8·423	8·963	9·461	9·926
		1/3	*6·431*	*7·219*	*7·887*	*8·474*	*9·015*	*9·510*	*9·973*
0	0·9	0·30	5·111	5·729	6·250	6·710	7·126	7·510	7·868
		1/3	*5·172*	*5·786*	*6·303*	*6·760*	*7·174*	*7·556*	*7·912*
0	1	0·30	4·073	4·543	4·936	5·282	5·594	5·880	6·147
		1/3	*4·128*	*4·594*	*4·985*	*5·329*	*5·639*	*5·925*	*6·190*
0·1	0	0·30	2·906	3·362	3·752	4·099	4·415	4·708	4·982
		1/3	*2·931*	*3·385*	*3·773*	*4·119*	*4·434*	*4·725*	*4·998*
0·1	0·1	0·30	3·072	3·524	3·912	4·257	4·573	4·865	5·139
		1/3	*3·100*	*3·549*	*3·935*	*4·279*	*4·593*	*4·884*	*5·157*
0·1	0·2	0·30	3·411	3·877	4·276	4·632	4·956	5·257	5·539
		1/3	*3·444*	*3·907*	*4·304*	*4·658*	*4·981*	*5·280*	*5·561*
0·1	0·3	0·30	3·917	4·417	4·843	5·222	5·567	5·886	6·185
		1/3	*3·958*	*4·454*	*4·877*	*5·254*	*5·597*	*5·915*	*6·212*
0·1	0·4	0·30	4·617	5·184	5·661	6·082	6·463	6·816	7·145
		1/3	*4·669*	*5·231*	*5·705*	*6·123*	*6·503*	*6·853*	*7·181*
0·1	0·5	0·30	5·455	6·144	6·719	7·222	7·674	8·088	8·474
		1/3	*5·518*	*6·203*	*6·774*	*7·274*	*7·724*	*8·137*	*8·520*
0·1	0·6	0·30	6·053	6·879	7·575	8·189	8·744	9·255	9·730
		1/3	*6·119*	*6·941*	*7·634*	*8·245*	*8·798*	*9·307*	*9·781*
0·1	0·7	0·30	5·841	6·665	7·365	7·986	8·550	9·070	9·557
		1/3	*5·898*	*6·718*	*7·415*	*8·032*	*8·594*	*9·112*	*9·597*
0·1	0·8	0·30	5·045	5·742	6·332	6·854	7·328	7·765	8·173
		1/3	*5·096*	*5·788*	*6·375*	*6·895*	*7·366*	*7·801*	*8·207*
0·1	0·9	0·30	4·175	4·728	5·195	5·607	5·980	6·325	6·646
		1/3	*4·222*	*4·772*	*5·236*	*5·646*	*6·018*	*6·361*	*6·681*
0·1	1	0·30	3·441	3·871	4·232	4·550	4·836	5·100	5·345
		1/3	*3·485*	*3·913*	*4·272*	*4·588*	*4·873*	*5·136*	*5·380*

TABLE 2

Values of Ω_1 as a function of D_θ/D_r and b/a for $M/M_p = 0{\cdot}2$ and $0{\cdot}3$

