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FUNDAMENTAL FREQUENCY OF TRANSVERSE VIBRATION OF CIRCULAR, ANNULAR PLATES OF POLAR ORTHOTROPY

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1. INTRODUCTION

The determination of dynamic parameters of vibrating circular plates of polar orthotropy† is of interest in many technological applications in view of the always increasing use of composite materials. Several papers have been written on the subject matter in the case of solid plates [1, 2] but the information is considerably more scarce in the case of circular annular plates specially when dealing with free edges where the problem becomes considerably more complicated if one satisfies exactly the natural boundary conditions.

The present study deals with clamped and simply supported plates at the outer edge and free at the inner boundary. The fundamental frequency coefficient is determined by means of an approximate approach whereby polynomial co-ordinate functions which yield excellent accuracy in the case of solid plates are used. The frequency coefficient is then determined by means of the optimized Rayleigh–Ritz method [3]. It is shown that in the case of isotropic annular plates the frequency coefficients possess remarkably good accuracy [4].

2. APPROXIMATE SOLUTION

Following Lekhnitskii's standard notation [5] the problem is governed by the functional

$$J(W) = \iint_{\bar{P}} \left[D_r W''^2 + D_\theta \left(\frac{W'}{\bar{r}} \right)^2 + 2 D_r \nu_\theta \frac{W' W''}{\bar{r}} \right] \bar{r}\, \mathrm{d}\bar{r}\, \mathrm{d}\theta$$

$$- D_r 2\pi a \left[W''(a) + \nu_\theta \frac{W'(a)}{a} \right] W'(a) - \rho h \omega^2 \iint_{\bar{P}} W^2\, \bar{r}\, \mathrm{d}\bar{r}\, \mathrm{d}\theta, \qquad (1)$$

†They are also called plates with cylindrical anisotropy [5].

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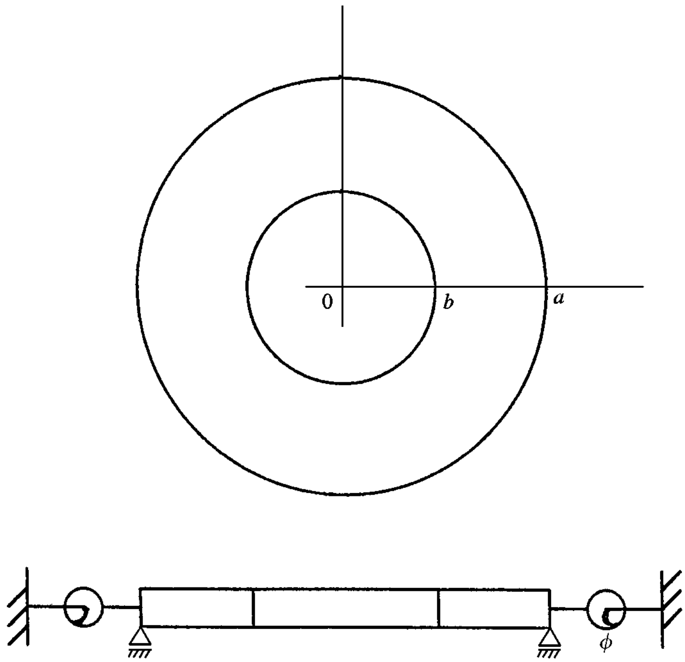

Figure 1. Vibrating structural system under study: ϕ is the edge flexibility coefficient.

subject to the following boundary conditions at $r = a$ (see Figure 1):

$$W(a) = 0, \quad W'(a) = -\phi D_r \left[W''(a) + \nu_\theta \frac{W'(a)}{a} \right]. \tag{2a,b}$$

Equation (2b) is the constitutive relation which defines the flexibility coefficient ϕ. The natural boundary conditions at $\bar{r} = b$ are not taken into account [6]. Introducing the dimensionless variable $r = \bar{r}/a$ and substituting in equations (1) and (2) one obtains

$$\frac{a^2}{2\pi D_r} J(W) = \int_{r_b}^{1} \left[W''^2 + \frac{D_\theta}{D_r} \frac{W'^2}{r^2} + 2\nu_\theta \frac{W' W''}{r} \right] r \, \mathrm{d}r \, \mathrm{d}\theta$$

$$- [W''(1) + \nu_\theta W'(1)] \, W'(1) - \Omega^2 \int_{r_b}^{1} W^2 r \, \mathrm{d}r, \tag{3}$$

$$W(1) = 0, \quad W'(1) = -\phi' [W''(1) + \nu_\theta W'(1)], \tag{4a,b}$$

where $\Omega^2 = (\rho h/D_r) \, a^4 \omega^2$, $r_b = b/a$ and $\phi' = \phi \, D_r/a$.

The following approximation is conveniently used:

$$W \cong W_a = \sum_{j=1}^{N} C_j \varphi_j(r) = \sum_{j=1}^{N} C_j (a_j r^{p+j-1} + b_j r^{j+1} + 1), \tag{5}$$

where the a_j's and b_j's are obtained substituting each co-ordinate function in equations (4). The parameter "p" which appears in equation (5) is Rayleigh's optimization parameter [3].