			D_θ/D_r						
$\frac{M}{M_p}$	$\frac{b}{a}$	ν_θ	0·5	0·75	1	1·25	1·5	1·75	2
0·2	0	0·30	2·906	3·362	3·752	4·099	4·415	4·708	4·982
		1/3	*2·931*	*3·385*	*3·773*	*4·119*	*4·434*	*4·725*	*4·998*
0·2	0·1	0·30	3·072	3·524	3·911	4·257	4·573	4·865	5·139
		1/3	*3·100*	*3·549*	*3·935*	*4·279*	*4·593*	*4·884*	*5·157*
0·2	0·2	0·30	3·406	3·873	4·273	4·629	4·953	5·254	5·536
		1/3	*3·439*	*3·903*	*4·300*	*4·654*	*4·978*	*5·277*	*5·558*
0·2	0·3	0·30	3·895	4·398	4·826	5·206	5·552	5·872	6·172
		1/3	*3·935*	*4·435*	*4·860*	*5·238*	*5·582*	*5·901*	*6·199*
0·2	0·4	0·30	4·531	5·109	5·593	6·019	6·405	6·761	7·092
		1/3	*4·581*	*5·155*	*5·636*	*6·060*	*6·444*	*6·797*	*7·127*
0·2	0·5	0·30	5·167	5·867	6·453	6·968	7·432	7·857	8·251
		1/3	*5·224*	*5·920*	*6·504*	*7·016*	*7·478*	*7·901*	*8·294*
0·2	0·6	0·30	5·385	6·170	6·837	7·427	7·963	8·457	8·919
		1/3	*5·438*	*6·219*	*6·883*	*7·471*	*8·005*	*8·498*	*8·958*
0·2	0·7	0·30	4·976	5·709	6·333	6·887	7·390	7·855	8·290
		1/3	*5·021*	*5·750*	*6·371*	*6·922*	*7·424*	*7·887*	*8·320*
0·2	0·8	0·30	4·288	4·903	5·426	5·889	6·310	6·698	7·061
		1/3	*4·329*	*4·941*	*5·461*	*5·922*	*6·341*	*6·727*	*7·089*
0·2	0·9	0·30	3·603	4·099	4·520	4·891	5·229	5·540	5·831
		1/3	*3·642*	*4·136*	*4·554*	*4·924*	*5·260*	*5·570*	*5·860*
0·2	1	0·30	3·025	3·419	3·750	4·042	4·306	4·549	4·775
		1/3	*3·062*	*3·454*	*3·784*	*4·075*	*4·338*	*4·580*	*4·806*
0·3	0	0·30	2·906	3·362	3·752	4·099	4·415	4·708	4·982
		1/3	*2·931*	*3·385*	*3·773*	*4·119*	*4·434*	*4·725*	*4·998*
0·3	0·1	0·30	3·071	3·523	3·911	4·257	4·572	4·865	5·138
		1/3	*3·099*	*3·548*	*3·934*	*4·278*	*4·593*	*4·884*	*5·157*
0·3	0·2	0·30	3·401	3·869	4·269	4·625	4·950	5·252	5·534
		1/3	*3·434*	*3·899*	*4·297*	*4·651*	*4·975*	*5·274*	*5·555*
0·3	0·3	0·30	3·872	4·379	4·809	5·190	5·537	5·858	6·158
		1/3	*3·912*	*4·415*	*4·843*	*5·222*	*5·567*	*5·886*	*6·185*
0·3	0·4	0·30	4·443	5·029	5·521	5·953	6·343	6·702	7·036
		1/3	*4·491*	*5·074*	*5·562*	*5·992*	*6·380*	*6·737*	*7·070*
0·3	0·5	0·30	4·897	5·595	6·183	6·701	7·169	7·600	8·000
		1/3	*4·948*	*5·642*	*6·228*	*6·744*	*7·211*	*7·640*	*8·039*
0·3	0·6	0·30	4·873	5·608	6·234	6·789	7·296	7·763	8·200
		1/3	*4·917*	*5·649*	*6·273*	*6·827*	*7·332*	*7·798*	*8·234*
0·3	0·7	0·30	4·398	5·060	5·624	6·124	6·579	7·000	7·393
		1/3	*4·436*	*5·094*	*5·655*	*6·153*	*6·607*	*7·026*	*7·417*
0·3	0·8	0·30	3·788	4·343	4·815	5·233	5·613	5·965	6·293
		1/3	*3·824*	*4·375*	*4·845*	*5·261*	*5·640*	*5·990*	*6·316*
0·3	0·9	0·30	3·212	3·665	4·048	4·388	4·696	4·981	5·247
		1/3	*3·247*	*3·696*	*4·078*	*4·416*	*4·724*	*5·007*	*5·272*
0·3	1	0·30	2·727	3·091	3·398	3·669	3·914	4·140	4·350
		1/3	*2·760*	*3·123*	*3·429*	*3·698*	*3·942*	*4·167*	*4·376*

TABLE 3

Values of Ω_1 as a function of D_θ/D_r and b/a for $M/M_p = 0{\cdot}4$ and $0{\cdot}5$

$\frac{M}{M_p}$	$\frac{b}{a}$	ν_θ	D_θ/D_r 0·5	0·75	1	1·25	1·5	1·75	2
0·4	0	0·30	2·906	3·362	3·752	4·099	4·415	4·708	4·982
		1/3	*2·931*	*3·385*	*3·773*	*4·119*	*4·434*	*4·725*	*4·998*
0·4	0·1	0·30	3·071	3·523	3·911	4·257	4·572	4·864	5·136
		1/3	*3·098*	*3·548*	*3·934*	*4·278*	*4·592*	*4·884*	*5·156*
0·4	0·2	0·30	3·397	3·865	4·266	4·622	4·947	5·249	5·531
		1/3	*3·430*	*3·895*	*4·293*	*4·648*	*4·972*	*5·272*	*5·553*
0·4	0·3	0·30	3·849	4·359	4·791	5·174	5·522	5·843	6·144
		1/3	*3·889*	*4·395*	*4·824*	*5·205*	*5·551*	*5·871*	*6·171*
0·4	0·4	0·30	4·353	4·947	5·445	5·882	6·276	6·638	6·975
		1/3	*4·399*	*4·989*	*5·485*	*5·920*	*6·312*	*6·673*	*7·009*
0·4	0·5	0·30	4·651	5·338	5·920	6·434	6·900	7·330	7·730
		1/3	*4·696*	*5·380*	*5·960*	*6·473*	*6·937*	*7·365*	*7·764*
0·4	0·6	0·30	4·474	5·162	5·750	6·272	6·747	7·187	7·598
		1/3	*4·512*	*5·197*	*5·782*	*6·302*	*6·776*	*7·214*	*7·624*
0·4	0·7	0·30	3·981	4·587	5·104	5·562	5·980	6·365	6·725
		1/3	*4·014*	*4·617*	*5·131*	*5·587*	*6·003*	*6·387*	*6·746*
0·4	0·8	0·30	3·429	3·937	4·369	4·753	5·102	5·425	5·408
		1/3	*3·460*	*3·966*	*4·406*	*4·778*	*5·126*	*5·447*	*5·747*
0·4	0·9	0·30	2·925	3·343	3·697	4·011	4·296	4·560	4·806
		1/3	*2·956*	*3·371*	*3·724*	*4·036*	*4·321*	*4·583*	*4·829*
0·4	1	0·30	2·502	2·842	3·128	3·381	3·610	3·821	4·018
		1/3	*2·532*	*2·870*	*3·155*	*3·407*	*3·636*	*3·846*	*4·042*
0·5	0	0·30	2·906	3·362	3·752	4·099	4·411	4·708	4·982
		1/3	*2·931*	*3·385*	*3·773*	*4·119*	*4·434*	*4·725*	*4·998*
0·5	0·1	0·30	3·070	3·523	3·910	4·256	4·572	4·864	5·138
		1/3	*3·098*	*3·548*	*3·933*	*4·278*	*4·592*	*4·883*	*5·156*
0·5	0·2	0·30	3·392	3·861	4·262	4·619	4·944	5·246	5·528
		1/3	*3·425*	*3·891*	*4·289*	*4·644*	*4·968*	*5·269*	*5·550*
0·5	0·3	0·30	3·825	4·338	4·772	5·157	5·506	5·828	6·130
		1/3	*3·865*	*4·374*	*4·806*	*5·188*	*5·535*	*5·856*	*6·156*
0·5	0·4	0·30	4·264	4·862	5·364	5·806	6·204	6·570	6·910
		1/3	*4·308*	*4·903*	*5·403*	*5·842*	*6·239*	*6·603*	*6·942*
0·5	0·5	0·30	4·428	5·100	5·671	6·177	6·636	7·060	7·456
		1/3	*4·469*	*5·133*	*5·707*	*6·211*	*6·669*	*7·092*	*7·486*
0·5	0·6	0·30	4·154	4·801	5·354	5·845	6·292	6·706	7·092
		1/3	*4·188*	*4·831*	*5·381*	*5·871*	*6·316*	*6·728*	*7·114*
0·5	0·7	0·30	3·662	4·224	4·703	5·128	5·515	5·878	6·206
		1/3	*3·692*	*4·251*	*4·727*	*5·151*	*5·536*	*5·892*	*6·225*
0·5	0·8	0·30	3·155	3·626	4·027	4·384	4·708	5·007	5·287
		1/3	*3·181*	*3·649*	*4·048*	*4·403*	*4·725*	*5·023*	*5·302*
0·5	0·9	0·30	2·702	3·092	3·422	3·716	3·982	4·228	4·459
		1/3	*2·730*	*3·118*	*3·447*	*3·739*	*4·004*	*4·250*	*4·479*
0·5	1	0·30	2·324	2·643	2·913	3·151	3·366	3·565	3·750
		1/3	*2·351*	*2·669*	*2·938*	*3·175*	*3·390*	*3·588*	*3·773*