Substituting equation (5) in equation (3) and making use of the classical Rayleigh–Ritz method one obtains

$$\frac{a^2}{2\pi D_r}\frac{\partial J}{\partial C_i} = \left\{ \sum_{j=1}^{N} \int_{r_b}^{1} \left[\varphi_j'' \varphi_i'' + \frac{D_\theta}{D_r}\frac{\varphi_j' \varphi_i'}{r^2} + v_\theta \frac{\varphi_j'' \varphi_i' + \varphi_j' \varphi_i''}{r} \right] r \, \mathrm{d}r \right.$$

$$- \frac{1}{2}\sum_{j=1}^{N} [\varphi_j'(1)(\varphi_i'(1) + v_\phi \varphi_i'(1)) + (\varphi_j''(1) + v_\theta \varphi_j'(1))\varphi_i'(1)]$$

$$\left. - \Omega^2 \int_{T_b}^{1} \varphi_j \varphi_i r \, \mathrm{d}r \right\} C_j = 0. \qquad (6)$$

The non-triviality condition leads to a frequency determinant whose lowest root constitutes the fundamental frequency coefficient $\Omega_1 = \sqrt{(\rho h/D_r)}\,\omega_1 a^2$. Minimizing Ω_1 with respect to "p" one determines an optimized value of Ω_1.

3. NUMERICAL RESULTS

The numerical determinations have been performed making $v_\theta = 0{\cdot}30$ and taking $N = 5$ in equation (5).

Table 1 shows a comparison of eigenvalues obtained using the present approach and the exact values determined in reference [4] in the case of isotropic, circular annular plates: (1) simply supported and (2) clamped, at the outer edges. The agreement is very good for all the relations of $r_b = b/a$ considered, the maximum difference being less than 1% for $r_b = 0{\cdot}1$.

TABLE 1

Comparison of fundamental eigenvalues $\Omega_1 = \sqrt{(\rho h/D)}\,\omega_1 a^2$ in the case of isotropic circular annular plates ($D = D_r = D_\theta$)

	$r_b = 0$	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
Simply supported external boundary										
(1)	4·935	4·898	4·737	4·668	4·765	5·077	5·711	6·931	9·555	17·709
(2)	4·9351	4·8532	4·7177	4·6640	4·7640	5·0768	5·7107	6·9309	9·5554	17·7087
Clamped external boundary										
(1)	10·216	10·244	10·437	11·428	13·603	17·715	25·674	43·142	93·035	360·350
(2)	10·2158	10·1592	10·4080	11·4237	13·6027	17·7145	25·6742	43·1422	93·0351	360·3503

(1) Present approximate results.
(2) Exact eigenvalues [5].

TABLE 2

Values of $\Omega_1 = \sqrt{(\rho h/D_r)}\ \omega_1 a^2$ in the case of circular annular plates of polar orthotropy simply supported at the outer boundary

D_θ/D_r	$r_b = 0$	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
0·50	4·075	3·835	3·403	3·230	3·244	3·430	3·844	4·658	6·416	11·887
0·75	4·542	4·434	4·159	4·033	4·086	4·338	4·870	5·906	8·139	15·082
1	4·935	4·898	4·737	4·668	4·765	5·077	5·711	6·931	9·555	17·709
1·25	5·281	5·286	5·213	5·201	5·344	5·714	6·439	7·821	10·787	19·993
1·50	5·593	5·625	5·621	5·664	5·855	6·280	7·089	8·618	11·890	22·042

TABLE 3

Values of $\Omega_1 = \sqrt{(\rho h/D_r)}\ \omega_1 a^2$ in the case of circular annular plates of polar orthotropy clamped at the outer boundary

D_θ/D_r	$r_b = 0$	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
0·50	9·324	9·083	9·167	10·333	12·697	16·966	25·052	42·619	92·591	359·969
0·75	9·806	9·732	9·858	10·909	13·164	17·347	25·366	42·882	92·813	360·160
1	10·216	10·244	10·437	11·428	13·603	17·715	25·674	43·142	93·035	360·350
1·25	10·577	10·672	10·936	11·901	14·019	18·070	25·977	43·400	93·256	360·541
1·50	10·904	11·044	11·375	12·334	14·412	18·415	26·275	43·656	93·477	360·731

Tables 2 and 3 depict values of $\Omega_1 = \sqrt{(\rho h/D_r)}\ \omega_1 a^2$ for circular annular plates of polar orthotropy of outer simply supported and clamped edges, respectively, for $D_\theta/D_r = 0{\cdot}50, 0{\cdot}75, \ldots, 1{\cdot}50$.

For $r_b \geqslant 0{\cdot}7$ one observes clearly the "dynamic stiffening" effect [7] in the case of an outer simply supported edge when $D_\theta/D_r = 0{\cdot}5$ while for $D_\theta/D_r = 1{\cdot}5$ the phenomenon is observed for $r_b \geqslant 0{\cdot}1$. On the other hand, when the outer edge is clamped the "dynamic stiffening" effect takes place for $r_b \geqslant 0{\cdot}3$ for $D_\theta/D_r = 0{\cdot}5$ and for $r_b \geqslant 0{\cdot}1$ for $D_\theta/D_r = 1{\cdot}5$. In other words, both tables give an approximate indication of the possibility of increasing the fundamental frequency of the plate by introducing a hole, by simultaneous consideration of the parameter r_b and D_θ/D_r.

The present approach appears as an advantageous one specially in the case of plates of non-uniform thickness. It can be extended in a straightforward fashion to antisymmetric modes by incorporating the azimuthal variable in the co-ordinate functions.

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