TABLE 4

Maximum values of Ω_1 as a function of M/M_p and D_θ/D_r and the corresponding values of b/a

		$\nu_\theta = 0{\cdot}30$		$\nu_\theta = 1/3$	
$\frac{M}{M_p}$	$\frac{D_\theta}{D_r}$	$\frac{b}{a}$	Ω_1	$\frac{b}{a}$	Ω_1
0	0·5	0·675	7·323	0·675	7·408
0	0·75	0·677	8·239	0·677	8·317
0	1	0·679	9·013	0·679	9·086
0	1·25	0·681	9·698	0·680	9·768
0	1·5	0·682	10·320	0·681	10·386
0	1·75	0·683	10·894	0·682	10·958
0	2	0·684	11·430	0·683	11·492
0·1	0·5	0·625	6·080	0·624	6·150
0·1	0·75	0·629	6·930	0·628	6·990
0·1	1	0·632	7·646	0·631	7·703
0·1	1·25	0·634	8·280	0·634	8·334
0·1	1·5	0·636	8·856	0·636	8·907
0·1	1·75	0·638	9·387	0·637	9·437
0·1	2	0·639	9·884	0·639	9·931
0·2	0·5	0·582	5·397	0·581	5·451
0·2	0·75	0·587	6·177	0·586	6·228
0·2	1	0·591	6·841	0·590	6·888
0·2	1·25	0·594	7·429	0·593	7·474
0·2	1·5	0·596	7·964	0·596	8·007
0·2	1·75	0·598	8·458	0·598	8·499
0·2	2	0·600	8·919	0·599	8·959
0·3	0·5	0·545	4·960	0·544	5·008
0·3	0·75	0·551	5·692	0·550	5·737
0·3	1	0·556	6·316	0·555	6·357
0·3	1·25	0·559	6·869	0·558	6·909
0·3	1·5	0·562	7·372	0·561	7·410
0·3	1·75	0·564	7·838	0·563	7·874
0·3	2	0·566	8·273	0·566	8·307
0·4	0·5	0·513	4·656	0·512	4·701
0·4	0·75	0·520	5·353	0·519	5·394
0·4	1	0·525	5·946	0·524	5·984
0·4	1·25	0·529	6·473	0·528	6·509
0·4	1·5	0·532	6·953	0·531	6·987
0·4	1·75	0·535	7·396	0·534	7·429
0·4	2	0·537	7·810	0·536	7·842
0·5	0·5	0·486	4·433	0·485	4·475
0·5	0·75	0·493	5·102	0·492	5·140
0·5	1	0·499	5·672	0·498	5·707
0·5	1·25	0·503	6·178	0·502	6·212
0·5	1·5	0·506	6·639	0·505	6·671
0·5	1·75	0·509	7·065	0·508	7·096
0·5	2	0·511	7·464	0·510	7·493

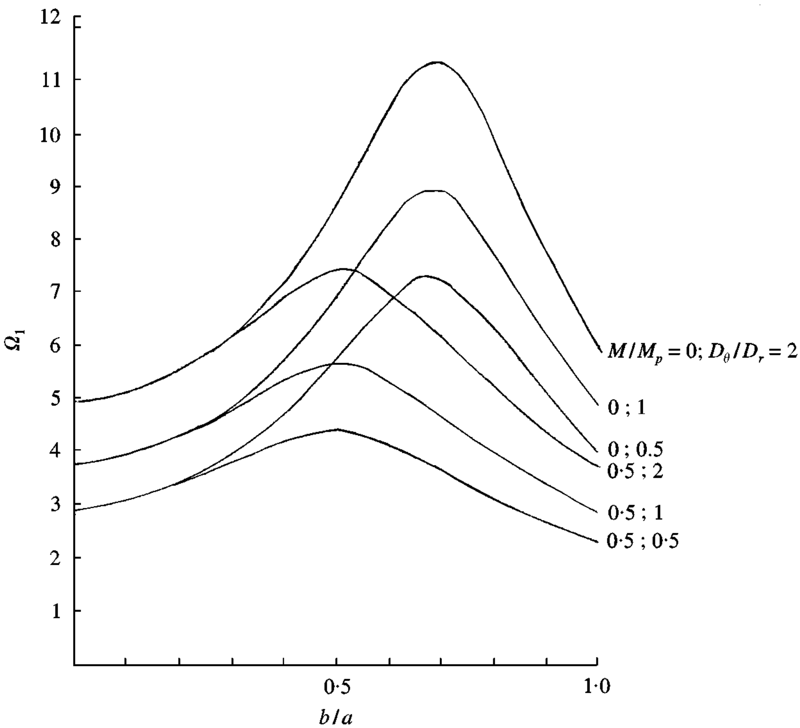

Figure 2. Variation of $\Omega_1\sqrt{(\rho h/D_r)}$ as a function of b/a for different values of M/M_p and D_θ/D_r ($\nu_\theta = 0{\cdot}3$).

Following previous works, the following approximate expression for W has been used,

$$W \cong W_a = C_1(\alpha_p r^p + \alpha_q r^q + 1) + C_2(\beta_p r^{p+1} + \beta_q r^{q+1} + 1), \tag{4}$$

where the α's and β's of each co-ordinate function are determined substituting them in equations (2) and (3). The exponential parameters "p" and "q" are Rayleigh's optimization exponents which allow for appropriate minimization of the eigenvalue under investigation [4].

Substituting equation (4) in equation (1) and requiring that

$$\frac{\partial J[W_a]}{\partial C_i} = 0 \quad (i = 1, 2), \tag{5}$$

one obtains a linear homogeneous system of equations in C_1 and C_2. The non-triviality condition leads to a determinantal equation whose lowest root constitutes the fundamental frequency coefficient $\Omega_1 = \sqrt{(\rho h/D_r)}\omega_1 a^2$.

Since

$$\Omega_1 = \Omega_1(p, q), \tag{6}$$

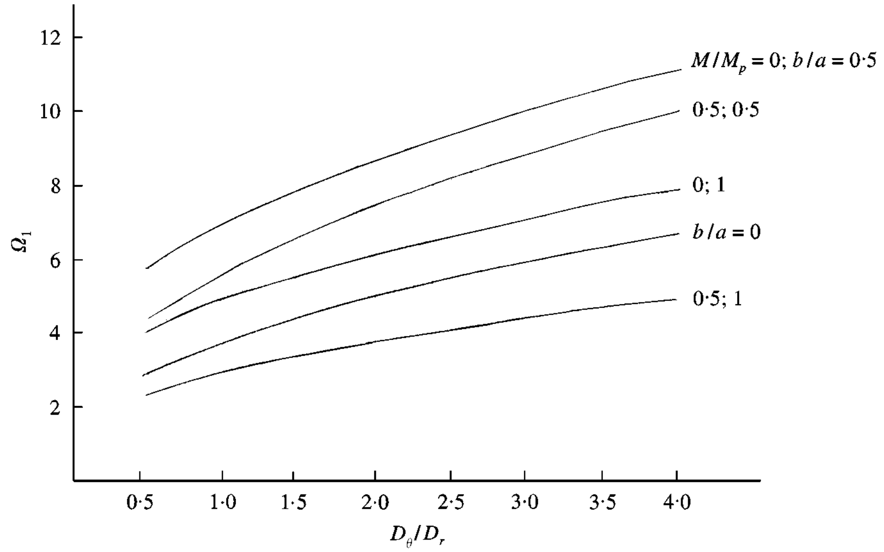

Figure 3. Variation of Ω_1 as a function of D_θ/D_r for different values of M/M_p and b/a ($\nu_\theta = 0{\cdot}3$).

by requiring

$$\frac{\partial \Omega_1}{\partial p} = \frac{\partial \Omega_1}{\partial q} = 0, \tag{7}$$

one obtains the optimized values of Ω_1.

3. NUMERICAL RESULTS

All the calculations have been performed for $\upsilon_\theta = 0{\cdot}3$ and $\frac{1}{3}$ and the frequency coefficients Ω_1 have been determined for the following ranges of the parameters b/a and D_θ/D_r: $b/a = 0$; $0{\cdot}1, 0{\cdot}2, \ldots, 1{\cdot}0$; $D_\theta/D_r = 0{\cdot}5$; $0{\cdot}75$, $1{\cdot}0, \ldots, 2{\cdot}0$.

Tables 1–3 depict values of Ω_1 for $M/M_p = 0$ and $0{\cdot}1$; $0{\cdot}2$ and $0{\cdot}3$ and $0{\cdot}4$ and $0{\cdot}5$ respectively. When $D_\theta/D_r = 1$ (isotropic case), the numerical values of $\sqrt{(\rho h/D)}\omega_1 a^2$ are in excellent agreement with results already available in the literature [3, 5].

Table 4 shows values of maximum values of Ω_1 as a function of the corresponding value of b/a for $\nu_\theta = 0{\cdot}3$ and $\frac{1}{3}$ for all the values of D_θ/D_r and M/M_p previously considered.

It can be easily seen that the value of b/a which yields a maximum value of Ω_1 is, from a practical viewpoint, mainly a function of M/M_p, as can be deduced from Figure 2.

Figure 3 depicts the variation of Ω_1 as a function of D_θ/D_r for $M/M_p = 0$, $b/a = 0{\cdot}5$; $M/M_p = 0{\cdot}5$, $b/a = 0{\cdot}5$; $M/M_p = 0$, $b/a = 0{\cdot}1$ and $M/M_p = 0{\cdot}5$, $b/a = 1$. For $b/a = 0$, the frequency coefficient is independent of M/M_p since the mass is placed directly on the support.

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REFERENCES

1. R. Y. BODINE 1959 *Journal of Applied Mechanics* **26**, 666–668. The fundamental frequencies of a thin circular plate simply supported along a circle of arbitrary radius.
2. S. G. LEKHNITSKII 1968 *Anisotropic Plates*. New York: Gordon and Breach Science Publishers, Inc.
3. A. W. LEISSA 1969 *NASA SP*160. Vibration of plates.
4. P. A. A. LAURA 1995 *Ocean Engineering* **22**, 235–250. Optimization of variational methods.
5. P. A. A. LAURA, R. H. GUTIERREZ, S. A. VERA and D. A. VEGA 1999 *Journal of Sound and Vibration* **223**, 842–845. Transverse vibrations of a circular plate with a free edge and a concentric circular support.